Delhi School of Economics MA Economics past year papers (2006-2020)

Econschool

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1 DSE 2006

1. Suppose A and B have to divide a box of chocolates and both of them prefer to have more chocolates to fewer chocolates. An allocation that gives all the chocolates to A is
   A. Pareto inefficient
   B. Pareto efficient
   C. Pareto unfair
   D. Pareto fair

2. Among the effects of urban rent control are
   A. a disincentive to build for rental purposes
   B. an implicit redistribution of wealth from those who do not have rent-controlled housing to those who do have rent-controlled housing
   C. a low degree of mobility in housing market
   D. all of the above

3. Processed and piped water is
   A. a public good
   B. a private good
   C. a private bad
   D. a public bad

4. A situation in which all electricity generators in a state can sell their output only to the State Electricity Board is described as
   A. a monopoly
   B. a monopsony
   C. an oligopolistic market
   D. a monopolistically competitive market

5. Consider a firm that produces single good using labor and capital. Let C, Q, L, K, w, r and p denote cost of production, output, labor input, capital input, the wage rate, the price of capital and the price of output respectively. An example of the cost function for the firm is
   A. $C = wL + rK$
   B. $C = \min\{wL, rK\}$
   C. $C = pQ/[wL + rK]$
   D. $C = Q\sqrt{wr}$

6. If the IS curve is downward sloping and the LM curve is vertical, a unit increase in government expenditure results in

A. crowding in and higher increase in equilibrium income

B. no crowding out and equivalent increase in the equilibrium income

C. partial crowding out and lower increase in equilibrium income

D. complete crowding out and no increase in equilibrium income

7. Monetary policy is completely ineffective in raising output if

A. the IS curve is horizontal and the LM curve is upward sloping

B. the IS curve is vertical and the LM curve is upward sloping

C. the IS curve is downward sloping and the LM curve is upward sloping

D. the IS curve is downward sloping and the LM curve is vertical

8. The price and quantity combinations of two services (trim and perm) sold at the "Be Handsome" beauty parlour are as follows (P_t and Q_t denote the price and quantity respectively of a given service at time t):

Service	P_0	P_1	Q_0	Q_1
Trim	50	60	10	15
Perm	150	170	8	12

The Laspeyres price index for time $t = 1$ is

A. 117.2

B. 115.3

C. 121.8

D. 119.6

9. Suppose we flip a fair coin three times. The probability of getting two heads is

A. 2/8

B. 1/3

C. 2/3

D. 3/8

10. Which one of the following is true?

A. A consumer never buys an inferior good.

B. A consumer's budget share for an inferior good is necessarily less than half.

C. A consumer's budget share for an inferior good is constant.

D. A consumer's budget share for an inferior good decreases with an increase in money income.

11. Consider a consumer with the utility function $u(x, y) = x^{1/2} + y$. Which one of the following is true?

A. Income elasticity of both goods x and y is 1.

B. Income elasticity of good x is 0.

C. Income elasticity of good y is 0.

D. Income elasticity of good x is 1 and of good y is 0.

12. Consider the experiment of tossing two unbiased coins in succession. What is the probability of obtaining two heads, given that at least one of the coins comes up heads?

A. 1/2

B. 1/4

C. 1/3

D. 2/3

13. Suppose all input prices as well as output price double. The level of output produced by a profit-maximizing firm which uses a decreasing returns to scale technology will

A. be double of its original level

B. increase, but not necessarily double

C. remain constant

D. change, but we cannot say in which direction

14. Consider an $n \times n$ matrix A with real entries. A is non-singular if and only if

A. the determinant of A is not equal to 0

B. the columns vectors of A are linearly independent

C. the row vectors of A are linearly independent

D. any of the above three conditions is satisfied

15. Consider a function $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. If f is increasing, i.e., $x \geq y$ implies $f(x) \geq f(y)$, then

A. f is concave

B. f is convex

C. f is quasi-convex

D. f is continuous

16. Consider a singular $n \times n$ matrix A with real entries. Interchanging the positions of a pair of adjacent columns of A

A. does not change the value of the determinant of A

B. changes the sign of the determinant

C. increases the value of the determinant

D. decreases the value of the determinant

17. There are three women on the platform of a train station. The train that they are waiting for has 5 coaches and each of them is equally likely to enter any coach. What is the probability that they will all enter the same coach?

    A. 12/25

    B. 3/5

    C. 3/125

    D. 9/25

    E. 1/25

18. Suppose a neighborhood has 90 Hindus and 10 Muslims. What is the probability that two randomly selected persons from that neighborhood will have the same religion?

    A. 0.5

    B. 0.81

    C. 0.9

    D. 0.82

19. Exchange rate overshooting occurs

    A. under fixed exchange rates when the central bank mistakenly buys or sells too much foreign exchange

    B. under fixed exchange rates as a necessary part of the adjustment process for any monetary shock

    C. under flexible exchange rates when the exchange rate rises (depreciates) above and then falls down to equilibrium after a monetary expansion

    D. under flexible exchange rates, so that large financial shocks in the domestic economy have very little impact on exchange rates

20. In an open economy with a system of fixed exchange rates

    A. monetary policy is an effective tool for stabilizing the economy

    B. fiscal policy is a by-product of exchange rate policy

    C. fiscal policy is an effective tool for stabilizing the economy

    D. both (a) and (b) above

    Answer 21, 22 and 23 for the following situation. Consider a competitive exchange economy with two agents (1 and 2) and two goods (X and Y). Agent 1's endowment of (X, Y) is $(0, 5)$ and Agent 2's endowment of (X, Y) is $(10, 0)$. An allocation for Agent i is denoted (x_i, y_i), where x_i is his allocation of X and y_i is his allocation of Y. Agent i's objective is to choose (x_i, y_i) to maximise his utility $\min\{x_i, y_i\}$

21. The allocation with $(x_1, y_1) = (3, 3)$ and $(x_2, y_2) = (7, 2)$ is

    A. a competitive equilibrium allocation and is Pareto efficient

B. not a competitive equilibrium allocation but is Pareto efficient

C. neither a competitive equilibrium allocation nor is Pareto efficient

D. a competitive equilibrium allocation but is not Pareto efficient

22. The allocation with $(x_1, y_1) = (10, 5)$ and $(x_2, y_2) = (0, 0)$ is

A. a competitive equilibrium allocation and is Pareto efficient

B. not a competitive equilibrium allocation but is Pareto efficient

C. neither a competitive equilibrium allocation nor is Pareto efficient

D. a competitive equilibrium allocation but is not Pareto efficient

23. Assuming the sum of prices is 1, the competitive equilibrium prices (p_1, p_2) are

A. $(1, 0)$

B. $(0, 1)$

C. $(1/2, 1/2)$

D. $(1/3, 2/3)$

24. Consider an economy which is demand constrained. The components of aggregate demand are: (i) Household Consumption; (ii) Government Investment Expenditure; and (iii) Private Investment. The government finances its expenditure by imposing a lump-sum tax on the households and always maintains a balanced budget. Households consume 1/2 of their disposable income. Private investment and government investment are complementary and the two are related in the following way: a unit investment by the government induces private investors to invest 2 units. Starting from an equilibrium situation, the government raises its investment expenditure by 10 units, what will be the corresponding increase in the equilibrium level of income for this economy?

A. 20 units

B. 10 units

C. 50 units

D. income remains the same

25. Consider an economy where 1/2 of the total income (output) is distributed to the workers and the other half is distributed to the capitalists. The capitalists save upto 50% of their income and consume the rest; the workers save 25% of their income and consume the rest. The aggregate demand consists of total consumption demand and total investment demand. Investment demand is autonomously given at 100 units. Output is demand determined. How does the equilibrium income value change if 3/4 of the total income is distributed to the workers and 1/4 to the capitalists?

A. equilibrium output increases by 160/3 units

B. equilibrium output increases by 100/3 units

C. equilibrium output increases by 800/3 units

D. equilibrium output does not change

26. The aggregate production function in an economy at any time period t is given by $Y_t = \min\{K_t/2, L_t/4\}$, where K_t and L_t are respectively the aggregate stock of capital and the available stock of labour at time t. In each period, 20% of the total output is saved and invested, which augments the next period's capital stock. Capital does not depreciate. Available labour stock grows by 4 units in every period. The economy currently has 200 units of capital and 420 units of labour. What is the current level of employment and what will be the level of employment tomorrow?

    A. Current Employment: 420; Employment tomorrow: 424

    B. Current Employment: 400; Employment tomorrow: 424

    C. Current Employment: 400; Employment tomorrow: 400

    D. Current Employment: 420; Employment tomorrow: 420

27. Members of the Gymkhana Club are charged quarterly fees on the basis of their average weight, rounded to the nearest kg. Of the 560 members, 120 weighed between 60 and 69 kg, 140 weighed between 70 and 79 kg, 170 weighed between 80 and 89 kg, and the remaining weighed between 90 and 99 kg. If members are charged Rs. 50 per kilo of their weight, on average how much must each member pay?

    A. Rs. 3800

    B. Rs. 3900

    C. Rs. 4000

    D. Rs. 4100

28. ICICI Bank collects data on 10000 respondents. Out of the 6800 men, 4200 have credit cards, and out of the 3200 women, 2500 have credit cards. Out of the men with credit cards, 1200 have unpaid balances, whereas out of the women with credit cards, 1400 have unpaid balances. What is the probability that an individual selected at random is a man without an unpaid balance?

    A. 0.68

    B. 0.56

    C. 0.12

    D. 0.84

29. A monopolist has two plants. In plant 1, the total cost function is $c_1(q_1) = 2q_1$ and in plant 2, the total cost function is $c_2(q_2) = q_2^2/2$, where q represents the quantity of good produced. The demand faced by the monopolist is $p = 10 - q$, where $q = q_1 + q_2$. How does the monopolist allocate its total production to serve the market?

    A. $q_1 = 6$ and $q_2 = 2$

    B. $q_1 = 2$ and $q_2 = 2$

    C. $q_1 = 4$ and $q_2 = 4$

D. $q_1 = 4$ and $q_2 = 10/3$

30. There are two individuals A and B. The utility function of both individuals are identical and given by $u(x, y) = \max\{x, y\}$. Each of them has 1 unit of good x and 1 unit of good y. Which of the following is a Pareto optimal allocation?

    A. A has $x = 1, y = 1$ and B has $x = 1, y = 1$

    B. A has $x = 2, y = 0$ and B has $x = 0, y = 2$

    C. A has $x = 0, y = 0$ and B has $x = 2, y = 2$

    D. A has $x = 3/2, y = 1/2$ and B has $x = 1/2, y = 3/2$

    Answer 31, 32 and 33 for the following information. Three players A, B and C take turns playing a game as follows. A and B play in the first round. The winner plays C in the second round, while the loser sits out. The winner of the second round plays the person who was sitting out. The game continues in this fashion, with the winner of the current round playing the next round with the person who sits out in the current round. The game ends when a player wins twice in succession; this player is declared the winner of the contest. For any of the rounds, assume that the two players playing the round each have a probability $1/2$ of winning the round, regardless of how the past rounds were won or lost.

31. The probability that A becomes the winner of the contest is

    A. $5/14$

    B. $1/2$

    C. $3/7$

    D. $7/16$

32. The probability that C becomes the winner of the contest is

    A. $1/7$

    B. $1/5$

    C. $1/8$

    D. $2/7$

33. The probability that the game continues indefinitely, with no one winning twice in succession, is

    A. $1/10^{23}$

    B. 0

    C. $1/2^{23}$

    D. $1/2^{16}$

34. Consider collecting a random sample that has two observations, from a population that is normally distributed with mean μ and variance 16. The variance associated with the distribution of twice the difference between these two observations equals

A. 64

B. 128

C. $32/\sqrt{2}$

D. $64/\sqrt{2}$

35. In an open economy with a system of flexible exchange rates and perfect capital mobility, an expansionary monetary policy

A. causes the domestic currency to appreciate

B. has a greater impact on income than in a closed economy

C. increases capital inflows into the country

D. induces a balance of payments deficit

36. Which of the following would make the LM curve flatter in the (Y, r) space?

A. an increase in income sensitivity of money demand

B. an increase in interest sensitivity of planned investment

C. an increase in the marginal propensity to consume

D. an increase in the interest sensitivity of money demand

37. Consider the graph of the function $f(x) = (x^2 + 2x - 1)/x$ defined over all positive real values of x. This graph has the following asymptotes

A. The line defined by $2x + 2$ and the vertical axis

B. One asymptote: the line defined by $y = x + 2$

C. The line defined by $y = x + 2$ and the vertical axis

D. The line defined by $y = 2x + 1$ and the vertical axis

38. Which of the following statements is correct? In a closed economy, fiscal policy is more effective

A. the smaller the induced change in interest rates and smaller the responsiveness of investment to these changes

B. the larger the induced change in interest rates and smaller the responsiveness of investment to these changes

C. the smaller the induced change in interest rates and larger the responsiveness of investment to these changes

D. the larger the induced change in interest rates and larger the responsiveness of investment to these changes

39. Along the long run supply curve of a perfectly competitive industry, all of the following may change, except

A. the prices of inputs used

B. the amounts of inputs used

C. the number of firms in the industry

D. the level of profits

Answer 40 and 41 for the following situation. Consider a market served by a pair of firms, 1 and 2. The inverse market demand is given by $p = 1 - (x_1 + x_2)$, where x_i is the output of Firm i. Let Firm 1's cost function be $C_1(x_1) = x_1/2$ and Firm 2's cost function be $C_2(x_2) = x_2/3$

40. If Firms 1 and 2 are Cournot duopolists, then Cournot equilibrium outputs are

A. $(x_1, x_2) = (4/18, 4/18)$

B. $(x_1, x_2) = (2/18, 5/18)$

C. $(x_1, x_2) = (5/18, 3/18)$

D. $(x_1, x_2) = (0, 1/3)$

41. Now suppose the situation changes so that Firm 1 chooses x_1 first and Firm 2 chooses x_2 after observing Firm 1's choice. Relative to Cournot situation, in this new situation,

A. x_1 and x_2 decrease

B. x_1 decreases and x_2 increases

C. x_1 increases and x_2 decreases

D. x_1 and x_2 increase

42. Suppose Asha's preferences between two commodities x_1 and x_2 can be represented by $u(x_1, x_2) = \min\{x_1 - 5, x_2 + 3\}$. Given an income of Rs. 73, and facing prices of Rs. 3 for x_1 and Rs. 4 for x_2, Asha's optimal consumption bundle of (x_1, x_2) will be

A. $(12.5, 4.5)$

B. $(10.42, 10.42)$

C. $(15, 7)$

D. $(3, 16)$

43. Consider the following macroeconomic model:

$$\begin{aligned} C &= 2000 + 0.4Y_D \\ I &= 500 - 10r + 0.4Y \\ G &= 400 \\ T &= 1000 \\ (M/P)^D &= 0.2Y - 50r \\ (M/P)^S &= 1000 \end{aligned}$$

where Y_D is disposable income. The equation of the IS curve for this model is

A. $Y = 9500 - 50r$

B. $Y = 12500 - 50r$

C. $Y = 14500 - 100r$

D. $Y = 14500 - 60r$

44. A negative supply shock (e.g., oil price increase) shifts the Phillips curve and _____ the natural rate of unemployment. If the government wants to keep the economy at the original rate of unemployment, it must have _____ inflation.

A. lowers, ever increasing

B. raises, ever decreasing

C. raises, ever increasing

D. does not change, ever increasing

45. An increase in foreign income _____ the equilibrium output of a small open economy with uncovered interest parity and flexible exchange rates.

A. increases

B. decreases

C. leaves unchanged

D. first increases then decreases

46. Amit has a box containing 6 red balls and 3 yellow balls. Amita has a box containing 4 red balls and 5 yellow balls. Amit randomly draws one ball from his box and puts it into Amita's box. Now Amita randomly draws one ball out of her box. What is the probability that balls drawn by Amit and Amita were of different colours?

A. 1/3

B. 2/15

C. 4/15

D. 7/15

47. Two patients share a hospital room for two days. Suppose that, on any given day, a person independently picks up an airborne infection with probability 1/4. An individual who is infected on the first day will certainly pass it to the other patient on the second day. Once contracted, the infection stays for at least two days. What is the probability that both patients have contracted the infection by the end of the second day?

A. 125/256

B. 121/256

C. 135/256

D. 131/256

48. A blood test detects a given disease with probability 8/10 given that the tested person actually has the disease. With probability 2/10, the test incorrectly shows the presence of the disease in a disease-free person. Suppose 1/10 of the population has the disease. What is the probability that the person tested actually has the disease if the test indicates the presence of the disease?

    A. 1

    B. 9/13

    C. 4/13

    D. 7/13

49. Your teacher knows 6 jokes and in each class tells 2 jokes; each joke has an equal chance of being selected. What is the probability that, in a given lecture, at least 1 joke is told which was not told in the previous class?

    A. 28/30

    B. 14/30

    C. 16/30

    D. 12/30

50. Suppose there are only two goods (X and Y), two countries (A and B), and labour is the only factor of production. The amount of labour required to produce a unit of the i-th good in the j-th country is a constant l_{ij}. Suppose $l_{XA}/l_{YA} < l_{XB}/l_{YB}$. If these two countries trade with each other, which of the following outcomes would definitely not occur in equilibrium?

    A. A produces only X and B produces only Y

    B. A produces both goods and B produces only Y

    C. A produces only X and B produces both goods

    D. Both countries produce both goods

51. Suppose that in a particular country, the (inverse) demand curve for a certain good is given by $P = a - Q$, where P is price, Q is quantity demanded and a is a positive constant. The market supply curve of a competitive domestic industry is given by $P = bQ$, where b is a positive constant. The country can import any amount of the same good at an exogenously given world price of P^*. If, in order to raise revenue, the government imposes a tariff of t per unit on imports, it will result in a deadweight welfare loss of

    A. $t(a - P^*)$

    B. $a - b(P^* + t)$

    C. t

    D. t^2

    E. $t^2(1 + b)/2b$

52. Consider a function $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. If f is strictly increasing (i.e., $x > y$ implies $f(x) > f(y)$) and differentiable, then the derivative of f

    A. may be less than 0 at some $x \in \mathbb{R}$

    B. may be infinite at some $x \in \mathbb{R}$

    C. is greater than or equal to 0 at every $x \in \mathbb{R}$

    D. is greater than 0 at every $x \in \mathbb{R}$

53. Consider the following claim: "There is some general election and some party such that all the candidates of that party in that election are honest." If this claim is false, then which of the following statements must be true?

    A. In every election, there exists a party such that all its candidates are dishonest.

    B. There is some general election and some party such that all the candidates of that party in that election are honest.

    C. In every election, all candidates of all parties are dishonest.

    D. In every election, every party has at least one dishonest candidate.

Answer 54, 55, 56 and 57 using the following information. Consider a society consists of individuals who may belong to various sets called Family and/or Gangs. The collections of Families and Gangs satisfy the following rules:

- The entire society is a Family.
- The empty subset of a Society is also a family.
- Given a collection of Families, the set of individuals who belong to every Family in that collection is also a Family.
- Given any two Families, the set of individuals who belong to either of the two Families is also a Family.
- A set of individuals is called a Gang if and only if the set of individuals not in it constitute a Family.

54. The intersection of two Gangs is necessarily

    A. a Family

    B. a Gang

    C. not a Family

    D. not a Gang

55. The union of a collection of Gangs is necessarily

    A. not a Family

    B. not a Gang

    C. a Family

D. a Gang

56. Which of the following statements is necessarily true?

A. A set of individuals cannot be a Gang and a Family.

B. There are at least two sets of individuals that are both a Family and a Gang.

C. The union of a Family and a Gang is a Gang.

D. The intersection of a Family and a Gang is a Family.

57. Suppose we are given a Family and a Gang. Then, the set of individuals who belong to the given Family but not to the given Gang necessarily constitute

A. a Family

B. a Gang

C. neither a Family, nor a Gang

D. a Family and a Gang

58. You have 10 pockets. What is the smallest number of one rupee coins you need to have a different number of coins in each pocket?

A. 45

B. 35

C. 60

D. 50

59. There are 3 red and 5 black balls in an urn. You draw two balls in succession without replacing the first ball. What is the probability that the second ball you draw is red?

A. 2/7

B. 3/8

C. 5/7

D. 1/4

60. Given that $\mathbb{R}$ denotes the set of real numbers, which of the following mappings is a one-to-one (i.e., injective) function?

A. $f(x) = \tan x$, where $x \in \mathbb{R}$ and $x \geq 0$

B. $f(x) = |x|$, where $x \in \mathbb{R}$

C. $f(x) = 1/x$, where $x \in \mathbb{R}$ and $x \geq 0$

D. $f(x) = |x|$, where $x \in \mathbb{R}$ and $x \geq 0$

2 DSE 2007

1. Consider a consumer having a utility function $U = 2x + y$ where x and y are two commodities. Suppose that the price of x and y are both Rs. 5 each and the consumer's income is Rs. 100. In equilibrium, the consumer will consume the bundle

   A. $(10, 10)$

   B. $(0, 20)$

   C. $(20, 0)$

   D. None of the above

2. With quantity of x on the horizontal axis, the Engel curve for commodity x corresponding to the utility function $U = y + f(x)$ will be

   A. a ray through the origin

   B. a vertical line

   C. a horizontal line

   D. a curve with an inflexion point

3. Suppose a producer has a fixed coefficient technology $Q = \min\{2K, 3L\}$, where Q is output and K and L are the inputs. At equilibrium

   A. $K = L$

   B. $K = (3/2)L$

   C. $K = (2/3)L$

   D. K and L are indeterminate

4. A monopolist will never produce a quantity where the price elasticity of demand is inelastic because:

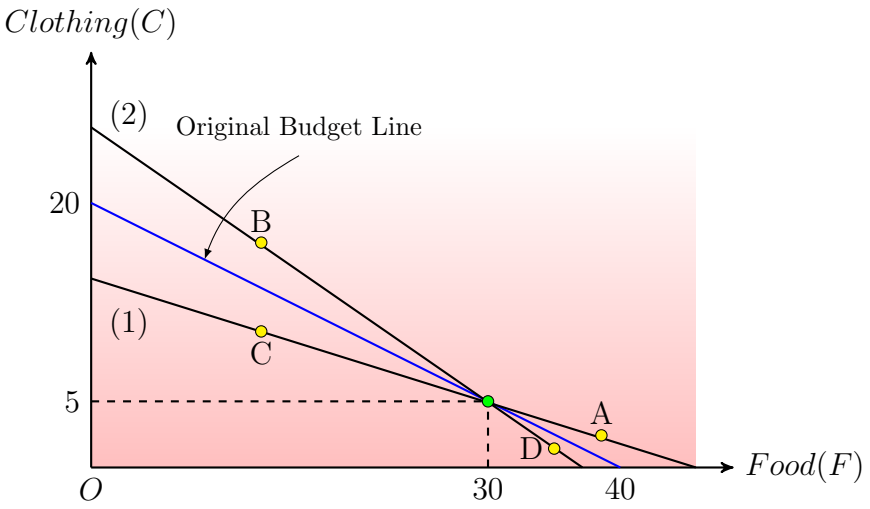
   A. $P >$ MC in such a case

   B. MR $>$ MC in such a case

   C. MR < 0 in such a case

   D. $P <$ ATC in such a case

5. In the figure below when the budget lie (drawn in blue) was originally $5F + 10C = 200$, Anita purchased 30 units of F and 5 units of C. At two later dates, lines (1) and (2) were the budget lines. Anita had the same preferences in all three instances. As a rational consumer, which market baskets might she have purchased at the later dates?



A. A, B

B. A, D

C. B, C

D. C, D

6. NAIRU implies

   A. the unemployment rate is zero

   B. inflation is contant

   C. the rate of growth of real GDP is constant

   D. none of the above

7. In an open economy with perfect capital mobility and fixed exchange rates, an open market operation will

   A. increase output and reduce interest rate

   B. increase the money supply

   C. lower the trade deficit

   D. change the composition of the monetary base

8. If a person is a borrower, a rise in the interest rate will

   A. always reduce borrowing

   B. always increase borrowing

   C. could increase borrowing depending on the relative strengths of income and substitution effects

   D. none of the above

9. The J-curve suggests that the effect of an appreciation of the exchange rate on the trade balance is to:

A. improve it in the short run

B. worsen it in the short run

C. leave it unchanged

D. improve it in the long run

10. Under classical supply conditions, a fiscal expansion

A. only raises the price level and the interest rate

B. expands supply at constant prices

C. expands supply at rising prices

D. leaves both supply and price level unchanged

11. Consider the following functions $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. Which of the following functions is quasi-convex?

A. $f(x) = x^2$

B. $f(x) = \cos x$

C. $f(x) = e^{-x}$

D. $f(x) = x^{-1}$ if $x \neq 0$; $f(x) = 0$ if $x = 0$

12. Consider an $n \times n$ matrix A with real entries. If matrix B is derived by adding the first column of A to the last column of A, then

A. det $A <$ det B

B. det $A >$ det B

C. det $A =$ det B

D. the sign of det A is the opposite of the sign of det B

13. Consider a strictly decreasing (i.e., $x > y$ implies $f(x) < f(y)$) and differentiable function $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. Denote the derivative of f at $x \in \mathbb{R}$ by $Df(x)$. Which of the following may be true about f?

A. $Df(x) > 0$ for some $x \in \mathbb{R}$

B. $Df(x) = 0$ for some $x \in \mathbb{R}$

C. $Df(x) = 0$ for every $x \in (a, b)$ where $a < b$

D. $Df(x) \geq 0$ for every $x \in \mathbb{R}$

14. If the statement "There exists a legislature and a party such that every legislator in that party pays taxes" is false, then which of the following statements must be true?

A. In every legislature and every party, all legislators do not pay taxes

B. There exists a legislature such that every legislator in every party does not pay taxes

C. In every legislature, there exists a party and a legislator in that party who does not pay taxes

D. In every legislature and every party, there exists a legislator who does not pay taxes

Hint: Negation of 'there exists' = 'every', Negation of 'every' = 'there exists'

15. If $f : \mathbb{R}^n \to \mathbb{R}$ is twice differentiable, concave and homogeneous of degree 1, then the Hessian matrix of f is

A. negative definite

B. positive definite

C. singular

D. non-singular

Example : $f(x, y) = \sqrt{xy}$

16. Consider an experiment in which five fibres having different lengths are subjecting to a testing process to learn which fibre will break first. The lengths of the five fibres are 1 inch, 2 inches, 3 inches, 4 inches and 5 inches. Suppose the probability that any fibre will be the first to break is proportional to the length of that fibre. The probability that the length of the fibre that breaks first is not more than 3 inches is

A. 3/5

B. 2/5

C. 1/3

D. 2/3

17. Which of the following statements is WRONG: A probability density function $f(x)$

A. can be used to model a continuous variable

B. integrates to 1 over the range of x

C. increases steadily from 0 to 1

D. must be greater than or equal to 0 over the range of x

18. A primary school has 6 classes, numbered from 1 to 6. Class 1 has twice as many students as each of the other classes. What is the probability that a randomly selected student is from an odd-numbered class?

A. 10/21

B. 3/7

C. 8/21

D. 4/7

19. A traffic light on the way to the University is red 40% of the time. What's the probability of getting a red light (i) 2 days in a row; (ii) any 2 out of the 3 days.

A. (i) 0.16; (ii) 0.20

B. (i) 0.16; (ii) 0.29

C. (i) 0.24; (ii) 0.29

D. none of the above

20. Let Y denote the number of heads obtained when 3 coins are tossed. The variance of Y^2 is

A. 6.5

B. 7.0

C. 7.5

D. 8.0

21. A firm has the production function $Q = 12L - L^2$, where L is labour input and Q is output. If the firm is a monopolist with a demand curve $P(Q) = 100 - Q$, what is the MRP_L (Marginal Revenue Product of Labour) curve?

A. $1200 - 288L + 72L^2 - 4L^3$

B. $1200 - 288L - 72L^2 + 4L^3$

C. $1200 - 488L + 72L^2 - 4L^3$

D. $1200 - 488L - 72L^2 + 4L^3$

22. Sudhir lives in Gujarat. His total wealth next year, including his house, will be Rs. 5,00,000. There is a 10% chance that a big earthquake will occur next year and completely destroy his house, valued at Rs. 2,00,000.

(i) What is Sudhir's expected wealth next year if he chooses not to buy house insurance?

(ii) Suppose Sudhir's utility function is given by $U(W) = W^{0.5}$, where W represents total wealth in thousands of rupees. Is Sudhir risk-averse, risk-loving, or risk-neutral?

A. (i) Rs. 4,50,000 and (ii) risk-averse

B. (i) Rs. 4,50,000 and (ii) risk-neutral

C. (i) Rs. 4,80,000 and (ii) risk-loving

D. (i) Rs. 4,80,000 and (ii) risk-averse

23. Consider an airport that produces noise (N) that declines as the distance (d) in kilometres, from the airport increases: $N(d) = 1/(d^2)$. Praful works at the airport. Praful's damage from noise is Rupee 1 per unit of noise and is associated with where Praful lives. His costs of commuting are Rupee 1 per kilometre (each way). The closest he can live to the airport is $d = 0.1$ km.

(i) What is the distance Praful will live from the airport in the absence of compensation for the noise?

(ii) Suppose Praful is compensated for his damage, wherever he may live. How much will he be compensated?

A. (i) 1 km and (ii) Rs. 100

B. (i) 0.1 km and (ii) Rs. 100

C. (i) 1 km and (ii) Rs. 1

D. (i) 0.1 km and (ii) Rs. 100

24. Suppose the car market is perfectly competitive and each firm has the same cost structure. There are 1000 firms in the industry. The market demand and cost schedules are given below.

Market Demand

Price (lacs per car)	3.65	5.20	6.80	8.40	10.00	11.60	13.20
Quantity (thousands per year)	500	450	400	350	300	250	200

Costs per firm (Rs. lacs)

Output	200	250	300	350	400	450	500
MC	6.40	7.00	7.65	8.40	10.00	12.40	12.70
AVC	7.80	7.00	7.10	7.20	7.50	8.00	9.00
ATC	12.80	11.00	10.43	10.06	10.00	10.22	11.00

In the long run

A. Some firms will exit the industry, but others will remain

B. Some firms will enter the industry

C. The number of firms will remain the same

D. The industry will close down

25. Abhik's and Brinda's demand curves for apples are given by

$$p = 20 - q \quad \text{(Abhik)}$$
$$p = 5 - (q/2) \quad \text{(Brinda)}$$

where $p, q \geq 0$ for each.
Suppose these are the only two consumers in the market, and the market supply function is given by: $p = 2 + Q$. Then the equilibrium quantity in the market is:

A. 12 apples

B. 9 apples

C. 6 apples

D. 3 apples

26. A farmer's demand for wheat is given by the function:

$$q = 25 + m/(20p)$$

where m is his money income and p is the price of the wheat. Let the farmer's money income be Rs. 1000. Suppose that initially, the farmer purchases wheat in the market for Rs. 10 per kg. Now, consider a situation where the farmer obtains a BPL card which allows him to purchase unto 35 kg of wheat for Rs. 5 per kg. What will be the increase in his demand for wheat attributable to the Slutsky substitution effect?

A. 3.5 kg

B. 4.0 kg

C. 4.5 kg

D. 5.0 kg

Answer 27, 28, 29 and 30 using the following information. An economy has two agents (1 and 2) and 3 units of each of two goods, X and Y. An *allocation* in this economy is defined as a tuple $((x_0, y_0), (x_1, y_1), (x_2, y_2))$ of non-negative real numbers, where (x_1, y_1) and (x_2, y_2) are the amounts of goods X and Y consumed by Agents 1 and 2 respectively, and (x_0, y_0) are the amounts of the two goods left over after the consumption of the two agents (i.e., $x_0 = 3 - x_1 - x_2$, $y_0 = 3 - y_1 - y_2$). The agents' ability to consume is limited, so that for each Agent $i = 1, 2$, (x_i, y_i) must satisfy $x_i^2 + y_i^2 \leq 12$. Agent 1 is higher on the social ladder and gets to choose his consumption bundle (x_1, y_1) first. Agent 2 chooses her consumption bundle from what remains of the two goods after Agent 1 has chosen. An *equilibrium* for this economy is an allocation such that

(1) (x_1, y_1) maximises Agent 1's utility subject to the constraints: $x_1 \leq 3$, $y_1 \leq 3$, $x_1^2 + y_1^2 \leq 12$, and

(2) (x_2, y_2) maximises Agent 2's utility subject to the constraints: $x_2 \leq 3 - x_1$, $y_2 \leq 3 - y_1$, $x_2^2 + y_2^2 \leq 12$, and

It is given that Agent 1's utility function $u_1(x_1, y_1) = x_1^2 y_1$ and Agent 2's utility function $u_2(x_2, y_2) = ax_2 + y_2$, where '$a$' is a given positive number.

27. Which one of the allocations $((x_0, y_0), (x_1, y_1), (x_2, y_2))$ specified below is an equilibrium for this economy?

A. $((0, 0), (3, \sqrt{3}), (0, 3 - \sqrt{3}))$

B. $((0, 0), (\sqrt{6}, \sqrt{6}), (3 - \sqrt{6}, 3 - \sqrt{6}))$

C. $((0, 0), (2.5, \sqrt{5.75}), (0.5, 3 - \sqrt{5.75}))$

D. $((0, 0), (\sqrt{8}, 2), (3 - \sqrt{8}, 1))$

28. Let $a = 1$, for Agent 2's utility function. Then, the equilibrium allocation in the above question is

A. Pareto efficient, because the slopes of the Agents' indifference curves are tangent at that allocation

B. Not Pareto efficient, because the slopes of the Agents' indifference curves are not tangent at that allocation

C. Pareto efficient, because there is a bound on Agent 1's ability to consume

D. Not Pareto efficient, as Agent 2 can be made better off without making Agent 1 worse off

29. Suppose there was no bound on the ability of any agent to consume, and you could choose the value of the parameter 'a' in Agent 2's utility function. For what value of 'a' would the equilibrium in Q 27 be efficient?

A. 1

B. $\sqrt{2}$

C. $\sqrt{8}$

D. 2

30. Now suppose that Agent 2 is higher on the social ladder than Agent 1, so Agent 2 gets to choose her consumption bundle before Agent 1 does. All other features of the economy are unchanged. If $a = 2$, which one of the allocations$((x_0, y_0), (x_1, y_1), (x_2, y_2))$ below is an equilibrium for this modified economy?

A. $((0,0), (3, \sqrt{3}), (0, 3 - \sqrt{3}))$

B. $((0,0), (3 - \sqrt{48/5}, 3 - \sqrt{12/5}), (\sqrt{48/5}, \sqrt{12/5}))$

C. $((0,0), (0, 3 - \sqrt{3}), (3, \sqrt{3}))$

D. $((0,0), (2, 3 - \sqrt{8}), (1, \sqrt{8}))$

The following information applies to the questions 31, 32, 33 and 34. Consider an economy where the nominal wage rate (W) is set by the workers and it depends on the expected price level (P^e) and the rate of unemployment (u) in the following way: $W = P^e(1 - \alpha u)$, where α is a parameter; $0 < \alpha < 1$. The actual price level (P) on the other hand is set by the producers who charge a mark up over the nominal wage rate so that: $P = (1 + \lambda)W$, where $\lambda > 0$ is another parameter.

31. If the workers are always able to correctly guess the price level then the rate of unemployment will always be maintained at a level which is given by:

A. $\frac{1}{1+\lambda}$

B. $\frac{\alpha}{1+\lambda}$

C. $\frac{1}{(1+\lambda)\alpha}$

D. $\frac{1+\alpha}{1+\lambda}$

E. $\frac{\lambda}{\alpha(1+\lambda)}$

32. There will always be full employment of the aggregate stock of labour force if and only if the expected and the actual price level are related in the following way:

A. $\frac{P}{P^e} = 1$

B. $\frac{P}{P^e} = 1 + \lambda$

C. $\frac{P}{P^e} = \frac{1+\lambda}{1-\alpha}$

D. $\frac{P}{P^e} = (1+\lambda)(1-\alpha)$

33. An increase in output ceteris paribus leads to

A. an increase in the actual price level

B. a decrease in the actual price level

C. the actual price level will remain unchanged

D. given information is not sufficient to conclude anything about the change in the actual price level

34. An increase in the aggregate stock of labour force ceteris paribus leads to

A. an increase in the actual price level

B. a decrease in the actual price level

C. the actual price level will remain unchanged

D. given information is not sufficient to conclude anything about the change in the actual price level

The following information applies to the questions 35 and 36. Consider the following macroeconomic model. The goods market is characterised by the following set of equations:

$$\begin{aligned} C &= \alpha_0 + \alpha_1 Y \\ I &= \beta_0 + \beta_1 Y - \beta_2 i \\ G &= \overline{G} \text{ (exogenously given)} \end{aligned}$$

where α_0, α_0, α_1, α_2, β_0, β_1, β_2 are all positive constants and $\alpha_1 + \beta_1 < 1$.
The money market equilibrium condition is given by:

$$\frac{\overline{M}}{P} = \gamma_1 Y - \gamma_2 i; \quad \gamma_1, \gamma_2 \text{ positive constants.}$$

Answer the following two questions on the basis of this model:

35. If interest rate (i) increases by one unit , to maintain equilibrium in the goods market, income level (Y) has to decrease by

A. $\frac{\beta_2}{1-(\alpha_1+\beta_1)}$units

B. $\frac{1}{1-(\alpha_1+\beta_1)}$units

C. $\frac{1}{1-(\alpha_1+\beta_1)+\frac{\gamma_1}{\gamma_2}\beta_2}$units

D. $\frac{1}{1-(\alpha_1+\beta_1)+\frac{\gamma_1}{\gamma_2}\alpha_2}$units

36. A unit increase in government expenditure (G) will increase the equilibrium income level (Y) by

A. $\frac{\beta_2}{1-(\alpha_1+\beta_1)}$units

B. $\frac{1}{1-(\alpha_1+\beta_1)}$units

C. $\frac{1}{1-(\alpha_1+\beta_1)+\frac{\gamma_1}{\gamma_2}\beta_2}$units

D. $\frac{1}{1-(\alpha_1+\beta_1)+\frac{\gamma_1}{\gamma_2}\alpha_2}$units

The following information applies to the next two questions. Savitri has got a job in a houschold for 2 months with wages w_1 and w_2 respectively. She plans for her consumption in the two months (denoted by, c_1 and c_2 respectively) so as to maxinize her utility given by $\log c_1 + \frac{1}{1+\delta}\log c_2$, where $\delta > 0$ is the discount rate. She is also allowed to borrow from her employer in the first month which she has to repay, with interest at the rate r, from her second month's earnings. Alternatively she can save some of her wages in the first month and keep it in the bank to earn an interest at the same rate r. In this framework answer the following two questions:

37. Savitri will optimally decide to borrow from her employer in the first month if and only if the following condition is satisfied:

A. $w_2 > (1+r)w_1$

B. $(1+\delta) > (1+r)$

C. $(1+\delta)w_2 > (1+r)w_1$

D. $(1+\delta) < (1+r)$

38. Assuming that the above condition is satisfied, when the discount rate (δ) goes up, her borrowing

A. goes up

B. goes down

C. does not change

D. cannot say on the basis of the given information

39. Consider two open economies with fixed exchange rates, when the exchange rate is unity.

The economies are characterised by the following set of equations, where $i = 1, 2$:

$$\begin{aligned} C_i &= c_{01} + c_{1i}(Y_i - T_i) \\ I_i &= \overline{I}_i \\ G_i &= \overline{G}_i \\ T_i &= t_i Y_i \\ M_i &= m_{0i} + m_{1i} Y_i \end{aligned}$$

If $c_{01} = c_{02} = 200$; $c_{11} = c_{12} = 0.5$; $I_1 = I_2 = 250$; $G_1 = 114$; $G_2 = 120$; $t_1 = t_2 = 0.4$; $m_{01} = m_{02} = 40$; $m_{11} = 0.05$; $m_{12} = 0.3$ then which of the following is true ?

A. Country 1 has a higher equilibrium level of output than country 2 and it runs a trade deficit vis-à-vis Country 2

B. Country 1 has a lower equilibrium level of output than country 2 and it runs a trade deficit vis-à-vis Country 2

C. Country 1 has a higher equilibrium level of output than country 2 and it runs a trade surplus vis-à-vis Country 2

D. Country 1 has a lower equilibrium level of output than country 2 and it runs a trade surplus vis-à-vis Country 2

40. In the Mundell-Fleming model of a small open economy with flexible exchange rates and perfect capital mobility, suppose the economy is initially in equilibrium. If lump sum taxes are increased, what happens to the equilibrium levels of the country's (i) GDP; (ii) interest rate and (iii) exchange rate?

A. (i) falls, (ii) falls, (iii) appreciates

B. (i) and (ii) remain unchanged; (iii) depreciates

C. (i) falls, (ii) and (iii) remain unchanged

D. All three remain unchanged

41. $f : \mathbb{R}^n \to \mathbb{R}$ is convex if and only if

A. $\{(x, r)\mathbb{R}^n \times \mathbb{R} | f(x) \leq r\}$ is convex

B. $\{(x, r)\mathbb{R}^n \times \mathbb{R} | f(x) \geq r\}$ is convex

C. $\{x \in \mathbb{R}^n | f(x) \leq r\}$ is convex for every $r \in \mathbb{R}$

D. $\{x \in \mathbb{R}^n | f(x) \geq r\}$ is convex for every $r \in \mathbb{R}$

42. If $f : \mathbb{R}_+ \to \mathbb{R}$ is defined by $f(x) = \int_x^{x^2} e^t dt$, then the derivative of f at $x > 0$ is

A. $2xe^{x^2} - e^x$

B. $2xe^{x^2} + e^x$

C. $(2x - 1)e^{x^2}$

D. $(2x - 1)e^x$

43. The sequence (x_n), where $x_n = (-1)^n(1 + n^{-1})$ and $n = 1, 2, ...$

    A. converges to 1

    B. converges to -1

    C. converges to 1 and -1

    D. converges to neither 1 nor -1

44. The set $\cap_{n=1}^{\infty}(-1 - n^{-1}, 1 + n^{-1})$ is identical to

    A. $(-1, 1]$

    B. $[-1, 1)$

    C. $(-1, 1)$

    D. $[-1, 1]$

45. The set $\cup_{n=1}^{\infty}[-1 + n^{-1}, 1 - n^{-1}]$ is identical to

    A. $(-1, 1]$

    B. $[-1, 1)$

    C. $(-1, 1)$

    D. $[-1, 1]$

46. For $x, y \in \mathbb{R}^n$, let $d(x, y) = \max\{|x_i - y_i||i = 1, ..., n\}$. Which of the following relations holds for all $x, y, z \in \mathbb{R}^n$?

    A. $d(x, z) = d(x, y) + d(y, z)$

    B. $d(x, z) > d(x, y) + d(y, z)$

    C. $d(x, z) \geq d(x, y) + d(y, z)$

    D. $d(x, z) \leq d(x, y) + d(y, z)$

47. Given sets X, Y, let $X \times Y = \{(x, y)|x \in X \text{ and } y \in Y\}$, and let $X - Y = \{x \in X|x \notin Y\}$. Which of the following formulae is generally correct for sets A, B and C?

    A. $(A - B) \times C = (B \times C) - (A \times C)$

    B. $(A - B) \times C = (A \times C) - (B \times C)$

    C. $(A - B) \times C = (A \times C) - B$

    D. $(A - B) \times C = A - (B \times C)$

48. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by $z = xy$. At $(0, 0)$ this function

    A. Attains a maximum

    B. Attains a minimum

    C. Attains saddle point

    D. Is undefined

49. What is the maximum value of the function in the previous question, subject to the constraints $x > 0$, $y > 0$, and $2x + y = 6$?

    A. 2.5

    B. 4

    C. 4.5

    D. 9

50. "Beautiful cities repel people. Look at India's most beautiful cities - they are among the least populated cities of the country". Which of the following statements illustrates a method of reasoning most similar to the one in the argument above?

    A. Children who are spanked tend to be more naughty than other children. Hence if a child is not spanked, that child is less likely to be naughty.

    B. Pesticides are known to cause asthma among some people. However, most asthmatic people tend to live in regions where pesticides are not commonly used.

    C. This software program helps increase the work efficiency of its users. As a result these users have more free time for other activities.

    D. During warm weather, my crops suffer from pests more than during cooler weather. Therefore, a warm environment must help pests to multiply.

51. The average travel time to a distant city is w hours by train or z hours by bus. A man cannot decide weather to take the train or the bus, so he tosses a coin. What is his expected travel time?

    A. $2(w + z)$

    B. $2(w + z)/w - z$

    C. $(w + z + 2)/2$

    D. None of the above

52. A class has 6 students, randomly divided into two teams, A and B, for a race. What is the probability that the three runners in team A will come in first, second and third?

    A. $3!/6!$

    B. $1/6!$

    C. $(3!)^2/6!$

    D. $3/6!$

53. Suppose that each person out of a group of 4 friends is randomly assigned to one of 6 classes. What is the probability that no class has more than one person from this group?

    A. $6!/4!$

    B. $5/18$

C. 4/6

D. 6/4!

54. A continuous random variable has the probability density function $f(x) = 1/3$, for x between -1 and $+2$, and 0 elsewhere. Its mean, variance and median are:

A. $(1, \frac{3}{4}, \frac{1}{2})$

B. $(\frac{1}{2}, \frac{3}{4}, \frac{1}{2})$

C. $(\frac{1}{2}, 1, 1)$

D. $(\frac{1}{2}, 1, \frac{1}{2})$

55. The joint probability distribution of A and W for a given population is as follows, where A is the individual's age in years and W is the weight she can lift.

		A				
		15 years	25 years	40 years	60 years	80 years
	10 kilos	0.03	0.16	0.14	0.12	0.01
W	20 kilos	0.02	0.13	0.11	0.09	0.00
	30 kilos	0.01	0.09	0.08	0.01	0.00

The expectation of the Marginal distribution of A is:

A. 36.7 years

B. 37.6 years

C. 35.6 years

D. 38.6 years

56. In a survey of 400 likely voters, 215 responded that they would vote for the ruling party and 185 responded that they would vote for the opposition party. Let p denote the fraction of *all* likely voters who preferred the ruling party at the time of the survey, and let $\hat{p}$ be the fraction of survey respondents who preferred the ruling party. The standard error of $\hat{p}$ is:

A. 0.025

B. 0.035

C. 0.045

D. 0.055

57. Let X and Y be two random variables with $\mathbb{E}[X] = 3$, $\mathbb{E}[Y] = 0$, $\mathbb{V}[X] = 4$, $\mathbb{V}[Y] = 25$ and $\mathbb{E}[XY] = 5$. Find (i) $\mathbb{C}(X, Y)$; (ii) the Pearsonian coefficient of correlation between X and Y (ρ_{XY}); (iii) $\mathbb{V}[X + Y]$

A. $0.5, 5, 39$

B. $5, 0.5, 34$

C. $5, 5, 39$

D. $5, 0.5, 39$

58. If Y is a binomial random variable with parameters $n = 4$ and $p = 0.78$, calculate $\Pr(Y \geq 3)$ to 2 decimal places.

    A. 0.15

    B. 0.21

    C. 0.64

    D. 0.79

    For questions 59 and 60, please note that the two-tailed 5% critical value for a standardised normal distribution is ± 1.96 and the corresponding value for a one-tailed test is ± 1.645. At the 1% level of significance, the critical value for a two-tailed test is ± 2.58 and the corresponding value for a one-tailed test is ± 2.33.

59. An examination was given to two classes consisting of 40 and 50 students respectively. In the first class the mean grade was 74 with a standard deviation of 8, while in the second class the mean grade was 78 with a standard deviation of 7. On the basis of a two-tailed test, is there a significant difference between the performance of the two classes at a level of significance of 5% and 1%?

    A. There is no significant difference between the performance of the two classes at the 5% and 1% levels of significance

    B. There is a significant difference between the performance of the classes at the 5% level but no significant difference at the 1% level of significance

    C. There is a significant difference between the performance of the classes at the 1% level but no significant difference at the 5% level of significance

    D. There is a significant difference in performance at both the 5% and 1% levels

60. A sample poll of 300 voters from District A and 200 voters from District B showed that 56% and 48% respectively were in favour of a given candidate. At a level of significance of 5%, test the hypothesis that (a) there is a difference between the districts, (b) the candidate is preferred in District A.

    A. There is no significant difference between the districts and the candidate is not preferred in District A

    B. There is no significant difference between the districts and the candidate is preferred in District A

    C. There is a significant difference between the districts and the candidate is not preferred in District A

    D. There is a significant difference between the districts and the candidate is preferred in District A

3 DSE 2008

1. In the context of *ideal* price index numbers, consider the following statements:

   S1: The index number should be invariant to the choice of base, i.e. $P_{rs} \times P_{sr} = 1$.

   S2: An index that satisfies the circularity condition $P_{rs} \times P_{st} = P_{rt}$, $r \neq t$, need not satisfy S1.

   S3: If all prices change in the same proportion λ, then the index should equal λ.

   S4: If we change the units of measurement of the prices, but not those of the quantity weights, that should not affect the index.

   Given these statements, which of the following is true?

   A. S1 and S2

   B. S2 and S4

   C. S1, S2 and S3

   D. S1 and S3

2. Suppose that the probability that any particle emitted by a radioactive material will penetrate a given shield is .01. If ten particles are emitted, what is the probability that exactly one of the particles will penetrate the shield?

   A. $(.01)(0.99)^9$

   B. $(0.1)(0.99)^9$

   C. 0.1

   D. 1/9

3. Consider a symmetric 90% confidence interval for the population mean of a normal distribution with unknown variance, constructed using a random sample of 400 observations. Which of the following changes, ceteris paribus, would shorten the length of the confidence interval by the greatest amount?

   A. The confidence level is changed to 99%.

   B. The sample mean is half its original value.

   C. The sample size is four times its original value.

   D. The sample standard deviation is one third its original value.

4. Let A and B be any two events, each of which has a positive probability of occurring. Consider the following statements:

   I. If A and B are independent, they must be mutually exclusive.

   II. If A and B are mutually exclusive, they must be independent.

   III. If A and B are independent, they cannot be mutually exclusive.

IV. If A and B are mutually exclusive, they cannot be independent.

Which of the above statements are true?

A. I and IV.

B. II and III.

C. III and IV.

D. None of the statements are true.

5. Consider a random variable X which can take on only nonzero integer values from -20 to $+20$, and whose probability distribution is symmetric around 0. Suppose the $f(x)$, called the *probability mass function* of X, gives the probability that $X = x, \forall\ x = -20, \ldots, -1, 1, \ldots, 20$. Now consider the random variable $Y = X^2$. Which of the following would be an appropriate definition for $g(y)$, the probability mass function for Y?

A. $g(y) = f(\sqrt{y})\ \forall\ y = 1, 4, \ldots, 400$ and $g(y) = 0$ otherwise.

B. $g(y) = [f(\sqrt{y})]^2\ \forall\ y = 1, 4, \ldots, 400$ and $g(y) = 0$ otherwise.

C. $g(y) = 2f(\sqrt{y})\ \forall\ y = 1, 4, \ldots, 400$ and $g(y) = 0$ otherwise.

D. $g(y) = \sqrt{f(\sqrt{y})}\ \forall\ y = 1, 4, \ldots, 400$ and $g(y) = 0$ otherwise.

6. A consumer has a utility function $U(x, y) = 10(x^2 + 4xy + 4y^2) + 20$. Which one of the following statements must be true?

A. The goods are imperfect substitutes.

B. The goods are perfect substitutes.

C. The goods are perfect complements.

D. None of the above.

7. Suppose a consumer has a utility function $U(x, y) = \min\{x + y, 2y\}$. He maximises his utility subject to his budget constraint and consumes$(x^*, y^*) = (3, 3)$. Which one of the following statements must be true?

A. price of good x is necessarily equal to price of good y.

B. price of good x is double the price of good y.

C. price of good x is less than or equal to price of good y.

D. none of the above.

8. Suppose a monopolist sells his product in two separate markets. After the product is sold in one market, there is no possibility of it being resold in the other market. Which one of the following statements must be true?

A. Prices in both markets must be equal when the marginal cost of output is constant.

B. Prices must be higher in the market with higher price elasticity of demand.

C. Prices must be higher in the market with lower price elasticity of demand.

D. None of the above.

9. Consider a firm producing a single good with the cost function

$$C(x) = \begin{cases} 5, & \text{if } x = 0 \\ 10 + 10x, & \text{if } x > 0 \end{cases}$$

This firm's sunk cost and fixed cost are respectively

A. 10 and 10

B. 10 and 0

C. 0 and 10

D. 5 and 10

10. The elasticity of substitution of the production function $f(x, y) = cx^a y^b$ is

A. c/ab

B. ab/c

C. $a + b$

D. 1

11. In the standard IS-LM framework if you introduce endogenous money supply such that money supply depends positively on the nominal rate of interest, the corresponding LM curve

A. becomes steeper

B. becomes flatter

C. becomes horizontal

D. remains unchanged

12. Consider a simple Keynesian model where equilibrium output is determined by aggregate demand. Investment is autonomous and a constant proportion of the income is saved. In this framework an increase in the savings propensity has the following effect:

A. it leads to higher level of output in the new equilibrium

B. it leads to lower level of output in the new equilibrium

C. the level of output in the new equilibrium remains unchanged

D. the level of output in the new equilibrium may increase or decrease depending on the degree of increase in the savings propensity

13. In the Solow model of growth, an increase in the savings propensity has the following impact:

A. it leads to a higher steady state rate of growth

B. it leads to a lower steady state rate of growth

C. the steady state rate of growth remains unchanged

D. the steady state rate of growth may increase or decrease depending on the degree of increase in the savings propensity

14. If covered interest parity prevails between two countries,

A. nominal interest rates of the two countries must be equal.

B. expected currency depreciation must equal the interest rate differential.

C. expected currency depreciation must equal the interest rate differential plus the risk premium.

D. interest rate differential must equal the risk premium.

15. Which of the following constitute the "impossible trinity" which cannot be simultaneously ensured in an open economy?

1. Fixed exchange rate
2. Balance in the balance of payments
3. Free international capital mobility
4. Independent monetary policy

A. 1, 2 and 3

B. 1, 2 and 4

C. 1, 3 and 4

D. 2, 3 and 4

16. Consider the following two functions mapping points on the plane back to-points on the plane.

(i) $f(x_1, x_2) = (x_1 + 1, x_2 + 1)$ and

(ii) $g(x_1, x_2) = (x_2, x_1)$.

Which of the above functions is a linear function?

A. Both functions are linear.

B. Neither function is linear.

C. f is a linear function, g is not linear.

D. g is a linear function, f is not linear.

17. Consider the equation $x^y + y^z + z^x = k$, defined for all positive values of x and y, and where k is a given positive constant. The partial derivative $\partial z/\partial x$ then equals

A. $-(xz^{x-1} + y^z \ln y)/(yx^{y-1} + z^x \ln z)$

B. $-(yx^{y-1} + z^x \ln z)/(xz^{x-1} + y^z \ln y)$

C. $-(zy^{z-1} + x^y \ln x)/(xz^{x-1} + y^z \ln y)$

D. $-(zy^{z-1} + z^x \ln z)/(xz^{x-1} + z^x \ln z)$

18. The derivative of the function defined by $f(x) = \sin^2 x + \cos^2 x$ is

    A. an increasing function of x

    B. an oscillating function of x

    C. a constant

    D. a decreasing function of x

19. Consider an $n \times n$ real matrix A with $n > 4$. Interchanging the positions of two columns

    A. will change the sign of $\det A$

    B. will not change the sign of $\det A$

    C. may or may not change the sign of $\det A$, depending on the value of n

    D. may or may not change the sign of $\det A$, depending on the positions of the two columns

20. Suppose we have a chair with n legs and it stands with all its legs touching the floor, regardless of the floor quality, i.e., evenness, smoothness, etc. Then, n is

    A. 2

    B. 3

    C. 4

    D. 5

21. The frequency distribution of variable X (monthly family expenditure in Rs. '000) for 100 household is as follows:

X	10 - 15	5 - 9.9999	2 - 4.9999	Below 2
No. of households	10	15	45	30

    The median monthly expenditure is in the range:

    A. Rs. 2000 to 3000

    B. Rs. 3000 to 4000

    C. Rs. 4000 to 5000

    D. Rs. 5000 to 6000

22. The frequency distribution of variable Y, the number of heads when two damaged coins are tossed, is:

Y	0	1	2
F(y)	0.3	0.45	0.25

To two decimal places, the mean and variance of Y (in that order) are:

A. 0.95 and 0.50

B. 0.50 and 0.95

C. 0.95 and 0.55

D. 0.95 and 0.45

23. For variables X and Y we have the data: $\sum XY = 350$, $\sum X = 50$, $\sum Y = 60$, $\overline{X} = 5$, $\sigma_X^2 = 4$, and $\sigma_Y^2 = 9$. Which of the following holds?

A. A one unit change in X is associated with a 1.25 unit change in Y, and a one unit change in Y is associated with a 0.6 unit change in X.

B. A one unit change in X is associated with a 0.6 unit change in Y, and a one unit change in Y is associated with a 1.25 unit change in X.

C. The covariance between X and Y exceeds $\overline{Y}$.

D. The regression of Y on X passes through the origin.

24. The life of a cycle tyre is normally distributed with mean 350 days and variance 64. It is true that:

A. The probability that the life of the tyre will be less than 336.84 days is greater than 5%

B. The probability that the life of the tyre will be greater than 363.16 days is greater than 5%

C. The probability that the life of the tyre will be between 336.84 and 363.16 days is 90%

D. The probability that the life of the tyre will be less than 334.32 is greater than 3%

25. In a random sample of 400 mangoes selected from a large consignment, 30 were found rotten. The null hypothesis is that the proportion π of rotten mangoes in the consignment is 10%. It is ture that:

A. Given $H_A : \pi \neq 0.1$, we can't reject the null at the 1% level of significance, and the probability of error type I of this test is 0.005

B. Given $H_A : \pi < 0.1$, we reject the null at the 5% level of significance, and the probability of error type I of this test is 0.05

C. Given $H_A : \pi \neq 0.1$, we reject the null at the 10% level of significance, and the probability of error type I of this test is 0.1

D. If $H_A : \pi > 0.1$, the power of the associated test is higher than if $H_A : \pi < 0.1$

26. In a sample of 1000 mangoes, the mean weight of a mango is 210 g, and the standard deviation 9.5 g. In another sample of 1200 mangoes, the mean is 180 g and the standard deviation 11.5 g. Assume that the respective populations from which these samples are drawn have the same variances. Given the null hypothesis $H_0 : \mu_1 - \mu_2 = 0$, where μ_1 and μ_2 are the population means, it is true that:

A. If $H_A : \mu_1 - \mu_2 \neq 0$, we reject the null hypothesis at the 15%, 13%, 12% and 8% levels of significance.

B. If $H_A : \mu_1 - \mu_2 \neq 0$, we do not reject the null hypothesis at the 1% level of significance.

C. The appropriate estimator for testing weather the samples are from essentially the same population is $\mu_1 - \mu_2$.

D. If $H_A : \mu_1 - \mu_2 \neq 0$, and we conduct a test at the 7% level of significance, the probability of error type I of this test is 0.035.

27. Data on India's exports of Jute and Tea for the years 2000-2003 are as follows:

	2000	**2001**	**2002**	**2003**
Jute				
Quantity('000 t)	871	706	724	627
Price(Rs '00000/1000 t)	202	320	311	302
Tea				
Quantity('000 t)	199	179	214	288
Price(Rs '00000/1000 t)	577	767	884	799

Which of the following are true?

A. The chain-base price indices with 2000 as base year are $\overline{P}_{01} = 146.3$, $\overline{P}_{02} = 152.2$, and $\overline{P}_{03} = 144.9$

B. The chain-base price indices with 2000 as base year are $\overline{P}_{01} = 148.3$, $\overline{P}_{02} = 152.2$, and $\overline{P}_{03} = 146.9$

C. The chain-base price indices with 2000 as base year are $\overline{P}_{01} = 148.3$, $\overline{P}_{02} = 154.2$, and $\overline{P}_{03} = 144.9$

D. The chain-base price indices with 2000 as base year are $\overline{P}_{01} = 148.3$, $\overline{P}_{02} = 156.2$, and $\overline{P}_{03} = 146.9$

28. Suppose that 80% of all statisticians are shy, whereas only 15% of all economists are shy. Suppose also that 90% of the people at a large gathering are economists and the other 10% are statisticians. If you meet a shy person at random at the gathering, what is the probability that the person is a statistician?

A. 8/9

B. 0.8

C. .08

D. 80/215

29. Each cell of the following table provides the probability of the joint occurrence of the corresponding pair of values of the random variables X and Y.

$X \downarrow$ $Y \rightarrow$	1	2	3	4
1	.1	0	.1	0
2	.3	0	.1	.2
3	0	.1	0	.1

Consider the following statements about X and Y:

I. $\Pr(Y = 2) > \Pr(X = 1)$

II. $\Pr(Y = 1|X = 2) = \Pr(Y = 1|X = 1)$

III. The events $X = 3$ and $Y = 3$ are mutually exclusive.

IV. X and Y are independent.

Which of the above statements are true?

A. only I and II.

B. only II and III.

C. only III and IV.

D. only II, III and IV.

30. A survey of asset ownership in poor households in rural UP and Bihar found that 40% of the households own a radio, 15% own a television and 60% own a bicycle. It also found that 5% of the households own both a radio and a television, 26% own both a radio and a bicycle, 5% own both a television and a bicycle, and 1% own all three. If a randomly selected poor household in these areas is found to own exactly one of these three assets, what is the probability that it is a bicycle?

A. 20/23

B. 17/23

C. 15/23

D. 12/23

31. Consider an exchange economy with two persons and two goods. Person 1's utility function is $u_1(x, y) = x + y$ and person 2's utility function is $u_2(x, y) = e^{x^2+y^2+2xy}$. Person 1's endowment is (1, 0) and person 2's endowment is (0, 1). Denote person 1's allocation by (x_1, y_1) and person 2's allocation by (x_2, y_2). The set of efficient allocations $((x_1, y_1), (x_2, y_2))$ is such that

A. $(x_2, y_2) = (1 - x_1, 1 - y_1)$ for all $x_1, x_2 \in [0, 1]$

B. $(x_2, y_2) = (1 - x_1, 1 - y_1)$ for all $x_1, x_2 \in (0, 1)$

C. $(x_1, y_1) = (1/2, 1/2) = (x_2, y_2)$

D. $(x_1, y_1) = (e^a, e^b)$ and $(x_2, y_2) = (1 - e^a, 1 - e^b)$ for all $a, b \in (-\infty, 0)$

32. Consider an exchange economy with persons 1 and 2 and goods x and y. Person 1's utility function is $u_1(x, y) = x^2y^2$ and person 2's utility function is $u_2(x, y) = e^{x+y}$. The total endowment of the economy is (2, 1). The allocation $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (1, 0)$ is

A. Pareto inefficient

B. Pareto efficient or inefficient depending on the endowments of the two persons

C. Pareto efficient or inefficient depending on the state of the world

D. Pareto efficient

33. Consider the situation of the preceding question. If person 1's endowment is (1, 1) and 2's endowment is (1, 0), then the following allocation is a competitive equilibrium:

A. $(x_1, y_1) = (3/2, 1/2)$ and $(x_2, y_2) = (1/2, 1/2)$

B. $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (1, 0)$

C. $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (0, 1)$

D. $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (0, 0)$

34. Consider the situation of the preceding question. Which of the following is an equilibrium price vector?

A. $(p_1, p_2) = (1, 0)$

B. $(p_1, p_2) = (0, 1)$

C. $(p_1, p_2) = (1, 1)$

D. none of the above

35. Consider an exchange economy with the same two agents and utility functions as the last three questions, but now the endowments of the two persons are (0, 1) and (2, 0). Which of the following pre-trade lump-sum transfers of wealth will lead to allocations $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (1, 0)$ and prices $(p_1, p_2) = (1, 1)$ being a competitive equilibrium?

A. subsidies of 1 to both persons

B. taxes of 1 to both persons

C. a subsidy of 1 to person 1 and a tax of 1 on person 2

D. a subsidy of 1 to person 2 and a tax of 1 on person 1

36. There are 3 items of choice, x, y, z, and Ms. A has 4 possible choice situations: in 3 of them, she is asked to choose one or more items from the 3 possible pairs of items $\{x, y\}, \{y, z\}$ and $\{x, z\}$. She chooses the items x, y and z respectively in these 3 situations. In the 4th situation she must choose one or more items from the set $\{x, y, z\}$; we are not told directly what her choice is. Which of the following is correct?

A. Ms A's choice violate the weak axiom of revealed preference.

B. Ms A's choices are consistent with the weak axiom of revealed preference.

C. We can't say (a) or (b) because we don't know her choice from the set $\{x, y, z\}$

D. We can't say (a) or (b), even though we can deduce her choice from the set $\{x, y, z\}$

Questions 37-39 use the following information: A chemical factory produces a chemical K and an effluent E which it dumps in a river. A downstream fishery produces fish F and its costs are affected by the level of effluent in the river. The two firms are competitive and face unit prices $P_K = 10$ and $P_F = 20$ for the chemical and the fish respectively. The cost function of the chemical factory is $C(K, E) = K^2[(5 - E)^2 + 1]$. The cost function of the fishery is $\widetilde{C}(F, E) = F^2E^2$.

37. If the chemical factory chooses levels of chemical and effluent to maximize profits, and the fishery chooses the level of fish to maximize profits, the chosen level of the chemical, effluent and fish (i.e. K, E, F) are respectively

    A. 5, 0, 5/8

    B. 3, 0, 2/5

    C. 5, 5, 2/5

    D. 5, 5, 5/8

38. Suppose the socially optimal levels of the chemical, effluent and fish, denoted by $\overline{K}, \overline{E}, \overline{F}$ respectively, are the levels that maximise the joint profit of an integrated firm consisting of the chemical factory and the fishery. Then $\overline{E}$ is equal to

    A. $5\overline{K}^{\,2}/(\overline{K}^{\,2} - \overline{F}^{\,2})$

    B. $5\overline{F}^{\,2}/(\overline{K}^{\,2} + \overline{F}^{\,2})$

    C. $5\overline{F}^{\,2}/(\overline{K}^{\,2} - \overline{F}^{\,2})$

    D. $5\overline{K}^{\,2}/(\overline{K}^{\,2} + \overline{F}^{\,2})$

39. Suppose a government knows the firms' production functions and their output prices, and can costlessly monitor the amount of effluent released by the chemical factory. If it sets a tax t per unit of effluent produced by the chemical factory, the level of this tax that will result in the socially optimal effluent level $\overline{E}$ is equal to

    A. $t = 10(\overline{K})(\overline{F})^2/(\overline{K}^{\,2} + \overline{F}^{\,2})$

    B. $t = 10(\overline{K})^2(\overline{F})^2/(\overline{K}^{\,2} - \overline{F}^{\,2})$

    C. $t = 10(\overline{K})^2(\overline{F})^2/(\overline{K}^{\,2} + \overline{F}^{\,2})$

    D. $t = 5(\overline{K})(\overline{F})^2/(\overline{K}^{\,2} + \overline{F}^{\,2})$

40. Consider a firm with one output and two possible choices of capital stock, say 1 and 2. The associated cost functions are $C(x, 1) = 2 + 2x$ and $C(x, 2) = 4 + x$. Before choosing its capital stock, the firm's cost function is

    A. $C^*(x) = \begin{cases} 2 + 2x, & \text{if } x \in [0, 2] \\ 4 + x, & \text{if } x > 2 \end{cases}$

    B. $C^*(x) = 2 + 2x$

    C. $C^*(x) = 4 + x$

    D. $C^*(x) = \begin{cases} 4 + x, & \text{if } x \in [0, 2] \\ 2 + 2x, & \text{if } x > 2 \end{cases}$

Questions 41-46 are related and share a common information set. The complete set of information is revealed gradually as you move from one question to the next.
Consider a macro-economy where the aggregate output in the short run is given by:
$Y = AL^{\alpha}(\overline{K})^{1-\alpha}$, where
L is the total employment of labour;
$\overline{K}$ is the total capital stock (which is fixed in the short run);
and $A > 0$ and $0 < \alpha < 1$ are parameters of the system.
Let P and W denote the aggregate price level and the money wage rate respectively. Assume that the producers in the economy maximise profit in a perfectly competitive set up.

41. The corresponding demand for labour schedule as a function of the real wage rate is given by

A. $L^d = \left[\dfrac{Y}{A\left(\overline{K}\right)^{1-\alpha}}\right]^{\frac{1}{\alpha}}$

B. $L^d = \overline{K}\left(A\alpha\right)^{\frac{1}{1-\alpha}}\left[\dfrac{W}{P}\right]^{\frac{1}{\alpha-1}}$

C. $L^d = \left[\dfrac{1}{A\alpha\left(\overline{K}\right)^{1-\alpha}}\right]^{\frac{1}{\alpha}}\left[\dfrac{W}{P}\right]^{\frac{1}{1-\alpha}}$

D. None of the above

42. If there is a one-shot increase in the stock of capital $(\overline{K})$, the demand for labour schedule, as derived above,

A. shifts up

B. shifts down

C. does not shift

D. the information available so far is not adequate to answer this question

Suppose the above economy is characterised by a single household which takes the aggregate price level and the money wage rate as given and decides on its consumption and labour supply by maximising its utility subject to its budget constraint. The household has a total endowment of $\overline{L}$ units of labour time, of which it supplies L^S units to the market at the money wage rate W, and enjoys the rest as leisure. Its utility depends on its consumption and leisure in the following way: $U = (C)^{\beta} + (\overline{L} - L^S)\beta\,; 0 < \beta < 1$. The only source of income of the household is the wage income and it spends its entire wage earning in buying consumption goods at the price P.

43. The corresponding supply of labour schedule as a function of the real wage rate is given by:

A. $L^s = \left[\dfrac{\overline{L}\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}\right]$

B. $L^s = \left[\dfrac{\overline{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}\right]$

C. $L^s = \left[\dfrac{\left(\frac{W}{P}\right)^{\beta}}{\overline{L}+\left(\frac{W}{P}\right)^{\beta}}\right]$

D. None of the above

Maximize $U = (C)^{\beta} + (\overline{L} - L^S)\beta$ such that $wL^s = PC$
Find first derivative conditions wrt (C, L^s)

44. If there is an exogenous increase in the total endowment of labour time $(\overline{L})$, the supply for labour schedule, as derived above,

A. shifts up

B. shifts down

C. does not shift

D. the information available so far is not adequate to answer this question

45. Let $\alpha = \frac{1}{2}$; $\overline{K} = 10; \overline{L} = 10; A = 4$. Given these parameter values, the unique non-negative equilibrium value of real wage rate that clears the labour market is given by:

A. $\dfrac{W}{P} = \left[\dfrac{1}{2\sqrt{2}}\right]^{1/2}$

B. $\dfrac{W}{P} = \left[\dfrac{1+\sqrt{2}}{2}\right]^{1/2}$

C. $\dfrac{W}{P} = [2 - 2\sqrt{2}]^{1/2}$

D. $\dfrac{W}{P} = [2 + 2\sqrt{2}]^{1/2}$

46. Given the labour demand and the labour supply schedule as derived above, the aggregate supply curve (output supplied as a function of the aggregate price level) is

A. upward sloping

B. downward sloping

C. vertical

D. horizontal

Questions 47 to 50 are related and share a common information set. The complete set of information is given below. Attempt all of them together.
Consider a closed economy where the general price level is exogenously fixed at $\overline{P}$. The goods market clearing condition is given by: $Y = C + I + G$, where the consumption demand (C) is a function of the disposable income $(Y - T)$ such that $C = c.(Y -$

$T), 0 < c < 1$; the investment demand (I) is a function of the real interest r such that $I = \overline{I} - d.r, d > 0$; total tax revenue in the economy (T) is a function of the aggregate real income such that $T = \tau.Y, 0 < \tau < 1$; and the government expenditure (G) is autonomous such that $G = \overline{G}$. The money market clearing condition is given by: $\frac{M}{\overline{P}} = L$, where supply of money (M) is exogenous such that $M = \overline{M}$; and the demand for real balance (L) is a function of the real income (Y) and the nominal interest rate (i) such that $L = a.Y - b.i, a, b > 0$. Finally, the nominal interest rate is the sum of the real rate of interest and the expected rate of inflation (π^e) such that $i = r + \pi^e$.
Assume that the economy starts from an equilibrium situation where both the goods market and the money market clear.

47. In *ceteris paribus*, a unit increase in government expenditure has the following impact on the equilibrium income level:

A. it increases by $\frac{1}{b[1-c(1-\tau)]+ad}$units

B. it decreases by $\frac{1}{b[1-c(1-\tau)]+ad}$units

C. it increases by $\frac{b}{b[1-c(1-\tau)]+ad}$units

D. it decreases by $\frac{b}{b[1-c(1-\tau)]+ad}$units

48. In *ceteris paribus*, a unit increase in the expected rate of inflation has the following impact on the equilibrium value of the real interest rate:

A. it increases by $\frac{[1-c(1-\tau)]}{b[1-c(1-\tau)]+ad}$units

B. it decreases by $\frac{a}{a[1-c(1-\tau)]+ad}$units

C. it increases by $\frac{b[1-c(1-\tau)]}{b[1-c(1-\tau)]+ad}$units

D. it decreases by $\frac{b[1-c(1-\tau)]}{b[1-c(1-\tau)]+ad}$units

49. In *ceteris paribus*, a unit increase in the general price level has the following impact on the equilibrium value of the real interest rate:

A. it increases by $\frac{[\overline{M}/(\overline{P})^2]}{b[1-c(1-\tau)]+ad}$units

B. it decreases by $\frac{[\overline{M}/(\overline{P})^2]}{b[1-c(1-\tau)]+ad}$units

C. it increases by $\frac{[1-c(1-\tau)]\,[\overline{M}/(\overline{P})^2]}{b[1-c(1-\tau)]+ad}$units

D. it decreases by $\frac{[1-c(1-\tau)]\,[\overline{M}/(\overline{P})^2]}{b[1-c(1-\tau)]+ad}$units

50. Suppose we now allow the general price level to be flexible. Given the above characteristics of the macro-economy, the corresponding aggregate demand curve (output demanded as a function of the general price level) is

    A. upward sloping

    B. downward sloping

    C. vertical

    D. horizontal

51. Fix an $m \times n$ matrix A and an m-vector b. A condition that ensures the existence of a solution (an n-vector x) of the equation $Ax = b$ is

    A. $\det A \neq 0$

    B. $m = n$

    C. the columns of A are linearly independent

    D. the rows of A are linearly independent

52. The real-valued function $f(x) = x^4$ is

    A. strictly convex

    B. strictly concave

    C. neither strictly concave nor strictly convex

    D. convex but not strictly convex

53. Consider the equation $Ax = 0$, where A is an $n \times n$ matrix such that $a_{ij} \neq 0$ for every $i \in \{1, \ldots, n\}$ and $a_{ij} = 0$ whenever $i > j$. This equation has

    A. n distinct solutions

    B. an $n-1$ dimensional vector space of solutions

    C. exactly one solution

    D. an n dimensional vector space of solutions

54. There is a pile of 17 matchsticks on a table. Players 1 and 2 take turns in removing matchsticks from the pile, starting with Player 1. On each turn, a player has to remove a number of sticks that equals the square of a positive integer, such that the number of matchsticks that remain on the table equals some non-negative integer. The player who cannot do so, when it is his/her turns, loses. Which of the following statement is true?

    A. If Player 2 plays appropriately, he can win regardless of how 1 actually plays.

    B. If Player 1 plays appropriately, she can win regardless of how 2 actually plays.

    C. Both players have a chance to win, if they play correctly.

    D. The outcome of the game cannot be predicted on the basis of the data given.

55. If A is a set of real numbers, let g_A be the function such that $g_A(x) = 1$ if $x \in A$, and $g_A(x) = 0$ if $x \notin A$. With this notation, consider the infinite sequence of functions f_n, where $f_n(x) = ng_{[0,1/n]}(x)$ (i.e., n multiplied by $g_{[0,1/n]}(x)$) for all real numbers x and for each $n = 1, 2, 3 \ldots$ Then

    A. For every x, the sequence of numbers $(f_n(x))_{n=1}^{\infty}$ has a limit in the space of real numbers.

    B. $\lim_{n\to\infty} f_n(x)$ does not exist, for any x.

    C. When $\lim_{n\to\infty} f_n(x)$ exists, the actual limit depends on the x in question.

    D. $\lim_{n\to\infty} f_n(x)$ exists for all but a finite set of real numbers x.

56. Continuing with the sequence of functions above, we consider the sequence of real numbers $(\int f_n)_{n=1}^{\infty}$ (that is, the sequence of their integrals). This sequence of integrals is

    A. an increasing sequence

    B. a decreasing sequence

    C. a constant sequence

    D. an oscillating sequence

57. For each positive integer $a = 1, 2, 3, \ldots$, let S_a be the set of points lying on the curve $y = 1/x^a$, for all positive real numbers x. Then the intersection of these sets over all a, (that is, $\cap_{a=1}^{\infty} S_a$) is

    A. a set with infinitely many points

    B. a set having a single point

    C. a set having exactly 2 points

    D. a set having more than 2, but a finite number of points

58. Which of the following two numbers is larger e^{π} or π^e?

    A. e^{π}

    B. π^e

    C. they are equal

    D. it depends on the value of e

59. The function $f(x) = \begin{cases} \sin(1/x), & \text{if } x > 0 \\ a, & \text{if } x = 0 \end{cases}$

    A. is continuous or discontinuous at $x = 0$, depending on the value of a

    B. is continuous at $x = 0$

    C. is discontinuous at $x = 0$

    D. is upper hemicontinuous at $x = 0$

60. Suppose v_1, v_2 and v_3 are three vectors in 3-dimensional space, and are linearly dependent. Then the vectors $v_1 + v_2$, $v_2 + v_3$, and $v_1 + v_3$

A. are linearly independent

B. may be linearly dependent

C. are linearly dependent

D. are linearly independent, except when one of the vectors is zero

4 DSE 2009

1. There are 4 married couples in a club. A 3-member committee must be formed among them, such that no married couple is part of the committee. The number of ways in which this committee can be formed is

   A. 16

   B. 44

   C. 32

   D. 56

2. For any sets E & F, $E - F = \{x \in E | x \notin F\}$, $\cup$ and $\cap$ refer to the Union and intersections of the sets. Consider the following statements for the sets A, B and C

   i. $A - (B \cup C) = (A - B) \cup (A - C)$

   ii. $A - (B \cup C) = (A - B) \cap (A - C)$

   iii. $A - (B \cup C) = (A - B) \cap C$

   A. i. is true

   B. ii. is true

   C. iii. is true

   D. none of them is necessarily true

3. $\int_{1.96}^{\infty} e^{-x^2/2} dx$ is approximately

   A. 0.025

   B. $\sqrt{2\pi}$

   C. $0.025/\sqrt{\pi}$

   D. $0.025/\sqrt{2\pi}$

4. Consider maximising $f(x, y) = x^2 - y^2$ subject to the constraint $x + y = 1$, where x and y are real numbers. This problem has

   A. no solution

   B. a unique solution

   C. 2 solutions

   D. an infinity of solutions

5. Suppose $P(x)$ and $Q(x)$ are real polynomials of degree m and k respectively, where both m and k are less than or equal to the positive integer n. Suppose the equation $P(x) = Q(x)$ has at least $(n + 1)$ distinct solutions. Which of the following choices best describes what this situation implies?

   A. $m = k = n$

B. $m = k < n$

C. $P(x)$ and $Q(x)$ are identical

D. $P(x)$ and $Q(x)$ are linear

6. There are three alternative definitions of a consistent estimator

   I. An estimator is consistent if its probability limit equals its true parameter value as sample size approaches infinity.

   II. An estimator is consistent if its mean squared error goes to zero as sample size approaches infinity.

   III. An estimator is consistent if it is unbiased, and its variance goes to zero as sample size approaches infinity.

   A. Only I is correct

   B. Only II is correct

   C. Only III is correct

   D. All three are correct

7. Given the data $\sum XY = 350$, $\sum X = 50$, $\sum Y = 60$, $N = 10$, $V(X) = 4$, $V(Y) = 9$, where $V(.)$ refers to the population variance. The correlation coefficient between X and Y, regression (slope) coefficient of Y on X, and the regression (slope) coefficient of X on Y are, respectively:

   A. $35/36, 35/16, 35/81$

   B. $5/6, 5/4, 5/9$

   C. $5/6, 35/16, 35/81$

   D. $\sqrt{35/36}, 35/16, 35/81$

8. In a surprise check in a local bus, 20 passengers were caught without tickets. The sum of squares and the population standard deviation of the amount in their pockets were Rs. 2000 and Rs. 6, respectively. If the total fine equals the total amount discovered on them, and a uniform fine is imposed, then the fine imposed on an individual is:

   A. Rs. 8

   B. Rs. 6

   C. Rs. 10

   D. Rs. 12

9. In a linear regression of Y on X, changing the units of measurement of the Y variable will affect all of the following except:

   A. the estimated intercept parameter

   B. the estimated slope parameter

   C. the Total Sum of Squares for the regression

D. R squared for the regression

10. A *fair* dice has numbers 1, 2, 3, 4, 5 and 6 on its side. It is tossed *once*. I win Rs. 1 if an odd number shows up; otherwise I lose Rs. 1. Let X be the number that shows up and Y the money I win. [Note: $Y < 0$ if I lose money.]
Which of the following is incorrect?

A. Prob $(X > Y) = 1$

B. Prob $(X = 3|Y = 1) = 3$

C. $\mathbb{E}(Y) = 0$

D. Prob $(Y = 1|X = 5) = 1$

11. Your budget is such that if you spend your entire income on two goods, x and y, you can afford either 4 units of x and 6 units of y or 12 units of x and 2 units of y. If you spent all your income on x, how many units of x could you buy?

A. 7

B. 16

C. 15

D. 18

12. The demand function for lemonade is $Q_d = 100 - p$, and the supply function is $Q_s = 10 + 2p$, where p is the price in rupees. The government levies a sales tax on lemonade after which the volume of sales drops to 60. then the per unit tax on lemonade is

A. Rs. 20

B. Rs. 15

C. Rs. 10

D. Rs. 5

13. There are only two price taking firms in a market. Their cost functions are $C_1 = x_1^2$ and $C_2 = 2x_2^2$, where x_i is the output of the i^{th} firm. Market supply is sum of the two firms output. Then the market supply function is

A. $x = \dfrac{3p}{4}$

B. $x = \dfrac{4p}{3}$

C. $x = \dfrac{p}{2}$

D. $x = 2p$

14. A monopoly faces the demand curve $P = 8 - Q$. The monopoly has a constant unit cost equal to 5 for $Q \leq 2$ and a constant unit cost equal to 3 for $Q > 2$. Its profit maximising output equals:

A. 3/2

B. 2

C. 5/2

D. Both 3/2 and 5/2

15. A firm has the production function $y = \min\{L + 2K, 2L + K\}$, where y is quantity of output, and L & K are the quantities of labour and capital inputs respectively. If the input price of L is Rupee 1 and the input price of K is Rupees 2, then to produce $y = 12$ costs the firm at least

A. 10 Rupees

B. 12 Rupees

C. 14 Rupees

D. 16 Rupees

16. The opportunity cost of holding money (that yields zero nominal return) vis-à-vis some interest bearing bond is:

A. the real interest rate

B. the nominal interest rate

C. the real interest rate when measured in real terms and the nominal interest rate when measured in nominal terms

D. None of the above

17. In the IS-LM framework, an increase in the expected rate of inflation results in

A. an increase in the equilibrium value of income and an increase in the equilibrium value of real interest rate

B. a decrease in the equilibrium value of income and a decrease in the equilibrium value of real interest rate

C. an increases in the equilibrium value of income and a decrease in the equilibrium value of real interest rate

D. a decrease in the equilibrium value of income and an increase in the equilibrium value of real interest rate

18. When the nominal wage rate is rigid, the aggregate supply schedule (in the output-price space) is:

A. horizontal

B. vertical

C. downward sloping

D. upward sloping

19. In IS-LM framework with an external sector i.e., the IS equation now includes a net export term, an appreciation of the (real) exchange rate

A. would necessarily result in a decrease in the equilibrium value of income

B. would result in a decrease in the equilibrium value of income only if the LM curve is vertical

C. would result in a decrease in the equilibrium value of income only if the Marshall-Lerner condition is satisfied

D. would result in a decrease in the equilibrium value of income only if the government maintains a balanced budget

20. According to the Baumol-Tobin Model, if income rises by ten percent, the transactions demand for money should rise by

A. Five percent

B. Ten Percent

C. Between five and ten percent

D. None of the above

21. Consider the following statements:
(i) $5^{44} > 4^{53}$ (ii) $2^{100} + 3^{100} < 4^{100}$

A. Both (i) and (ii) are false.

B. (i) is true, (ii) is false.

C. Both (i) and (ii) are true.

D. (i) is false, (ii) is true.

22. The WOW Language has only 2 letters in its alphabet, O and W; the language obeys the following rules: (i) deleting successive letters WO from any word which has more than 2 letters, gives another word with the same meaning. (ii) inserting OW or WWOO in any place in a word yields another word with the same meaning. O, OWOOW, WOO and OWW are 4 words in this language. Which of the following statements is FALSE?

A. the words WOO and OWW necessarily have the same meaning.

B. WOO and OWW may not have same meaning.

C. O and OWOOW must have the same meaning.

D. (b) and (c) are true.

23. Consider the system of equations in the unknowns x and y:

$$ax + y = a^2$$
$$x + ay = 1$$

Sets of all values of a for which this system has (i) no solution, (ii) multiple solutions, and (iii) a unique solution are respectively

A. (i) $a < 1$ (ii) $a > 1$ (iii) $a = 1$

B. (i) $a = -1$ (ii) $a = 1$ (iii) $a > 1$

C. (i) $a = -1$ (ii) $a = 1$ (iii) all other values of a

D. (i) $a = -1$ (ii) $a = 1$ (iii) $-1 < a < +1$

24. Consider the system of equations:

$$\begin{aligned} x - y + z &= 1 \\ 3x + z &= 3 \\ 5x - 2y + 3z &= 5 \end{aligned}$$

This system has

A. the unique solution $(x, y, z) = (1, 0, 0)$

B. the solution set $\left\{(1 - \frac{z}{3}, \frac{2z}{3}, z) |\ z \in \Re\right\}$

C. the solution set $\left\{(1 - \frac{z}{3}, \frac{2z}{3}, z) |\ z \geq \Re\right\}$

D. multiple solutions, but not described by (b) or (c).

25. Consider the function $f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then the following is true about the derivative of f:

A. $f'(0) = -1$ and $f'(x)$ is continuous at $x = 0$

B. $f'(0) = -1$ and $f'(x)$ is discontinuous at $x = 0$

C. $f'(0) = 0$ and $f'(x)$ is discontinuous at $x = 0$

D. $f'(x)$ is not defined at $x = 0$

26. For all set S, let S^2 denote the cartesian product $S \times S$. A binary relation R on S is a subset of S^2.
R is transitive if $(x, y) \in R$ and $(y, z) \in R$ implies $(x, z) \in R$.
R is negatively transitive if $(x, y) \notin R$ and $(y, z) \notin R$ implies $(x, z) \notin R$.
Inverse of R is defined as follows, $R^{-1} = \{(y, x) \mid (x, y) \in R\}$
Consider the following statements:
Statement A: R can not be transitive and negatively transitive at the same time.
Statement B: If R is transitive then R^{-1} must be transitive.
Statement C: If R is transitive then R^{-1} must be negatively transitive.
How many of the above statements are true?

A. None

B. One

C. Two

D. Three

27. A function $f(x_1, x_2, T) = (y_1, y_2)$ is defined as follows, where x_1, x_2 and T are non negative real numbers and $x_1 + x_2 \geq T$.
$y_1 = \min(\alpha, x_1)$, $y_2 = \min(\alpha, x_2)$ such that $y_1 + y_2 = T$. Find $f(5, 2, 6)$.

A. (3, 3)

B. (5, 1)

C. (0, 6)

D. (4, 2)

28. A function f is defined as follows. Here a, b and c are constants.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq c \\ ax + b & \text{if } x > c \end{cases}$$

Find values of a and b such that $f\prime(c)$ exists.

A. $a = 2c\, b = -c^2$

B. $a = c\, b = -2c^2$

C. $a = \dfrac{3}{2c}, b = -\dfrac{1}{c}$

D. $a = 1\, b = \ln c$

29. $f(c) = \max\limits_{2x+y=c, x\geq 0, y\geq 0}(x + 2y)$. Find $f'(c)$.

A. c

B. 0

C. 2

D. $c/2$

30. There are 3 persons, A, B and C. One of them is a Truth-teller (always tells the truth), another is a Liar (always lies) and the third is a normal person (sometimes lies, other times speaks the truth). They all know of each others and their own type.
A said: "I am a normal person."
B said: "A and C sometimes tell the truth."
C said: " B is a normal person."

A. These statements are insufficient to determine who is a Liar.

B. A is a normal person, B is a Truth-teller, C is a Liar.

C. Who is normal, or Liar or Truth-teller cannot be ascertained from the statements.

D. A is a Liar, B is a normal person, C is a Truth-teller.

31. The nine digits $1, 2, \ldots, 9$ are arranged in random order to form a nine digit number, which uses each digit exactly once. Find the probability that 1, 2 and 3 appear as neighbours in the increasing order.

A. 1/12

B. 1/72

C. 1/84

D. $(2/3)^9$

32. In a survey of 102 Timarpur residents in 2009, the average income was found to be Rs. 4635 per month. Previous studies show the population variance of income in this locality to be Rs. 12342 per month. It is asserted that the average monthly income is Rs. 4650 in this locality. Which conclusion below can be asserted from this information?

    A. The assertion is rejected at the 10% level.

    B. The assertion is rejected at the 5% level.

    C. The assertion is not rejected at the 10% level.

    D. None of the above.

33. Let X denote the absolute value of the difference between the numbers obtained when two dice are tossed. The expectation of X is:

    A. $1\frac{32}{36}$

    B. $1\frac{33}{36}$

    C. $1\frac{17}{18}$

    D. $1\frac{16}{18}$

34. Let Y denote the number of heads obtained when 3 coins are tossed. The variance of Y^2 is:

    A. 9.5

    B. 8.5

    C. 6.5

    D. 7.5

35. A company has 100 employees, 40 men and 60 women. There are 6 male executives. How many female executives should there be for gender and rank to be independent?

    A. 9

    B. 6

    C. 10

    D. 8

36. Consider two events A and B with Pr(A) = 0.4 and Pr(B) = 0.7. The maximum and minimum values of Pr(A∩B) respectively are:

    A. (0.4; 0.1)

    B. (0.7; 0.4)

    C. (0.7; 0.1)

D. (0.4; 0)

37. Jai and Vijay are taking a exam in statistics. The exam has only three grades A, B and C. The probability that Jai gets a B is 0.3, the probability that Vijay gets a B is 0.4, the probability that neither gets an A, but at least one gets a B is 0.1. What is the probability that neither gets a C but at least one gets a B?

    A. 0.1

    B. 0.6

    C. 0.8

    D. Insufficient data to answer the question

38. You've been told that a family has two children and one of these is a daughter . What is the probability that the other child is also a daughter?

    A. 1/2

    B. 1/3

    C. 1/4

    D. 3/4

Questions 39 and 40. Suppose least squares is used to fit a line relating y and x, namely $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$. Assume that in our data not all the x's are identical, so that at least some of the x's are different from their sample mean $\overline{x}$. Now consider the following possible assumptions about our data.

(1) $\mathrm{E}(\varepsilon_i) = 0$

(2) $\mathrm{Cov}(x_i, \varepsilon_i) = 0$

(3) Homoskedasticity: $\mathrm{Var}(\varepsilon_i) = \sigma^2$, a constant

(4) No autocorrelation: $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$

Indicate the one best answer to each question below.

39. When are the least-squares estimators unbiased?

    A. Only if our data satisfy assumptions (1) and (2).

    B. Only if our data satisfy assumptions (1), (2), (3), and (4).

    C. Only if our data satisfy assumptions (1), (2), and (3).

    D. Only if our data satisfy (1), (2), and (4).

40. When are the least-squares estimators "best" (lowest variance) of any unbiased estimators

    A. Only if our data satisfy assumptions (1) and (2).

    B. Only if our data satisfy assumptions (1), (2), (3), and (4).

    C. Only if our data satisfy assumptions (1), (2), and (4).

D. Cannot be determined, without additional assumptions.

41. A consumer spends an income of Rs. 100 on two goods, dosas and pizzas. Let x denote the number of dosas and y the number of pizzas consumed (fractions allowed). The consumer?s utility function is $U = e^{x^2+y^2}$. If the price of a dosa is Rs. 5, and the price of a pizza is Rs. 10, then the number of pizzas this consumer will buy is

    A. 0

    B. 10

    C. 5

    D. 8

42. Romeo and Juliet have 96 chocolates to divide between them. Romeo has the utility function $U = R^8J^4$ and Juliet has the utility function $U = R^4J^8$ where R is Romeo's chocolate consumption and J is Juliet's chocolate consumption. Which of the following is true

    A. Romeo would want to give Juliet some chocolates if he had more than 62.

    B. Juliet would want to give Romeo some chocolates if she had more than 60.

    C. Romeo and Juliet would never disagree about how to divide the chocolates.

    D. Juliet would want to give Romeo some chocolates if she had more than 64 chocolates.

43. A consumer spends an income of Rs. 100 on only two goods, A and B. Assume non satiation, i.e., more of any good is preferred to less. Suppose the price of B is fixed at Rs. 20. When the price of A is Rs. 10, the consumer buys 3 units of B. When the price of A is Rs. 20, she buys 5 units of A. From this we can conclude

    I. A is an inferior good

    II. A is a Giffen good

    III. B is a complement of A

    A. I only

    B. I and II

    C. I and III

    D. I, II and III

Questions 44 and 45. 'Suraksha' is the sole producer and supplier of security systems in India and the sole employer of locksmiths in the labour market. The demand for security systems is $D(p) = 100 - p$, where p is the price. The production of security systems only requires locksmiths and the production function is given by $f(L) = 4L$, where L is the number of locksmiths employed. The supply curve for locksmiths is given by $L(w) = \max\left[0, \frac{w}{2} - 20\right]$, where w is the wage rate.

44. How many locksmiths will 'Suraksha' employ?

A. 5

B. 10

C. 15

D. 20

45. If the government sets the minimum wage is 70 , how many locksmiths will Suraksha employ?

A. 5

B. 10

C. 15

D. 20

Questions 46 and 47. Suppose that a typical graduate student at the Delhi School of Economics lives in a two good world, books (x) and movies (y), with utility function $u(x, y) = x^{1/5}y^{4/5}$. Prices of books and movies are 50 and 10 respectively. Suppose the University is considering the following schemes.

Scheme 1: 750 is paid as fellowship and additional 250 as book grant. Naturally, book grant can only be spent on books. Scheme 2: 1000 as scholarship and gets one movie free on each book they purchase.

Believing that books and movies are perfectly divisible, compute the optimal consumption bundle under each scheme.

46. Optimal consumption bundle under scheme 1 is

A. (4 books, 80 movies)

B. (5 books, 75 movies)

C. (6.5 books, 57.5 movies)

D. (10 books, 50 movies)

47. Optimal consumption bundle under scheme 2 is

A. (4 books, 80 movies)

B. (4 books, 84 movies)

C. (5 books, 75 movies)

D. (5 books, 80 movies)

48. Let X stand for the consumption set and let R, I, P respectively stand for the weak preference relation, indifference relation and strict preference relation of a consumer. The weak preference relation R is said to satisfy Quasitransitivity if and only if for all x, y, z belonging to X, xPy and yPz $\rightarrow$ xPz. Which of the following preference relations over X = {x,y,z} satisfy Quasitransitivity?

A. xPy & yPz & zPx

B. xPy & yPz & zIx

C. xPy & yIz & zIx

D. yPx & yIz & xPz

49. Consider an exchange economy with two consumers (A & B) and two goods (x & y). Assume that total amount of x available is 4 and total amount of y available is 2 which is to be optimally distributed between A & B. A's utility function is $U_A = x_A^2 + 4x_A y_A + 4y_A^2$ and B's utility function is $U_B = x_B + y_B$. The contract curve for this exchange economy will be:

A. the entire boundary of the edgeworth box

B. allocations satisfying $(x_A = 0, 0 \leq y_A \leq 2)$ and $(0 \leq x_B \leq 4, y_B = 0)$

C. allocations satisfying $(0 \leq x_A \leq 4, y_A = 0)$ and $(x_B = 0, 0 \leq y_B \leq 2)$

D. all points inside the edgeworth box.

50. Consider the exchange economy in the above question. Suppose A is endowed with 3 units of good 1 and 1 unit of good 2, and B is endowed with 1 unit of each good. A competitive equilibrium is described by the following prices (of goods X and Y respectively) and allocation of goods.

A. Prices = (1,2) and $(x_A, y_A) = (2.5, 2)$, $(x_B, y_B) = (1.5, 0)$

B. Prices = (2,1) and $(x_A, y_A) = (2.5, 1.5)$, $(x_B, y_B) = (1.5, 0.5)$

C. Prices = (1,1) and $(x_A, y_A) = (2, 2)$, $(x_B, y_B) = (2, 0)$

D. None of the four option is correct

Questions 51 to 54. The following set of questions use a common set of information given below. Read the information carefully and then answer the questions sequentially. Consider an economy which is described by the following two relationship between aggregate income (Y), aggregate price level (P), domestic interest rate (r), and the real exchange rate (e):

(i) Goods market equilibrium condition: $Y = C(Y) + I(r) + G + NX(Y, e)$

(ii) Money market equilibrium condition: $\dfrac{M}{P} = L(Y, r)$

where

$$\begin{aligned} C &= \overline{C} + \sigma(Y - T) \; ; 0 < \sigma < 1 \\ T &= \tau Y \; ; 0 < \tau < 1 \\ G &= \overline{G} \\ I &= \overline{I} - \delta r \; ; \delta > 0 \\ NX &= \overline{X} - \mu Y - \gamma e \; ; 0 < \mu < 1 \; ; \gamma > 0 \\ L(Y, r) &= \overline{L} + \alpha Y - \beta r \; ; \alpha, \beta > 0 \\ r &= \overline{r} \\ M &= \overline{M} \\ P &= \overline{P} \end{aligned}$$

Suppose now you draw the IS and the LM relationship in the (Y, e) plane with Y in the horizontal axis and e in the vertical axis.

51. In this case
    A. the IS curve is upward sloping and the LM curve is downward sloping
    B. the IS curve is downward sloping and the LM curve is vertical
    C. the IS curve is vertical and the LM curve is upward sloping
    D. the IS curve is downward sloping and the LM curve is upward sloping

52. An increase in $\overline{r}$ shifts
    A. the IS curve to the left and the LM curve to the right
    B. the IS curve to the right and the LM curve to the left
    C. both the IS and the LM curves to the left
    D. both the IS and the LM curves to the right

53. An increase in $\overline{P}$ results in
    A. an increase in the equilibrium value of Y and a decrease in the equilibrium value of e
    B. a decrease in the equilibrium value of Y and an increase in the equilibrium value of e
    C. a decrease in the equilibrium value of Y and a decrease in the equilibrium value of e
    D. none of the above

54. If the government arbitrarily fixes the real exchange rate at some $\overline{e}$
    A. the two markets can be simultaneously in equilibrium only under special parametric restrictions
    B. the two markets can be simultaneously in equilibrium if the government follows an accommodating interest rate policy
    C. the two markets can be simultaneously in equilibrium if the government follows an accommodating money supply rule
    D. all of the above

55. A fall in the interest rate
    A. will reduce savings unambiguously
    B. will have an ambiguous effect on savings because of an ambiguous substitution effect
    C. will reduce savings unambiguously only for a borrower
    D. will reduce savings unambiguously only for a lender

56. An increase in the rate of depreciation, according to the neoclassical theory of investment, will

    A. lower investment by raising the user cost of capital

    B. raise investment by lowering the user cost

    C. raise investment because now more capital is depreciating

    D. none of the above

57. An increase in the saving rate in the Solow model

    A. increases the growth rate of the economy permanently

    B. increases the growth rate of the economy in the transition to the steady state but not in the steady state

    C. reduces the growth rate because aggregate demand falls

    D. none of the above

58. The "golden rule of accumulation" is the

    A. savings ratio that generates the highest growth rate of the economy

    B. the savings ratio that generates the highest capital-labour ratio

    C. the savings ratio where consumption (per capita) is maximized both in the transition to the steady state and in the steady state

    D. the savings ratio where consumption (per capita) is maximized in the steady state

59. At the "golden rule of accumulation"

    A. all wages invested , all profits consumed

    B. all wages as well as profits invested

    C. all wages consumed, all profits invested

    D. all wages as well as profits consumed

60. Consider the following three definitions for a country's current account surplus. Which of them is correct?

    (i) equal to its trade balance plus net income from abroad

    (ii) equal to its trade balance plus foreign direct investment

    (iii) equal to the change in its claims against the rest of the world

    A. (i) and (ii)

    B. (ii)and (iii)

    C. (i) and (iii)

    D. None

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1. A number, say X_1, is chosen at random from the set $\{1, 2\}$. Then a number, say X_2, is chosen at random from the set $\{1, X_1\}$. The probability that $X_1 = 2$ given that $X_2 = 1$ is

   A. 1

   B. 1/2

   C. 1/3

   D. 1/4

2. Suppose a random variable X takes values -2, 0, 1 and 4 with probabilities 0.4, 0.1, 0.3 and 0.2 respectively.

   A. The unique median of the distribution is 1

   B. The unique median of the distribution is 0

   C. The unique median of the distribution lies between 0 and 1

   D. The distribution has multiple medians

3. Two persons A and B, shoot at a target. Suppose the probability that A will hit the target is 1/3 and the probability that B will hit the target on any shot is 1/4. Suppose A shoots first and takes turns shooting. What is the probability that the target is hit for the first time by $A's$ third shot?

   A. 1/24

   B. 1/12

   C. 1/6

   D. 1/3

4. A fair coin is tossed repeatedly until a head is obtained for the first time. Let X denote the number of tosses that are required. The value of the distribution function of X at 3 is

   A. 3/4

   B. 1/2

   C. 7/8

   D. 7/16

5. Using a random sample, an ordinary least squares regression of Y on X yields the 95% confidence interval $0.43 < \beta < 0.59$ for the slope parameter β. Which of the following statements is false?

   A. This interval contains the parameter β with probability 0.95

   B. The point estimate of β obtained from our regression always lies within this interval

C. The 90% confidence interval for β is a subset of the interval obtained above

D. If such intervals are constructed from repeated samples drawn from the population in question, then on average 95 out of 100 of these intervals are likely to contain the true parameter value

6. Consider a binary relation $\succeq$ defined on the set $A = \{x, y, z\}$. Define relations $\succ$ and $\sim$ on A by: for $a, b \in A$,

$$a \succ b \text{ if and only if } a \succeq b \text{ and not } b \succeq a$$
$$a \sim b \text{ if and only if } a \succeq b \text{ and } b \succeq a$$

Suppose $x \succ y,\ y \sim z$ and $x \sim z$. Then

A. $\succ$ is transitive

B. $\sim$ is transitive

C. both $\succ$ and $\sim$ are transitive

D. we cannot conclude anything about the transitivity of $\succ$ and $\sim$

7. The utility function $u(x, y) = (x + y)^{\frac{1}{2}}$, for $(x, y) = (0, 0)$, exihibits

A. diminishing marginal rate of substitution and diminishing marginal utilities

B. increasing marginal rate of substitution and diminishing marginal utilities

C. constant marginal rate of substitution and diminishing marginal utilities

D. increasing marginal rate of substitution and constant marginal utilities

8. Suppose there are just two goods, say x_1 and x_2. Consider a consumer who chooses $x_2 = 0$ for all income levels $w > 0$ and all prices $p_1 > 0$ and $p_2 > 0$. These choices are consistent with the consumer

A. having utility function $u(x_1, x_2) = x_1 + 2x_2$

B. having utility function $u(x_1, x_2) = 2x_1 + x_2$

C. lexicographically preferring x_2 to x_1

D. lexicographically preferring x_1 to x_2

9. Consider a Cournot duopoly with firms 1 and 2 that produces a homogeneous good. The inverse demand curve for this good is given by $P(x) = 5 - x$, where x is the total output of the two firms. Firm 1 has a constant average cost 5/2 and firm 2 has a constant average cost 3/2. In equilibirum,

A. only firm 1 produces a positive output

B. only firm 2 produces a positive output

C. both firms produces positive outputs with firm 1 producing more than firm 2

D. both firms produces positive outputs with firm 2 producing more than firm 1

10. If all input prices double, then what happens to the minimum cost of producing a given output?
    A. It doubles
    B. It more than doubles
    C. It less than doubles
    D. It depends on the production function

11. In models where expectations adjust slowly, a progressive income tax schedule is
    A. a means to maximize tax revenue
    B. a means to ensure tax compliance
    C. always an automatic stabilizer in the long run
    D. often an automatic stabilizer in the short run

12. Open market operations decrease the supply of base money by
    A. selling government bonds
    B. selling gold
    C. reducing foreign currency holdings
    D. all of the above

13. If an economy is experiencing hyperinflation, then
    A. government seigniorage goes up
    B. government seigniorage goes down
    C. government seigniorage remains unchanged
    D. the impact on government seigniorage is ambiguous

14. When the nominal interest rate changes but the real rate of interest remains unchanged
    A. it affects both the investment function as well the money demand function
    B. it affects the investment function but does not affect the money demand function
    C. it affects the money demand function but does not affect the investment function
    D. neither the investment function not the money demand function gets affected

15. *Ceteris paribus*, higher velocity of money circulation leads to
    A. an increase in both the real and the nominal demand for money
    B. an increase in the real demand for money and a decrease in the nominal demand for money
    C. a decrease in the real demand for money and an increase in the nominal demand for money

D. a decrease in both the real and the nominal demand for money

16. Consider a 3×3 nonsingular matrix A with real entries. If the matrix B is derived from A by interchanging the first and last columns of A, then the determinant of B, denoted det B, is equal to

    A. det A

    B. $-$ det A

    C. 0

    D. 1/det A

17. The sequence $(-1)^n(1 + 1/n)$, for positive integers n,

    A. has limit point 1

    B. has limit point -1

    C. has limit points 1 and -1

    D. has no limit points

18. Consider the following two games in strategic form.

		Player Y
		Hawk
Player X	Enter	$(-1, 1)$
	Not enter	$(0, 6)$

		Player Y	
		Hawk	Dove
Player X	Enter	$(-1, 1)$	$(3, 3)$
	Not enter	$(0, 6)$	$(0, 7)$

    In every payoff "x, y", x is the payoff of the row-player and y is the payoff of column-player. Analyze these games. These games illustrates that, in a strategic situation

    A. an expanded set of strategic options can be disadvantageous

    B. a contracted set of strategic options can be advantageous

    C. one should view the situation from one's own, as well as from one's opponent's, perspective

    D. all of the above are true

19. Ice-cream vendors A and B know that they have to locate simultaneously on a beach. The beach is identified with the interval $[0, 1]$ and at every point in $[0, 1]$ there is a person who wants exactly one ice-cream cone. Each person will buy the ice-cream from the nearest vendor; if there are equidistant vendors, then the buyer randomizes among them with equal probabilities. Each vendor wants to maximize his own expected market share. The vendors will locate at

    A. 0 and 1

    B. 1/4 and 3/4

C. 1/2 and 1/2

D. 1/3 and 2/3

20. Suppose there is only one future period and the (presently unknown) state of the world in that period can be either s_1 or s_2. The future return on a share of a given company is 5 in state s_1 and -1 in state s_2. The future return on a government bond is 1 independent of the state. Suppose a third asset is offered on the market whose return is 3 in state s_1 and 0 in state s_2. The current prices of the stock and the bond are 3 and 1 respectively. If the price of the new asset rules out the possibility of arbitrage profit (which arises when portfolio of assets that are identical in terms of returns have different prices), what is the price of the new asset?

    A. It depends on the probabilities of the future states

    B. Strictly between 2 and 3

    C. Strictly between 1 and 2

    D. 2

    The following notational conventions apply wherever the following symbols are used. $\Re$ denotes the set of real numbers. Given a function f, $Df(x)$ and $D^2f(x)$ denote the first and second derivatives of f (if they exist), respectively, evaluated at x.

21. Suppose X_1 and X_2 are real valued random variables with f as their common probability density function. Suppose (x_1, x_2) is a sample generated by these random variables. The expectation of observations in the sample that fall within a specified interval $[a, b]$ is

    A. $\left(\int_a^b f(x)dx\right)^2$

    B. $\int_a^b x^2 f(x)dx$

    C. $2\int_a^b f(x)dx$

    D. $\int_a^b xf(x)dx$

22. Suppose $X_1....X_n$ are observed completion times of an experiment with values in $[0, 1]$. Each of these variables is uniformly distributed on $[0, 1]$. If Y is the maximum observed completion time, then mean of Y is

    A. $\left(\frac{n}{n+1}\right)^2$

    B. $\frac{n}{2(n+1)}$

    C. $\frac{n}{n+1}$

    D. $\frac{2n}{n+1}$

23. Suppose the random variable X takes values in the set $\{-1, 0, 1\}$ and the probability of each value is equal. Let $Y = X^2$.Which of the following statements is true?

    A. X and Y are correlated but independent

    B. X and Y are uncorrelated but dependent

C. X and Y are dependent and have same mean

D. X and Y are correlated and have different means

24. Suppose player 1 has five coins and player 2 has four coins. Both players toss all their coins and observe the number that comes up heads. Assuming all the coins are fair, what is the probability that player 1 obtains more heads than player 2?

A. $\frac{1}{2}$

B. $\frac{4}{9}$

C. $\frac{5}{9}$

D. $\frac{4}{5}$

25. Suppose 10 athletes are running in a race and exactly 2 of them are taking banned drugs. An investigator randomly selects 2 athletes for drug testing. What is the opportunity that neither of the cheaters will be caught?

A. $\frac{16}{25}$

B. $\frac{4}{5}$

C. $\frac{3}{5}$

D. $\frac{28}{45}$

26. Suppose θ is a random variable with uniform distribution on the interval $[-\pi/2, \pi/2]$. The value of the distribution function of the random variable $X = \sin\theta$ at $x \in [-1, 1]$ is

A. $\sin^{-1}(x)$

B. $\sin^{-1}(x) + \frac{\pi}{2}$

C. $\frac{\sin^{-1}(x)}{\pi} + \frac{1}{2}$

D. $\frac{\sin^{-1}(x)}{\pi} + \frac{\pi}{2}$

27. Let X be a normally distributed random variable with mean 0 and variance σ^2.Then, the mean of X^2 is

A. 0

B. σ

C. 2σ

D. σ^2

The next three questions are based on the following data:
The number of loaves of bread sold by a bakery in a day is a random variable X.The distribution of X has a probability density function f given by

$$f(x) = \begin{cases} kx, & \text{if } x \in [0, 5) \\ k(10 - x), & \text{if } x \in [5, 10) \\ 0, & \text{if } x \in [10, \infty) \end{cases}$$

28. As f is a probability density function, the value of k must be

    A. 0

    B. $\frac{-2}{25}$

    C. $\frac{1}{25}$

    D. $\frac{2}{75}$

29. Let A be the event that $X \geq 5$ and let B be the event that $X \in [3, 8]$. The probability of A conditional on B is

    A. $\frac{16}{37}$

    B. $\frac{21}{37}$

    C. $\frac{25}{37}$

    D. 1

30. Events A and B are

    A. not independent

    B. independent

    C. conditionally independent

    D. unconditionally independent

The next four questions are based on the following data:
Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1's endowment is $(0, 1)$ (i.e. no good x and 1 unit of good y) and agent 2's endowment is $(2, 0)$ (i.e. 2 units of good x and no good y).The agents can consume only nonnegative amounts of x and y.

31. Suppose agent 1 lexicographically prefers x to y, i.e., between any two bundles of goods, she strictly prefers the bundle containing more of x, and if the bundles contain equal amounts of x, then she strictly prefers the bundle with more of y. Suppose agent 2 treats x and y as perfect substitutes, i.e., between any two bundles (x, y) and (x', y') she strictly prefers (x, y) if and only if $x + y > x' + y'$.
    The competitive equilibrium allocation for this economy is

    A. 1 gets $(0, 1)$ and 2 gets $(2, 0)$

    B. 1 gets $(2, 0)$ and 2 gets $(0, 1)$

    C. 1 gets $(\frac{3}{2}, 0)$ and 2 gets $(\frac{1}{2}, 1)$

    D. 1 gets $(1, 0)$ and 2 gets $(1, 1)$

32. Suppose agents 1 and 2 have the preferences described above. The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

    A. $p_x/p_y = 1$

    B. $p_x/p_y \geq 1$

C. $p_x/p_y \leq 1$

D. $p_x/p_y > 1$

33. Now Suppose agents 1 lexicographically prefers y to x and agent 2 treats x and y as perfect substitutes.The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

A. $p_x/p_y = 1$

B. $p_x/p_y \geq 1$

C. $p_x/p_y \leq 1$

D. $p_x/p_y > 1$

34. Now Suppose agents 2 lexicographically prefers x to y and agent 1 treats x and y as perfect complements. The set of competitive equilibrium allocations

A. includes the allocation $(1, 0)$ for agent 1 and $(1, 1)$ for agent 2

B. includes the allocation $(0, 1)$ for agent 1 and $(2, 0)$ for agent 2

C. is empty

D. includes all allocations $(x, 1)$ for agent 1 and $(2 - x, 0)$ for agent 2, where $x \in [0, 2]$

35. Consider a person who chooses among lotteries. Each lottery is of the form (p_1, p_2, p_3), where p_1 is the probability of getting Rs.5, p_2 is the probability of getting Rs.1 and p_3 is the probability of getting Rs.0. This person prefers lottery $(0, 1, 0)$ to $(0.1, 0.89, 0.01)$. If this person maximizes expected utility and is faced with the lotteries $(0, 0.11, 0.89)$ and $(0.1, 0, 0.9)$, which lottery should he prefer?

A. the lottery $(0, 0.11, 0.89)$

B. the lottery $(0.1, 0, 0.9)$

C. he should be indifferent between these lotteries

D. there is insufficient data to decide

36. Consider an economy with two agents, A and B, and two goods x_1 and x_2. Both agents treat x_1 and x_2 as perfect complements. Suppose the total endowment of x_1 is 4 and total endowment of x_2 is 2. Which of the following allocations is not Pareto optimal? (Note that a bundle (a, b) represents a units of x_1 and b units of x_2.)

A. A gets (1, 1) and B gets (1, 1)

B. A gets (2, 1) and B gets ($\frac{3}{2}$, 1)

C. A gets ($\frac{1}{2}$, $\frac{3}{2}$) and B gets (3, $\frac{1}{2}$)

D. A gets (3, 2) and B gets (0, 0)

37. A consumer has the utility function $u(x, y) = xy$. Suppose the consumer demands bundle (x^*, y^*). Now suppose the seller of good x offers a "buy one, get one free" scheme: for each unit of good x purchased, the consumer gets another unit of x for free. Given this

scheme, suppose the consumer buys bundle (x_d, y_d) and gets an additional x_d for free. Which of the following statements must be true?

A. $x_d > x^*$ and $y_d > y^*$

B. $x_d > x^*$ and $y_d = y^*$

C. $x_d > x^*$ and $y_d < y^*$

D. $x_d = x^*$ and $y_d = y^*$

38. Consider a Bertrand duopoly with firms 1 and 2 that produce a homogenous good and sets prices p_1 and p_2 respectively. Suppose p_1 and p_2 have to be positive integers. If $p_1 < p_2$ (resp. $p_1 > p_2$), then firm 1(resp. firm 2) sells $5 - p_1$ (resp. $5 - p_2$) and the other firm sells nothing. If $p_1 = p_2$, then each firm sells $(5 - p_1)/2$. Firm 1 has a constant average cost 5/2 and firm 2 has a constant average cost 3/2. In equilibrium

A. $p_1 = 2 = p_2$

B. $p_1 = 3 = p_2$

C. $p_1 = 3$ and $p_2 = 2$

D. $p_1 = 3$ and p_2 is 2 or 3

39. Consider a Stackelberg duopoly with firm 1 as the leader and firm 2 as the follower. If (q_1, q_2) is the Stackelberg equilibrium, then

A. firm 1's optimal isoprofit curve and firm 2's reaction curve intersect at (q_1, q_2) and are tangential at (q_1, q_2)

B. firm 2's optimal isoprofit curve and firm 1's reaction curve intersect at (q_1, q_2) and are tangential at (q_1, q_2)

C. isoprofit curves of the two firms intersect at (q_1, q_2) and are tangential at (q_1, q_2)

D. reaction curves of the two firms intersect at (q_1, q_2)

40. Firm 1 is the potential entrant into a market in which firm 2 is the incumbent monopolist. Firm 1 moves first and chooses to "enter" or "not enter". If it does "not enter", then firm 1 gets profit 0 and firm 2 gets the monopoly profit 10. If firm 1 "enters", then firm 2 chooses to "fight" or "not fight". If firm 2 fights, then firm 1's profit is -2 and firm 2's profit is 6. If firm 2 does "not fight", then firm 1's profit is 2 and firm 2's profit is 8. Firm 2's strategy of "fight" is best response as

A. a commitment

B. a non-credible threat

C. a punitive action

D. acquiescence

41. Consider a closed economy. If the nominal wage is flexible and nominal money supply is increased, then which of the following will be true in equilibrium?

A. Real wage decreases and real money supply decreases

B. Real wage decreases and real money supply increases

C. Real wage is unchanged and real money supply is unchanged

D. Real wage decreases and real money supply is unchanged

42. Suppose an economy is at less than full employment and it consists of an aggregate "worker" and aggregate "capitalist", with the former having a higher marginal propensity to consume from his disposable income. Suppose both agents pay income tax according to the same linear schedule. If the government's budget is in balance and a lump sum income transfer is made from the capitalist to the workers, then the government's

A. budget will go into deficit

B. budget will go into surplus

C. income and expenditure will be unchanged

D. income and expenditure will change but the budget will stay in balance

The next four questions are based on the following information.
Consider an economy with an aggregate production function $Y = \alpha K + \beta L$, where α and β are positive constants, K is capital, L is labor and Y is output. K is **fixed** in the short run. Perfectly competitive producers take the nominal wage rate W and the price level P as given, and employ labor so as to maximize profit. This generates the labor demand schedule. The labor supply schedule is $L^S = -\gamma + \delta W/P$, where γ and δ are positive constants. Producers and workers have perfect information about P and W

43. The labour market will clear if and only if

A. $\beta > \gamma/\delta$

B. $\beta < \gamma/\delta$

C. $\beta > \delta/\gamma$

D. $\beta < \delta/\gamma$

44. Assume that required parametric condition of the previous question holds and the nominal wage is fixed. The short run aggregate supply curve schedule for this economy, with P along the vertical axis and Y along horizontal axis, will look as follows:

A. for high values of P it will be horizontal; for some mid-range values of P it will be downward sloping; for low values of P it will be horizontal again

B. for high values of P it will be horizontal; for some mid-range values of P it will be upward sloping; for low values of P it will be horizontal again

C. for high values of P it will be vertical; for some mid-range values of P it will be downward sloping; for low values of P it will be vertical again

D. for high values of P it will be vertical; for some mid-range values of P it will be upward sloping; for low values of P it will be vertical again

45. If there is a one shot increase in the fixed stock of the capital stock, then the short run aggregate supply schedule will

A. shift up

B. shift down

C. shift to the left

D. shift to the right

46. If there is a one shot increase in the fixed nominal wage rate, then the short run aggregate supply schedule will

A. shift up

B. shift down

C. shift to the left

D. shift to the right

The next four questions are based on the following information: Consider a closed economy with a simple Keynesian model of goods market, where prices are fixed and output in equilibrium is determined by aggregate demand. Investment is fixed at I^*. There is no government sector. Suppose there are two groups of households, called A and B, and the total income Y is distributed among these two groups in such a way that group A gets $Y^A = \lambda Y$, and group B gets $Y^B = (1-\lambda)Y$, where $\lambda \in (0,1)$ is a constant. The consumption function of group A is $C^A = c + c_A Y^A$ and the consumption function of group B is $C^B = c + c_B Y^B$, where $0 < c_A < c_B < 1$, i.e., the two groups have different consumption propensities.

47. The value of the investment multiplier in this economy is given by

A. $\frac{1}{1-\lambda c_A - \lambda c_B}$

B. $\frac{1}{1-(1-\lambda)c_A - (1-\lambda)c_B}$

C. $\frac{1}{1-c_B+\lambda(c_B-c_A)}$

D. $\frac{1}{1-c_A+(1-\lambda)(c_B-c_A)}$

48. If there is a one shot increase in the parameter λ,

A. equilibrium output unambiguously increases

B. equilibrium output unambiguously decreases

C. equilibrium output remains unchanged

D. equilibrium output increases or decreases depending on whether $\lambda \geq 1/2$

49. If there is a one shot increase in the parameter c_A,

A. equilibrium output unambiguously increases

B. equilibrium output unambiguously decreases

C. equilibrium output remains unchanged

D. equilibrium output increases or decreases depending on whether $\lambda \geq 1/2$

50. If there is a one shot increase in the parameter c_B,

    A. equilibrium output unambiguously increases

    B. equilibrium output unambiguously decreases

    C. equilibrium output remains unchanged

    D. equilibrium output increases or decreases depending on whether $\lambda \geq 1/2$

51. Suppose the function $f : \Re \to \Re$ is given by $f(x) = x^3 - 3x + b$. Find the number of points in the closed interval $[-1, 1]$ at which $f(x) = 0$.

    A. None

    B. At most one

    C. One

    D. At least one

52. Suppose a function $f : \Re \to \Re$ is differentiable at x. Consider the statements:

$$(i)\ Df(x) = \lim_{h \to 0} \frac{f(x) - f(x-h)}{h}$$
$$(ii)\ Df(x) = \lim_{h \to 0} \frac{f(x+2h) - f(x+h)}{2h}$$

    In general,

    A. (i) is true and (ii) is false

    B. (i) is false and (ii) is true

    C. Both are true

    D. Both are false

53. Consider the statements: for $x, y \in \Re$. Consider the statements:

$$(i)\ |x| - |y| \leq |x - y|$$
$$(ii)\ ||x| - |y|| = |x - y|$$

    In general,

    A. (i) is true and (ii) is false

    B. (i) is false and (ii) is true

    C. Both are true

    D. Both are false

54. Suppose the function $f : \Re^2 \to \Re$ is increasing in both arguments, i.e, $f(x, y)$ is increasing in x and increasing in y. For $x, y \in \Re$, let

$$x \wedge y = \begin{cases} x, & \text{if } x \leq y \\ y, & \text{if } x > y \end{cases}$$

Define $g : \Re^2 \to \Re$ by

$$g(x, y) = \begin{cases} f(x, y) - \frac{1}{2} f(x \wedge y, x \wedge y), & \text{if } x \geq y \\ \frac{1}{2} f(x \wedge y, x \wedge y), & \text{if } x < y \end{cases}$$

Which of the following statements is correct?

A. g is increasing in x and decreasing in y

B. g is increasing in both x and y

C. g is increasing in x but may or may not be increasing in y

D. g may or may not be increasing in x

The next three questions are based on the following definitions.
Consider the set $A = \{(x, y) \in \Re^2 \mid x^2 + y^2 \leq 1\}$. Given $(a, b) \in \Re^2$ such that $(a, b) \neq (0, 0)$ and $c > 1$, let

$$X = \{(x, y) + (a, b) \mid (x, y) \in A\}$$

$$Y = \left\{(2^{-1/2}(x + y), 2^{-1/2}(x - y)) \;\middle|\; (x, y) \in A\right\}$$

$$Z = \{c(x, y) \mid (x, y) \in A\}$$

55. Which set is **not** a disc (i.e., the region inside a circle)?

A. X

B. Y

C. Z

D. None

56. Which set has a larger area than A?

A. X

B. Y

C. Z

D. None

57. Which set does not contain all the points that belong to A?

A. X

B. Y

C. Z

D. None

58. Consider a twice differentiable function $f : \Re \to \Re$ and $a, b \in \Re$ such that $a < b$, $f(a) = 0 = f(b)$ and $D^2 f(x) + Df(x) - 1 = 0$ for every $x \in [a, b]$. Then,

A. f has a maximum but not a minimum over the open interval (a, b)

B. f has a minimum but not a maximum over (a, b)

C. f has neither a maximum not a minimum over (a, b)

D. f has a maximum and a minimum over (a, b)

59. Consider f as described in the previous question. Then,

A. $f(x) \leq 0$ for every $x \in [a, b]$

B. $f(x) \geq 0$ for every $x \in [a, b]$

C. $f(x) = 0$ for every $x \in [a, b]$

D. f must take positive and negative values on the interval $[a, b]$

60. Suppose $f : [0, 1] \to [0, 1]$ is a continuous nondecreasing function with $f(0) = 0$ and $f(1) = 1$. Define $g : [0, 1] \to [0, 1]$ by $g(y) = \min\{x \in [0, 1] \mid f(x) \geq y\}$. Then,

A. g is non-decreasing

B. If g is continuous, then f is strictly increasing

C. Neither (a) not (b) is true

D. Both (a) and (b) are true

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1. For the real-valued function $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$, (defined for all real numbers x), the point $x = 1$ is

   A. a local minimum

   B. a local maximum

   C. a point of inflection

   D. none of the above

2. Consider the function f mapping points of the plane into the plane, defined by $f(x, y) = (x - y, x + y)$. The range of this function is

   A. the 45 degree line

   B. a ray through the origin but not the 45 degree line

   C. the entire plane

   D. the first and third quadrants

3. Suppose $\{v_1, v_2, \cdots, v_n\}$ is a set of linearly dependent vectors , none of them being the zero vector. Suppose $c_1, c_2, \cdots, c_n$ are scalars, not all zero, such that $\sum_{i=1}^{n} c_i v_i = 0$. Then the minimum number of non-zero scalars is

   A. 1

   B. 2

   C. $n-1$

   D. cannot be determined

4. Suppose the non-zero $n \times 1$ column vector x solves the system of equations $Ax = b$, where A is an $m \times n$ matrix whose columns are the vectors $a_1, a_2, \cdots, a_n$ and b is an $m \times 1$ column vector. Then the set of vectors $\{a_1, a_2, \cdots, a_n, b\}$ is

   A. linearly independent

   B. linearly dependent

   C. linearly dependent only if $a_1, a_2, \cdots, a_n$ are linearly dependent

   D. linearly dependent only if $m = n$

5. For a system of linear equations $Ax = b$ with m equations and n variables, where $m > n$ and b is a given vector, the following is true.

   A. It can never have a unique solution

   B. It always has at least one solution

   C. It has at least a one-dimensional solution space

   D. If $Rank(A) = n$ and a solution exists, it must be unique

6. Suppose we are interested in estimating the mean height of M.A. students in Delhi University. An avergae height estimated from a random sample of size 30 is better than estimated from a random sample of size 20 because:
   A. The sample of size 20 is likely to be more biased because it is less representative
   B. The sample of size 30 is likely to yield more precise estimates of average height than the sample of size 20
   C. The Central Limit Theorem prescribes a minimum sample size of 30
   D. Both (a) and (b)

7. A doctor testing a diagnostic tool for a rare disease wants to minimize the chance that the test will find a patient to be healthy when she is in fact sick (the null hypothesis being that the patient is healthy). The doctor should minimize the probability of:
   A. Type I error, which would denote a false positive
   B. Type II error, which would denote a false positive
   C. Type I error, which would denote a false negative
   D. Type II error, which would denote a false negative

8. Suppose you have the following estimated regression equation: ln Y = 1.2 + 0.5X. Which of the following is a good interpretation of the estimated relationship between X and Y?
   A. A unit change in X is associated with a 50 percent change in Y
   B. A unit change in X is associated with a 0.5 percent change in Y
   C. A one percent change in X is associated with a 50 unit change in Y
   D. A one percent change in X is associated with a 0.5 unit change in Y

9. Let A_1, A_2, A_3 be independent events with probabilities p_1, p_2, p_3 respectively. The probability that none of these events occurs equals
   A. $1 - (p_1 + p_2 + p_3)$
   B. $1 - (p_1 + p_2 + p_3) + p_1p_2 + p_1p_3 + p_2p_3$
   C. $(1 - p_1)(1 - p_2)(1 - p_3)$
   D. $(1 - p_2)(1 + p_1p_3)$

10. The market for good X has the demand function $D(p) = 100 - 15p$ where p is the price of good X. There are ten price taking firms , each having a cost function $c(q) = q^2$, where q is the firm's own output. There is no new entry.
    A. Rs. 20
    B. Rs. 15
    C. Rs. 10
    D. Rs. 5

11. The market for good X has the demand function $D(p) = 100 - p$ and the supply function $S(p) = 20 + 3p$ where p is the price of good X. If the government imposes a sales tax of Rs. 10 *per* unit of the good on sellers, the equilibrium market price will increase by

    A. Rs. 10

    B. Rs. 7.5

    C. Rs. 5

    D. Rs. 0

12. A monopolist, who can produce output at a cost of Rs. 20 *per* unit faces an inverse demand curve given by $p = 60 - 2q$. The profit maximizing price for the monopolist is

    A. Rs. 40

    B. Rs. 20

    C. Rs. 30

    D. Rs. 60

13. Consider a duopoly market in which both firms choose quantities. Suppose we have the reaction curve of each firm, i.e , the curve that yields the firm's optimal quantity choice in response to a quantity choice in response to a quantity chosen by the other firm. If one firm is the Stackelberg leader and the other is the Stackelberg follower, then which of the following conditions characterizes the quantity chosen by the leader?

    A. The quantity at which the leader's isoprofit curve is tangential to the follower's reaction curve

    B. The quantity at which the follower's isoprofit curve is tangential to the leader's reaction curve

    C. The quantity where the leader's isoprofit curve attains a maximum

    D. The quantity where the two reaction curves intersect

14. A society has 3 individuals and 3 alternatives A, B and C. Individuals 1 and 2 strictly prefer A to B and B to C. Individual 3 strictly prefers C to B and B to A. A Rawlsian social planner would therefore choose

    A. A

    B. B

    C. C

    D. A or C

15. A natural monopoly is said to occur when

    A. the long run average cost curve is U-shaped

    B. the long run average cost curve is decreasing

    C. the long run average cost curve is flat

    D. the long run average cost curve is increasing

16. If everybody in the economy decides to consume a higher proportion of their income, the corresponding IS curve
    A. will be flatter
    B. will be steeper
    C. will remain unchanged
    D. will become horizontal

17. An increase in the expected rate of inflation will
    A. shift the LM curve up
    B. shift the LM curve down
    C. leave the LM curve unchanged
    D. have an ambiguous effect on the LM curve

18. A devaluation of domestic currency will improve the country's trade balance only if the following is true for price elasticities of demand:
    A. the sum of price elasticities of exports and imports equals 1
    B. the sum of price elasticities of exports and imports is less than 1
    C. the sum of price elasticities of exports and imports is greater than 1
    D. the price elasticity of exports exactly matches the price elasticity of imports

19. Under nominal wage rigidity, the short run aggregate supply schedule will be
    A. vertical
    B. horizontal
    C. upward sloping
    D. downward sloping

20. Okun's Law refers to
    A. a negative relationship between inflation and unemployment
    B. a negative relationship between unemployment and real GDP
    C. a negative relationship between interest rate and money demand
    D. a negative relationship between inflation and growth

    The following notational conventions apply wherever the following symbols are used. *$\Re$ denotes the set of real numbers. $\Re_{++}$ denotes the set of positive real numbers. $\mathcal{Q}$ denotes the set of rational numbers.*

21. Suppose the function $f : \Re_{++} \to \Re$ is given by $f(x) = \int_1^x t^{-1}dt$. Consider the following statements: for $x, y \in \Re_{++}$,

$$f(x+y) = f(x) + f(y)$$
$$f(xy) = f(x) + f(y)$$

In general,

A. (i) is true and (ii) is false

B. (i) is false and (ii) is true

C. Both are true

D. Both are false

22. Suppose a real valued function f is defined for all real numbers excepting 0, and satisfies the following conditions: $f(xy) = f(x) + f(y)$ for all x, y in the domain. Consider the statements:

$$f(1) = f(-1) = 0$$
$$f(x) = f(-x) \text{ for every } x$$

A. (i) is true and (ii) is false

B. (i) is false and (ii) is true

C. Both are true

D. Both are false

23. The closest point on the parabola $y = \frac{1}{4}x^2$ from a given point $(0, b)$ on the vertical axis, with $b > 0$, is the origin if and only if

A. $b < 3$

B. $b > 3$

C. $b < 2$

D. $b > 2$

24. Let f be a real valued differentiable function defined for all $x \geq a$. Consider the function F defined by $F(x) = \int_a^x f(t)dt$. If f is increasing on any interval, then on that interval F is

A. convex

B. concave

C. increasing

D. decreasing

25. Consider the function $f : \Re \to \Re$ defined by

$$f(x) = \begin{cases} x^2, & \text{if } x \in \mathcal{Q} \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{Q}$ is the set of rational numbers. Then, f is

A. discontinuous at every $x \neq 0$

B. is continuous at all points in $\mathcal{Q}$

C. continuous at multiple points

D. is discontinuous on a countable set of points

26. Suppose A and B are square matrices that satisfy

$$AB + BA = 0$$

where 0 is a square matrix of zeros. Then it must be that

A. $A^2B^3 = B^3A^2$

B. $A^2B^3 = B^2A^3$

C. $A^2B^3 = BA^4$

D. None of the above is necessarily true

Consider a Society consisting of individuals. These individuals may belong to various sets called Clubs and Tribes. The collections of Clubs and Tribes satisfy the following rules:

- *The entire Society is a Tribe*
- *The empty subset of Society is also a Tribe*
- *Given any collection of Tribes, the set of individuals who belong to each Tribe in that collection is also a Tribe*
- *Given any two Tribes, the set of individuals who belong to at least one of these Tribes is also a Tribe*
- *A set of individuals is called a Club if and only if the set of individuals not in it constitute a Tribe*

27. The intersection of two Clubs is necessarily

A. a Club

B. a Tribe

C. not a Club

D. not a Tribe

28. The union of a collection of Clubs is necessarily

A. not a Club

B. not a Tribe

C. a Club

D. a Tribe

29. Which of the following statements is necessarily true?

A. A set of individuals cannot be a Tribe and a Club

B. There are at least two sets of individuals that are both a Club and a Tribe

C. The union of a Club and a Tribe is a Tribe

D. The intersection of a Club and a Tribe is a Club

30. Suppose we are given a Club and a Tribe. Then, the set of individuals who belong to the given Tribe but not to the given Club necessarily constitute

A. a Club

B. a Tribe

C. neither a Club, not a Tribe

D. a Club and a Tribe

The next two questions are based on the following information. *Suppose X and Y are two random variables. X can take values -1 and 1. Y can take integer values between 1 and 6. The following is the joint probability distribution of X and Y.*

	$Y=1$	$Y=2$	$Y=3$	$Y=4$	$Y=5$	$Y=6$
$X=-1$	0.1	a	0.3	0	0	0
$X=1$	0	0	b	0.1	0.1	0.1

It is known that the expectations of the two random variables are $E(X) = -0.2$ and $E(Y) = 3.2$. Then

31. The value 'a' in the table equals

A. 0

B. 0.1

C. 0.2

D. 0.3

32. The value 'b' in the table equals

A. 0

B. 0.1

C. 0.2

D. 0.3

The next two questions are based on the following data. *Suppose X and Y are independent random variables that follow the uniform distribution on the [0,1] . Let $Z = min\{X, Y\}$.*

33. $\Pr(Z < -.5)$ (the probability that Z is less than 0.5) equals

A. 0.25

B. 0.5

C. 0.625

D. 0.75

34. $\Pr(Z < 0.5|X = 0.75)$ is
    A. 0.25
    B. 0.5
    C. 0.625
    D. undefined

35. Two players, A and B, will play a best of seven table tennis match (i.e. , the first to win 4 games will win the match, and the match will have at most 7 games).The two players are equally likely to win any of the games in the match. The probability that match will end in 6 games is
    A. less than the probability that it will end in 7 games
    B. equal to the probability that it will end in 6 games
    C. greater than the probability that it will end in 7 games
    D. None of (a), (b) or (c) is true

36. In order to join the 'Gamers Club', Mr. A must choose a box from the two identical boxes in a room, and draw one ball from the chosen box. All he knows is that both boxes are nonempty, and have a mix of red and green balls. If the ball that he draws from his chosen box is green, he is admitted to the club. You are given 2 identical boxes, 50 red balls and 50 green balls, and asked to allocate these balls to the two boxes in order to maximize Mr. A's probability of being admitted to the club, given that he will choose a box randomly. If you allocate these balls correctly, the probability that Mr. A gets admitted to the club equals
    A. 1
    B. 0.747
    C. 0.547
    D. 0.257

37. Consider the following equation to be estimated using ordinary least squares:

$$Y = a_0 + a_1 x_1 + a_2 X_2 + a_3(X_2 - X_1) + a_4 X_1 X_2 + u$$

    Which of the following statements is true?
    A. The parameters a_0, a_1, a_2, a_4 can be estimated, but not a_3
    B. The parameters a_0 and a_4 can be estimated, but not a_1, a_2, a_3
    C. The parameters a_0, a_1, a_2, a_3 can be estimated, but not a_4
    D. All parameters can be estimated

38. A researcher wants to test whether there are gender differences in the rates of immunization for boys and girls, after controlling for parental income, mother's education and access to health care facilities. The easiest way to test this would be to:

A. Include one dummy variable in the multiple regression

B. Include two dummy variables in the multiple regression

C. Include interaction dummies in the multiple regression

D. Run two separate regressions-one for boys and one for girls

39. Instead of estimating a production function $y = a_0 + a_1K + a_2L + u$, where y is output, K is capital input and L is labour input, a researcher estimates $y = a_0 + a_1K + u$. It is known that in the true model, $a_2 > 0$ and K and L are complements. Which of the following is true?

A. The estimated a_1 will be upwardly biased

B. The estimated a_1 will have a downward bias

C. The estimated a_1 will be biased, but the direction of bias cannot be determined

D. The estimated a_1 will be unbiased, but a_0 will be biased

40. A student has the opportunity to take a test at most thrice. The student knows that each time he takes the test, his score is an independent random draw from the uniform distribution on the interval [0,100]. After learning his score on a test, the student can either stop and accept it as his official score, or he can discard the result and retake the test. If the student rejects his score twice and takes the test a third time, that score will be his official score. If his objective is to maximize his expected official score, the student will decide to be retested after the very first test if and only if his score is less than

A. 50

B. 62.5

C. 75

D. 87.5

41. A consumer has utility function $u(x_1, x_2) = min\{2x_1 + x_2, x_1 + 2x_2\}$. Her income is $y = 100$, the prices are $p_1 = 20$ and $p_2 = 30$. The amount of x_1 in the utility maximizing bundle is

A. 7

B. 5

C. 2

D. 0

42. Consider the same utility function and income as above, but suppose the prices are $p_1 = 10$ and $p_2 = 30$. Then the amount of x_1 in the utility maximizing bundle is

A. 10

B. 5

C. 2.5

D. 0

43. A consumer spends Rs. 100 on only two goods, A and B. Assume non satiation, i.e. , more of any good is preferred to less. Suppose the price of B is fixed at Rs. 20. When the price of A is Rs. 10 , the consumer buys 3 units of B. When the price of A is Rs. 20, she buys 5 units of A. From this we can conclude that for the relevant price range

    A. A is an inferior good

    B. B is a complement of A

    C. A is a Giffen good

    D. All of the above

44. Consider a firm using two inputs to produce its output. It is known that greater use of both inputs increases output. Moreover, for any combination of positive input prices, the firm employs and input combination of the form $(x, \alpha x)$ where $\alpha > 0$ is a constant. Which of the following functions represents this firm's technology?

    A. $f(x, y) = min\{x^{\alpha}, y\}$

    B. $f(x, y) = min\{\alpha x, y\}$

    C. $f(x, y) = min\{x, \alpha y\}$

    D. $f(x, y) = min\{x, y^{\alpha}\}$

45. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. There are 6 units of X and 4 units of Y available. An allocation is denoted by (x_1, y_1), (x_2, y_2), where (x_1, y_1) is agent 1's consumption bundle, (x_2, y_2) is agent 2's consumption bundle, $x_1 + x_2 = 6$ and $y_1 + y_2 = 4$. Agent 1 has the utility function $u_1 = min\{x_1, y_1\}$ and agent 2 has the utility function $u_2(x_2, y_2) = min\{x_2, y_2\}$. Which of the following allocations is not Pareto efficient?

    A. (2,2), (4,2)

    B. (3,2), (3,2)

    C. (3,1), (3,3)

    D. (1,2), (5,2)

46. Consider an exchange economy with two goods and two agents have the same preferences as in the previous question. Agent 1's endowment is $(0, \alpha)$ and agent 2's endowment is $(\beta, 0)$. What is generally true?

    A. Any allocation in which each agent gets equal amounts of the two goods is an equilibrium allocation

    B. If $\beta < \alpha$, then the price of X is 0

    C. If $\beta > \alpha$, then the price of X is 0

    D. Any allocation in which agent 2 gets none of good Y is an equilibrium allocation

47. Consider an exchange economy with two goods and two agents. Agent 2's utility is as above. The equilibrium allocation is $(x_1, y_1) = (6, 1)$ and $(x_2, y_2) = (4, 4)$. The equilibrium prices are $(1, 1)$. What could be the endowment?

    A. Agent 1's endowment is (2,5) and agent 2's endowment is (8,0)

    B. Agent 1's endowment is (7,0) and agent 2's endowment is (3,5)

    C. Both (a) and (b)

    D. Neither (a) nor (b)

48. A firm has an order to supply 20 units of output. It can divide its production across different plants, 1 and 2, with cost function $c_1(q_1) = q_1^2$ and $c_2(q_2) = 3q_2^2$ respectively. The total order must be produced, i.e., $q_1 + q_2 = 20$. To meet the total production target at minimum cost, the amount of output the firm should produce in its first plant is

    A. 20 units

    B. 15 units

    C. 10 units

    D. 5 units

49. Consider an exchange economy with two agents and aggregate endowment (10,10). The agents' utility functions are $u_1(x_1, y_1) = x_1 + 2y_1$ and $u_2(x_2, y_2) = 2x_2 + y_2$. Examples of efficient allocations are

    A. $(x_1, y_1) = (5, 10)$ and $(x_2, y_2) = (5, 0)$

    B. $(x_1, y_1) = (0, 10)$ and $(x_2, y_2) = (10, 0)$

    C. $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (10, 5)$

    D. All of the above

50. Consider the following two games in strategic form.

$$\begin{array}{lcc} & Hawk & \\ Enter & -1 & 1 \\ Not\ enter & 0 & 6 \end{array} \qquad \begin{array}{lcc} & Hawk & Dove \\ Enter & -1,1 & 3,3 \\ Not\ enter & 0,6 & 0,7 \end{array}$$

    Compute the Nash equilibria of the two games. What general lesson can be drawn from these equilibria ?

    A. Eliminating a strategic option may be beneficial

    B. Having extra strategic option is not beneficial

    C. Having extra strategic option is beneficial

    D. Eliminating a strategic option is beneficial

51. Suppose the following equation holds for an economy, at every time t: $P_t = (1+a)\frac{W_t N_t}{Y_t}$. That is the price of output is a fixed markup over unit labour cost. Let π be the inflation rate, w be the wage inflation rate, and λ be the rate of growth in labour productivity. (Labour productivity $= Y_t/N_t$). Which of the following is true?

    A. $w = \pi - \lambda$

    B. $\pi = w + \lambda$

    C. $\pi = w - \lambda$

    D. $w = \lambda - \pi$

    The next three questions have to do with the following fixed price model of an economy.
    $C = C_0 + c_1(Y - T), 0 < c_1 < 1$
    $I = i_0 + i_1 r, i_1 < 0$
    $Y = C + I + G$
    $M^d = m_0 + m_1 Y + m_2 r, m_2 < 0 < m_1$
    $M^d = M,$
    where C, I, G, Y, M^d, M are respectively aggregate consumption, investment, government expenditure, output, money demand and money stock.

52. In this economy, an increase in G

    A. necessarily reduces I

    B. may increase I

    C. reduces I only if m_1 is large

    D. increases I only if m_1 is small

53. The more negative is m_2, ceteris paribus,

    A. the greater the effect of an increase in G on r

    B. the smaller the effect of an increase in G on r

    C. The magnitude of the effect of G on r does not depend on the magnitude of m_2

    D. r decreases as G increases, but the magnitude of this is indereminate

54. If c_1 is very close to 1, then an increase in M has

    A. a very large positive impact on r

    B. a very small positive effect on r

    C. a very small negative effect on r

    D. a very large negative impact on r

    The next six questions are based on the following information. *Consider an economy in which aggregate output is produced by using 2 factors, capital K and labour L. Aggregate production technology is given by the production function: $Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$, where $A > 0$ and $0 < \alpha < 1$. At each instant t, both factors are fully employed.*

A constant proportion s of total output is saved and invested in every period, which augments the capital stock in the next period. There is no depreciation of capital. Labour force grows at a constant rate n. Let $A = 20, \alpha = \frac{1}{2}, s = \frac{1}{4}, n = \frac{1}{10}$. Then

55. The steady state level of output per worker is
    A. 50
    B. 1000
    C. 2500
    D. 5000

56. The rate of growth of aggregate output in the steady state is
    A. 0
    B. $\frac{1}{4}$
    C. $\frac{1}{2}$
    D. $\frac{1}{10}$

57. The golden rule of capital per worker is
    A. 25
    B. 2500
    C. 5000
    D. 10000

58. The golden rule level of consumption per worker is
    A. 500
    B. 1000
    C. 250
    D. 25

59. An increase in the value of the parameter A from its current value
    A. will increase the steady state level of output per worker and increase the growth of aggregate output in the steady state
    B. will decrease the steady state level of output per worker and increase the growth of aggregate output in the steady state
    C. will increase the steady state level of output per worker and leave unchanged the growth of aggregate output in the steady state
    D. will decrease the steady state level of output per worker and leave unchanged the growth of aggregate output in the steady state

60. An increase in the value of the parameter n from its current value

A. will increase the steady state level of output per worker and increase the growth of aggregate output in the steady state

B. will decrease the steady state level of output per worker and increase the growth of aggregate output in the steady state

C. will increase the steady state level of output per worker and leave unchanged the growth of aggregate output in the steady state

D. will decrease the steady state level of output per worker and leave unchanged the growth of aggregate output in the steady state

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1. A rural landowner can deposit his savings in a commercial bank and receive an annual interest rate of 8%. Alternatively he can lend to villagers who need credit. If all loans are of the same size and only sixty per cent of them are repaid, the interest rate that would make his earnings the same as from depositing his savings in a bank is

   A. 8%

   B. 48%

   C. 80%

   D. 120%

2. Consider a two-person two-good exchange economy, where agents are denoted by A, B and goods are denoted by X, Y . A Pareto optimal allocation of this economy may **not** remain Pareto optimal if

   A. Everything else remaining the same, Agent A transfers a part of her endowment to Agent B

   B. Everything else remaining the same, Agent A gets additional endowment

   C. Everything else remaining the same, Agent A's utility function is monotonically transformed

   D. All of the above

3. Given the same production function and market demand, a monopolist earns at least as much profit as a competitive firm in the short run, because

   A. Monopolist is free to charge the competitive market price

   B. Short run profit of a competitive firm is zero

   C. Marginal cost of a monopolist is smaller than that of a competitive firm

   D. Average cost of a monopolist is smaller than that of a competitive firm

4. Mr. B thinks cheese is addictive - the more you eat, the more you want. Suppose x denotes the quantity of cheese. Mr. B's utility function can be represented by

   A. $u(x, y) = x^2 + y$

   B. $u(x, y) = lnx + lny$

   C. $u(x, y) = x + y$

   D. $u(x, y) = min\{x, y\}$

5. Sania and Saina are bargaining over how to split 10 Rupees. Both claimants simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 10$. If $s_1 + s_2 \leq 10$ then the claimants receive the shares they named; otherwise both receive zero. Find **all** pure strategy Nash Equilibria of this game

   A. $s_1 = 5, s_2 = 5$

B. $\{(s_1, s_2)|s_1 + s_2 = 10\}$

C. $\{(s_1, s_2)|s_1 + s_2 \leq 10\}$

D. There is no pure strategy Nash equilibrium

6. "Developing new antibiotics is expensive. It is also known that the more frequently bacteria are exposed to antibiotics, the more quickly the bacteria will develop resistance to the antibiotics. Yet, usually, each parent will press a doctor for an antibiotic if there is any chance it will heal a child quicker than that without drugs."
The above statement can be best understood as a problem of

   A. Public good: Bacteria are public bad

   B. Adverse selection: Doctors have information that is not available to parents

   C. Risk aversion: Parents are risk averse

   D. Short-run utility maximization: Most parents are myopic; they fail to see the long run effect

7. If the correlation between variables X and Y is 0, then

   A. The regressions of Y on X and X on Y intersect at right angles, and pass through $(\overline{X}, \overline{Y})$

   B. The regressions of Y on X and X on Y do not intersect at right angles but do pass through $(\overline{X}, \overline{Y})$

   C. The regressions of Y on X and X on Y intersect at right angles, but do not pass through $(\overline{X}, \overline{Y})$

   D. The regressions of Y on X and X on Y do not intersect at right angles, and do not pass through $(\overline{X}, \overline{Y})$

8. One reason why the sample median is used as an estimator of the population mean is that

   A. The average of all sample medians equals the population mean

   B. The sample median equals the population mean

   C. The sample median is unaffected by extreme values

   D. The sample median occurs more often than the mode or the mean

9. Suppose X is a random variable, which follows $Uniform[-1, 1]$. Find the covariance between X and X^2

   A. 1

   B. 1/4

   C. 1/8

   D. 0

10. Which of the following statements is **not** an indicator that multicollinearity among two or more variables is present in a multiple regression model estimated using OLS:

A. Small changes to the data can cause large changes to estimated coefficients

B. Coefficients are estimated with bias

C. The estimated coefficients have large standard errors even though the R^2 is high

D. A test of joint significance of two or more coefficients is significant even though individually they are not significant

11. Given the linear regression $Y = \alpha + \beta X$; a very high correlation between variables X and Y necessarily implies that

A. The slope coefficient β is statistically significant

B. The observations (x, y) lie along a straight line

C. Small changes in X cause large changes in Y

D. The regression line is steep

12. What would happen (other things being equal) to a confidence interval if you calculated a 99% confidence interval rather than a 95% confidence interval?

A. It will be narrower

B. It will not change

C. The sample size will increase

D. It will become wider

13. An $n-gon$ is a regular polygon with n equal sides. Find the number of diagonals (edges of an $n-gon$ are not considered as diagonals) of a 10-gon.

A. 20 diagonals

B. 25 diagonals

C. 35 diagonals

D. 45 diagonals

14. $\int_0^1 x^n sin(x)\,\mathrm{d}x$

A. Does not exist

B. Is necessarily greater than 1

C. Is greater than $\frac{1}{n+1}$

D. Is less than $\frac{1}{n+1}$

15. $\lim_{n\to\infty}(\sqrt{(n-1)} - \sqrt{n})$

A. Equals 1

B. Equals 0

C. Does not exist

D. Depends on n

16. The equation $x^7 = x + 1$

A. Has no real solution

B. Has no positive real solution

C. Has a real solution in the interval (0,2)

D. Has a real solution but not within (0,2)

17. The cumulative distribution function $F(x)$ of a random variable has a slope of 1 for x in the interval $[0, 1]$ and takes a constant value thereafter. Which of the following statements most accurately defines the probability density function of X?

A. It is zero in the interval $[0, 1]$ and 1 for all higher values of x

B. It is 1 in the interval $[0, 1]$ and zero for all higher values of x

C. It is increasing in the interval $[0, 1]$ and constant and positive for all higher values of x

D. It is increasing in the interval $[0, 1]$ and zero for all higher values of x

18. A and B are two non empty sets.

$$A - B = \{x \in A | x \notin B\} \text{ and } A + B = (A - B) \cup (B - A)$$

Consider the following statements:
Statement 1: $A + B = B$ implies $A \subset B$
Statement 2: $A + B = \phi$ implies $A = B$
Statement 3: $A + B = A \cup B$ implies $A \cap B = \phi$
How many of the above statements are correct?

A. 0

B. 1

C. 2

D. 3

19. The short-run aggregate supply curve is upward sloping because

A. A lower price level creates a wealth effect

B. Lower taxes motivate people to work more

C. Money wages do not immediately change when the price level changes

D. Most business firms operate with long-term contracts for output but not labour

20. The investment demand curve shifts rightward if

A. The expected profit rate increases

B. The real interest rate falls

C. Savers increase their thriftiness

D. The economy moves into a recession

21. A firm producing hockey sticks in Punjab has a production function given by $Q = 2\sqrt{KL}$ where K stands for capital, L stands for labour and Q stands for output. The rental rate of capital is Rs 1 and the wage rate is Rs 4. What will be the firm's cost function?

A. $2Q$

B. Q

C. Q^2

D. $2Q^2$

22. Antony and Cleopatra run a food stall together. Their joint profit is 10 Rupees per month, from which, Antony gets I_A Rupees and Cleopatra gets I_C Rupees. Utility functions of Antony and Cleopatra are $U_A = \sqrt{I_a I_C}$ and $U_C = I_C$, respectively. Consider the following divisions,

Division 1: $I_A = 7, I_C = 3$ Division 2: $I_A = 3, I_C = 7$

A. Both the divisions are Pareto optimal

B. None of the divisions is Pareto optimal

C. Only Division 1 is Pareto optimal

D. Only Division 2 is Pareto optimal

23. Consider a homogenous goods market with the demand function $Q = 30 - P$, where Q and P denote quantity and price respectively. There are two firms playing a price game in the following manner: firm 1 quotes a price and then firm 2 chooses a price. When they charge the same price they share the market equally and otherwise the market demand goes to the firm charging lower price. Firm 1 has a capacity constraint at the output level 5 units such that upto five units the marginal cost of production is Rs 3 per unit of output, however beyond 5 units it cannot produce any output. Firm 2 does not have any capacity constraint, it can produce any amount with the marginal cost Rs 6. What would be the equilibrium price in the market?

A. 3

B. 6

C. 6 - ϵ, where ϵ is very small positive number

D. 3 + ϵ, where ϵ is very small positive number

24. Consider the following two-player game. The players simultaneously draw one sample each from a continuous random variable X, which follows $Uniform[0, 100]$. After observing the value of her own sample, which is private information (that is, opponent does not observe it), players simultaneously and independently choose one of the following: $SWAP$, $RETAIN$. If both the players choose $SWAP$ then they exchange their

initially drawn numbers. Otherwise, if at least one person chooses $RETAIN$, both of them retain their numbers. A player earns as many Rupees as the number she is holding at the end of the game.
Find the probability that the players will exchange their initially drawn numbers

A. 1

B. 1/2

C. 1/3

D. 0

25. The productivity of a labourer depends on his daily wage. In some range of wage, the more he is paid, the better his health and the more work he is able to do in a given day. Suppose that the relationship between productivity and daily wage is as follows. No work is done for wage below Rs. 20 per day. Each rupee earned above 20 increases productivity by 5 units until the daily wage is Rs. 140 per day. Beyond this level of wage, productivity is constant. If a farmer needs to hire labour for a total of 6000 units of work per day, how many labourers is he likely to hire?

A. 4

B. 5

C. 6

D. 10

26. Consider a society in which half the population earns 100 rupees per day and the other half earns 200 rupees per day. The Gini coefficient of inequality for this society is given by

A. 1/2

B. 1/3

C. 1/6

D. 1/8

27. A and B live in an exchange economy. There are two goods X and Y. Utility functions of A and B are $U^A(x, y) = x + y$ and $U^B(x, y) = x^2 + y^2$, respectively. $A's$ endowment is 2 units of X and 1 unit of Y, while $B's$ endowment is 1 unit of X and 2 units of Y. Consider the following allocations
Allocation 1: A gets 1.5 units of X and zero unit of Y, B gets 1.5 units of X and 3 units of Y
Allocation 2: A gets 1.5 units of X and 1.5 unit of Y, B gets 1.5 units of X and 1.5 units of Y

A. Both the allocations are Pareto optimal

B. None of the allocations is Pareto optimal

C. Only allocation 1 is Pareto optimal

D. Only allocation 2 is Pareto optimal

28. Fill in the blanks: Given a downward sloping linear demand curve and constant marginal cost curves, if a per unit tax is imposed on a monopoly, a monopoly will ..... the quantity of its good and its revenue after tax will .....

    A. Increase ; Decrease

    B. Increase ; Increase

    C. Decrease ; Increase

    D. Decrease ; Decrease

29. The demand curve for electricity is $D(p) = 120 - p$. The marginal cost of electricity production is $MC_1(q) = 20+q$. The marginal cost of pollution due to electricity production is $MC_2(q) = 3q$. Find the competitive equilibrium output and the social optimum output.

    A. 50;20

    B. 50;30

    C. 60;20

    D. 60;30

30. Suppose that there are two agents in an economy with income x_1 and x_2. If the richer person transfers a portion of her income to the poorer person without changing the income ranking (that is the rich remains richer even after the transfer) then it is called a *progressive transfer*. Welfare of this economy is measured by a function $W(x_1, x_2)$. W is a 'good' measure of social welfare if the social welfare increases due to a progressive transfer. Consider the following candidates for W;

$$W_1(x_1, x_2) = x_1 + x_2 \quad ; \quad W_2(x_1, x_2) = x_1 x_2$$

    A. Only W_1 is a 'good' measure

    B. Only W_2 is a 'good' measure

    C. Both W_1 and W_2 are 'good' measures

    D. NeitherW_1 nor W_2 is a 'good' measures

31. Consider the following model, estimated using OLS

$$Y_i = \beta X_i + \epsilon_i; \quad i = 1, 2, \ldots, n$$

    where there is no intercept, and $Var(\epsilon_i) = \sigma^2$. Which of the following statements is **not** true?

    A. The R^2 from this regression can be large even if X and Y have low correlation.

    B. The least squares residuals need not sum to zero.

    C. The mean square error is given by $\frac{\sum(Y_i - \hat{Y})^2}{(n-1)}$

D. The least squares estimator of the slope coefficient is given by: $\frac{n\sum X_iY_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2}$

32. An analyst trying to estimate the demand for rice has estimated the following two models
Model 1: $D = 50 + 0.3Y + 0.1P + 12N; \quad R^2 = 0.7$
where D is the demand for rice per household, Y is income per household, P is the price of rice and N is household size. The standard errors associated with the coefficient Y is 0.1, that associated with P is 0.05, and of N is 3.
Model 2: $D/N = 50 + 0.2Y/N - 0.5P \; ; \quad R^2 = 0.9$
where the standard errors of the estimated slope coefficients are 0.1 and 0.2 respectively.
Which of the following statements is true?

    A. Model 2 is preferred to Model 1 because it has a higher R^2.

    B. Model 1 is preferred to Model 2 because the coefficients are all signifcant.

    C. The two models are not comparable in terms of fit

    D. None of the above

33. Consider the regression $Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i^* + u_i^*$, and $Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i$, where $Y_i^* = w_1 Y_i$ and $X_i^* = w_2 X_i$; w_1, w_2 are constants. Is it true that

    A. $\hat{\beta}_1^{\,*} = w_2 \hat{\beta}_1$

    B. $\hat{\beta}_2^{\,*} = \frac{w_2}{w_1} \hat{\beta}_2$

    C. $Var(\hat{\beta}_2^{\,*}) = (\frac{w_1}{w_2})^2 Var(\hat{\beta}_2)$

    D. $r_{xy}^2 \neq r_{x^*y^*}^2$, where r denotes correlation coefficient

34. For variables X and Y we have the data $\sum XY = 350, \sum X = 50, \sum Y = 60, \overline{X} = 5, \sigma_X^2 = 4, \sigma_Y^2 = 9$, where $\overline{X}$ denotes the mean of X and σ_X^2 denotes the variance of X. Which of the following holds

    A. A one unit change in X causes a 1.25 unit change in Y , and a one unit change in Y causes a 0.6 unit change in X

    B. A one unit change in X causes a 0.6 unit change in Y , and a one unit change in Y causes a 1.25 unit change in X

    C. A 10% change in X causes a 15% change in Y

    D. The regression of Y on X passes through the origin

35. X is a normally distributed random variable with unknown mean μ and standard deviation equal to 2. The value of the sample mean from a random sample of size 25 is 10. Which of the following values lie within the 95% confidence interval for μ?

    A. 9.3

    B. 9.8

    C. 10.6

    D. All of the above

36. An econometrician uses data from the Consumer Expenditure Survey conducted by the National Sample Survey Organization for the years 1991 and 2001 and plots the cumulative distribution function for real consumer expenditure per capita for these years. He finds that cumulative distribution function for 2001 is everywhere to the right of that for 1991. Consider the following,
Conclusion 1: The Gini coefficient for 2001 is higher than for 1991
Conclusion 2: Consumption expenditure of every individual has increased in 2001 compared to 1991
Conclusion 3: Real consumption expenditure per capita has increased in 2001 compared to 1991
Which of the above conclusions are correct

A. Only (iii) is correct

B. (i) and (ii) are correct

C. (ii) and (iii) are correct

D. None of the conclusions is correct

37. Let $X_1, X_2, \cdots, X_n$ be random samples from a normal distribution with parameters μ and σ^2. Then the random variable

$$\frac{(n-1)S^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

has a chi-squared ($\tilde{\chi}^2$) distribution with $n-1$ degrees of freedom. Here $\overline{X}$ denotes the mean of $X_1, X_2, \cdots, X_n$. Suppose area under a chi-squared curve with $n-1$ degrees of freedom to the right of $\tilde{\chi}^2_{v,n-1}$ is v . A $100(1-\alpha)\%$ confidence interval for the variance σ^2 is

A. $\left[\frac{(n-1)S^2}{\tilde{\chi}^2_{\frac{\alpha}{2},n-1}}, \frac{(n-1)S^2}{\tilde{\chi}^2_{1-\frac{\alpha}{2},n-1}}\right]$

B. $\left[\frac{(n-1)S^2}{\tilde{\chi}^2_{1-\frac{\alpha}{2},n-1}}, \frac{(n-1)S^2}{\tilde{\chi}^2_{\frac{\alpha}{2},n-1}}\right]$

C. $\left[\frac{nS^2}{\tilde{\chi}^2_{\frac{\alpha}{2},n-1}}, \frac{nS^2}{\tilde{\chi}^2_{1-\frac{\alpha}{2},n-1}}\right]$

D. $\left[\frac{nS^2}{\tilde{\chi}^2_{1-\frac{\alpha}{2},n-1}}, \frac{nS^2}{\tilde{\chi}^2_{\frac{\alpha}{2},n-1}}\right]$

38. Suppose you have 500 observations and you regress *wage* (measured in rupees per hour) on experience in the labour market, *exper* (measures in years), and on experience in the labour market squared, ($exper_2$). Your estimated OLS equation is

$$\hat{wage} = 3.73 + 0.298\ exper - 0.0061\ exper^2$$

$$(0.35) \quad (0.041) \quad (0.0009)$$

where the standard errors are in brackets. The estimated equation implies

A. The returns to experience is strictly increasing

B. The returns to experience is strictly diminishing

C. The returns to experience is constant

D. Experience has no statistically significant effect on wage

39. Suppose you have a sample of size one from one of the following densities

$$H_0 : f(x) = 2x \quad 0 \leq x \leq 1$$
$$H_1 : f(x) = 2 - 2x \quad 0 \leq x \leq 1$$

Let α and β denote type I error and type II error, respectively. Find the test procedure of the form "Reject H_0 if $x < k$" with $\alpha = 0.09$. Find β for this test.

A. $k = 0.5, \beta = 0.25$

B. $k = 0.5, \beta = 0.36$

C. $k = 0.3, \beta = 0.16$

D. $k = 0.3, \beta = 0.49$

40. An urn contains equal number of green and red balls. Suppose you are playing the following game. You draw one ball at random from the urn and note its colour. The ball is then placed back in the urn, and the selection process is repeated. Each time a green ball is picked you get 1 Rupee. The **first** time you pick a red ball, you pay 1 Rupee and the game ends. Your expected income from this game is

A. ∞

B. Positive but finite

C. Zero

D. Negative

41. Two women and four men are to be seated randomly around a circular table. Find the probability that the women are**not** seated next to each other.

A. 1/2

B. 1/3

C. 2/5

D. 3/5

42. A fair coin is tossed until a head comes up for the first time. The probability of this happening on an odd-numbered toss is

A. 1/2

B. 1/3

C. 2/3

D. 3/4

43. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, then the number of outcomes B must have so that A and B are independent, is

A. 4

B. 3 or 9

C. 6

D. 5 or 10

44. Consider the system of equations

$$\alpha x + \beta y = 0$$
$$\mu x + v y = 0$$

α, β, μ and v are i.i.d random variable. Each of them takes value 1 and 0 with equal probability.
Statement A: The probability that the system of equations has a unique solution is 3/8
Statement B: The probability that the system of equations has at least one solution is 1.

A. Both the statements are correct

B. Both the statements are false

C. Statement A is correct but B is false

D. Statement B is correct but A is false

45. $f(x, y) = x + y + xy$ where $x, y \in \Re_{++}$. For $c \in \Re_{++}$, let us define

$$L = \{(x, y) \in \Re^2 | f(x, y) \leq c\}$$
$$U = \{(x, y) \in \Re^2 | f(x, y) \geq c\}$$
$$I = \{(x, y) \in \Re^2 | f(x, y) = c\}$$

Which of the above sets are convex?

A. L

B. U

C. I

D. All of them

46. What is the total number of local maxima and local minima of the following function

$$f(x) = \begin{cases} (2+x)^3, & \text{if } -3 < x \leq -1 \\ x^{2/3}, & \text{if } -1 < x < 2 \end{cases}$$

A. 1

B. 2

C. 3

D. 4

47. Suppose that $f(x)$ is twice differentiable and strictly concave in x. Define,

$$g(x) = \lim_{t \to x} \frac{f(c+t)-f(c+x)}{t-x}$$

where c is a constant. Then $g(x)$ is

A. Decreasing function

B. Increasing function

C. Decreasing function when $c > 0$ and increasing function when $c < 0$

D. Increasing function when $c > 0$ and decreasing function when $c < 0$

48. A rectangle has its lower left hand corner at the origin and its upper right hand corner on the graph of $f(x) = x^2 + (1/x^2)$. For which x is the area of the rectangle minimized?

A. $x = 0$

B. $x = \infty$

C. $x = (1/3)^{1/4}$

D. $x = 2^{1/3}$

49. The real valued function $f(x) = x + |x| + (x-1) + |x-1|$, where $|x|, |x-1|$ stand for absolute values

A. Is differentiable everywhere except at $x = 0$

B. Is not continuous at 0

C. Is not differentiable at 1

D. Is not continuous at 1

50. Let $x = (x_1, x_2, \cdots, x_n)$ and $y = (y_1, y_2, \cdots, y_n)$ be two vectors in $\Re^n$. Define, $x \otimes y = \sum_{i=1}^{n} x_i|y_i|$, where $|y_i|$ stands for absolute value. How many of the following statements are correct
Statement 1: $x \otimes y = y \otimes x$
Statement 2: $x \otimes x = 0$ implies $x = (0, 0, \cdots, 0)$
Statement 3: $x \otimes (c, y) = c(x \otimes y)$, where $c \in \Re_{++}$ and $c.y = (cy_1, cy_2, \cdots, cy_n)$

A. 0

B. 1

C. 2

D. 3

The following set of information is relevant for the next six questions. Read the information carefully before answering the questions below.
Consider an economy where the nominal wage rate is set by a process of wage bargaining between the workers and the producers before actual production takes place. Thus at any period t , the nominal wage rate (W_t) is a function of the expected price level (P_t^e), the rate of unemployment (u_t) and the average productivity of the workers (A_t). The exact functional relationship is given below:

$$W_t = P_t^e F(u_t, A_t) \; ; \quad F_u < 0; F_A > 0$$

Once the nominal wage is determined, the producers set the actual price level (P_t) as a constant mark up (μ) over the nominal wage rate: $P_t = (1+\mu)W_t$. Define the actual rate of inflation as $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ and the expected rate of inflation as $\pi_t^e = \frac{P_t^e - P_{t-1}}{P_{t-1}}$

51. Given the above wage and price setting equations, derive the relationship between expected rate of inflation and actual rate of inflation. Which of the following equations represents this relationship?
    A. $\pi_t = (1+\pi_t)[F(u_t, A_t) - \mu] - 1$
    B. $\pi_t = (1+\pi_t)(1+\mu)F(u_t, A_t) - 1$
    C. $\pi_t = \pi_t^e(1+\mu)F(u_t, A_t)$
    D. None of the above

52. Suppose the average productivity of workers remains constant at a level $\overline{A}$. Given the relationship in previous question, which of the following equations defines the 'natural rate of unemployment'?
    A. $F(u_t, \overline{A}) = \mu$
    B. $F(u_t, \overline{A}) = \frac{1}{1+\mu}$
    C. $F(u_t, \overline{A}) = \frac{\mu}{1+\mu}$
    D. None of the above

53. A one-shot increase in the average productivity of worker, ceteris paribus, leads to
    A. An increase in the natural rate of unemployment
    B. A decrease in the natural rate of unemployment
    C. No change in the natural rate of unemployment
    D. Some change in the natural rate of unemployment but the direction is ambiguous

54. A one-shot increase in the producer's mark up, ceteris paribus, leads to
    A. An increase in the natural rate of unemployment
    B. A decrease in the natural rate of unemployment
    C. No change in the natural rate of unemployment

D. Some change in the natural rate of unemployment but the direction is ambiguous

55. Go back to the relationship between π_t and π_t^e above (first question of this list). Suppose expectations are static, i.e., $\pi_t^e = \pi_{t-1}$. Also let $F(u_t, A_t) = \frac{A_t}{u_t} - 1$ and $A_t = \overline{A}$ (a constant). The corresponding value of the non-accelerating inflation rate of unemployment is given by

A. $u_t = (\frac{\mu}{1+\mu})\overline{A}$

B. $u_t = (\frac{\mu}{2+\mu})\overline{A}$

C. $u_t = (\frac{1+\mu}{2+\mu})\overline{A}$

D. None of the above

56. Now suppose the workers productivity increases at a constant rate γ, that is , $\frac{A_t - A_{t-1}}{A_{t-1}} = \gamma$. To maintain a non-accelerating inflation rate, rate of unemployment has to increase at the rate

A. γ

B. μ

C. $\mu + \gamma$

D. $(\frac{\mu}{1+\mu})\gamma$

The following set of information is relevant for the next <u>four</u> questions. Read the information carefully before answering the questions below

Consider a small open economy with fixed nominal exchange rate (E), fixed domestic price level (P) and fixed foreign price level (P^*). Let ϵ be the corresponding real exchange rate. The goods market equilibrium condition is given by the following IS equation:

$$Y = C + I + G + X - \frac{IM}{\epsilon}$$

where
$C = c_0 + c_1 Y$ represents domestic consumption
$I = d_1 Y - d_2 r$ represents domestic investment
G represents government expenditure
$X = x_1 Y^* - x_2 \epsilon$ represents export
Y^* represents income of the foreign country
$IM = m_1 Y + m_2 \epsilon$ represents import

57. Suppose the rate of interest (r) as exogenously given. Then a unit increase in the foreign price level, ceteris paribus, increases domestic output by

A. $\left(\frac{\frac{m_1}{\epsilon} Y - \epsilon x_2}{1 - c_1 - d_1 + \frac{m_1}{\epsilon}}\right)\frac{1}{P^*}$

B. $\left(\frac{\frac{m_1}{\epsilon} Y - \epsilon x_2}{\epsilon(1 - c_1 - d_1) + m_1)}\right)$

C. $\left(\frac{\epsilon x_2 - \frac{m_1}{\epsilon}Y}{1 - c_1 - d_1 + \frac{m_1}{\epsilon}})\frac{1}{P^*}\right)$

D. None of the above

58. Write the goods market clearing level of output as a function of the real exchange rate (ϵ) and other parameters. If you plot this relationship between Y and ϵ in the Y, ϵ plane (with *epsilon* on the vertical axis), you will get

    A. A positively sloped schedule if the Marshall-Lerner condition is satisfied

    B. A negatively sloped schedule if the Marshall-Lerner condition is satisfied

    C. A positively sloped schedule irrespective of the Marshall-Lerner condition

    D. A negatively sloped schedule irrespective of the Marshall-Lerner condition

59. Let us now bring in an asset market and a foreign exchange market into the picture. Let the asset market equilibrium condition be represented by the following LM equation: $\overline{M} = l_1 Y - l_2 r$. Also, let the foreign exchange market equilibrium condition be represented by the following interest rate parity condition $r = r^* + \frac{1}{\epsilon}(\epsilon^e - \epsilon), r^*$ and ϵ^e being the foreign interest rate and the expected future exchange rate respectively. For any given value of r^* and ϵ^e , derive the level of output which will clear both the asset market and the foreign exchange market as a function of the real exchange rate (ϵ) and other parameters. If you plot this relationship between Y and ϵ in the Y, ϵ plane (with ϵ on the vertical axis), you will get

    A. A positively sloped schedule if the Marshall-Lerner condition is satisfied

    B. A negatively sloped schedule if the Marshall-Lerner condition is satisfied

    C. A positively sloped schedule irrespective of the Marshall-Lerner condition

    D. A negatively sloped schedule irrespective of the Marshall-Lerner condition

60. Let the equilibrium output and the equilibrium exchange rate be simultaneously determined by the intersection of the above two schedules (derived in the previous two questions). Suppose Marshall-Lerner condition is satisfied. Then

    A. An increase in G leads to an increase in the equilibrium value of ϵ, while an increase in $\overline{M}$ leads to a decrease in ϵ

    B. An increase in G leads to an decrease in the equilibrium value of ϵ, while an increase in $\overline{M}$ leads to an increase in ϵ

    C. Increase in either G or M leads to an increase in the equilibrium value of ϵ

    D. Increase in either G or M leads to a decrease in the equilibrium value of ϵ

8 DSE 2013

1. Starting from a stationary position, Sonia ran 100 meters in 20 seconds. Assuming that her distance from the starting point is a continuous and differentiable function $f(t)$ of time, she would definitely have run at a speed of y meters per second at some point of time, where y equals

   A. 4

   B. 20

   C. 6

   D. None of the above necessarily holds

2. Suppose $f : \mathbb{R} \to \mathbb{R}$ (i.e. it is a real-valued function defined on the set of real numbers). If f is differentiable, $f(3) = 2$, and $3 \leq f'(x) \leq 4$, for all x, then it must be that $f(5)$ lies in the following interval.

   A. $[8, 10]$

   B. $[0, 8)$

   C. $(10, \infty)$

   D. $[3, 4]$

3. The function defined by $f(x) = x(x-10)(x-20)(x-30)$ has critical points (i.e., points where $f'(x) = 0$)

   A. at some $x < 0$ and some $x > 30$.

   B. at an $x < 0$, and at some x between 0 and 30.

   C. between 0 and 10, 10 and 20, 20 and 30.

   D. None of the above captures all the critical points.

4. The area of the region bounded above by $f(x) = x^2 + 1$ and below by $g(x) = x - 6$ on the interval $[-1, 3]$ is

   A. 50/3

   B. 22

   C. 31

   D. 100/3

5. $\lim_{x \to 3} \left(\dfrac{x^2 - 5x - 2}{x - 2} \right)^{1/3}$ equals

   A. -2

   B. $-4/3$

   C. $(-4/3)^{1/3}$

   D. $(2/3)^{1/3}$

6. Consider an individual A with utility function $u(x) = 10\sqrt{x}$, where x denotes the amount of money available to her. Suppose, she has Rs 100. However, she has option of buying a lottery that will cost her Rs 51. If purchased, the lottery pays Rs 351 with probability p, and pays 0 (nothing) with remaining probability. Assume that A is expected utility maximizer. Which of the following statements is correct? A will

   A. not prefer to buy the lottery at all as long as $p < 1$

   B. certainly prefer to buy the lottery as long as $p > 0$

   C. prefer to buy the lottery if and only if $p > 51/351$

   D. prefer to buy the lottery if and only if $p > 51/221$

7. Find the equilibrium prices, when good 1 and good 2 are produced by two different monopolists

   A. $p_1 = p_2 = \dfrac{a+c}{2-b}$

   B. $p_1 = p_2 = \dfrac{a+c}{1-b}$

   C. $p_1 = \dfrac{a+c}{2-b}$, $p_2 = \dfrac{a+c}{1-b}$

   D. $p_1 = \dfrac{a+c}{1-b}$, $p_2 = \dfrac{a+c}{2-b}$

8. Find the equilibrium prices, when both the goods are produced by single monopolist.

   A. $p_1 = p_2 = \dfrac{a+c-bc}{2-b}$

   B. $p_1 = p_2 = \dfrac{a+c-bc}{1-b}$

   C. $p_1 = p_2 = \dfrac{a+c-bc}{2(1-b)}$

   D. $p_1 = p_2 = \dfrac{a+c-bc}{2}$

9. From the data in Table 1, we can conclude that the total number of poor people was

   A. Rising before 1991 but falling after 1991

   B. Falling before 1991 but rising after 1991

   C. Falling both before and after 1991, but at different rates

   D. Rising both before and after 1991, but at different rates

10. From the data in Table 2, we can conclude that both before and after 1991, mean consumption per-capita according to NAS was:

    A. Lower than mean consumption per-capita according to NSS

    B. Growing faster than mean consumption per-capita according to NSS

    C. Growing slower than mean consumption per-capita according to NSS

D. None of the above

11. If two balanced die are rolled, the sum of dots obtained is even with probability

A. 1/2

B. 1/4

C. 3/8

D. 1/3

12. A population is growing at the instantaneous growth rate of 1.5 per cent. The time taken (in years) for it to double is approximately

A. $\frac{\log 2}{0.15}$

B. $\frac{\log 2}{15}$

C. $\frac{\log 2}{0.015}$

D. $\frac{\log 2}{1.5}$

13. A linear regression model $y = \alpha + \beta x + \epsilon$ is estimated using OLS. It turns out that the estimated $\hat{\beta}$ equals zero. This implies that:

A. R^2 is zero

B. R^2 is one

C. $0 < R^2 < 1$

D. In this case R^2 is undefined

14. An analyst has data on wages for 100 individuals. The arithmetic mean of the log of wages is the same as:

A. Log of the geometric mean of wages

B. Log of the arithmetic mean of wages

C. Exponential of the arithmetic mean of wages

D. Exponential of the log of arithmetic mean of wages

15. A certain club consists of 5 men and 5 women. A 5 member committee consisting of 2 men and 3 women has to be constituted. How many ways are there of constituting this committee?

A. 20

B. 100

C. 150

D. None of the above

16. Exchange rate overshooting occurs:
    A. under fixed exchange rates when the central bank mistakenly buys or sells too much foreign exchange
    B. under fixed exchange rates as a necessary part of the adjustment process for any monetary shock
    C. under flexible exchange rates when the exchange rate rises (depreciates) above and then falls down to equilibrium after a monetary expansion
    D. decreases the value of the determinant

17. In an open economy with a system of flexible exchange rates and perfect capital mobility, an expansionary monetary policy:
    A. causes the domestic currency to appreciate
    B. has a greater impact on income than in a closed economy
    C. increases capital inflows into the country
    D. induces a balance of payments deficit

18. The short-run aggregate supply curve is upward sloping because
    A. lower price level creates a wealth effect
    B. lower taxes motivate people to work more
    C. money wages do not immediately change when the price level changes
    D. most business firms operate with long-term contracts for output but not labour

19. The term 'seignorage' is associated with
    A. inflation generated by printing new money
    B. real revenue created by printing new money
    C. public indebtedness created by printing new money
    D. none of the above

20. If money demand is stable, an open market purchase of government securities by the central bank will:
    A. increase both the level of income and the interest rate
    B. decrease both the level of income and the interest rate
    C. increase the level of income and lower the interest rate
    D. decrease the level of income and raise the interest rate

21. The function defined by $f(x) = x^5 + 7x^3 + 13x - 18$
    A. may have 5 real roots
    B. has no real roots
    C. has 3 real roots

D. has exactly 1 real root

22. The function $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^3 - 3x^2 + 6$ is

A. concave on $(-\infty, 2)$ and convex on $(2, \infty)$.

B. concave on $(-1, 2)$, convex on $(-\infty, -1)$ and $(2, \infty)$.

C. convex on $(-1, 2)$, concave on $(-\infty, -1)$ and $(2, \infty)$.

D. convex on $(-\infty, 2)$ and concave on $(2, \infty)$.

23. Consider the function $f(x) = \frac{9}{2}x^{(2/3)} - \frac{3}{5}x^{(5/3)}$ for all x in the closed interval $[-1, 5]$. $f(5)$ is approximately equal to 4.386. $f(x)$ attains a maximum on interval $[-1, 5]$ at

A. $x = -1$

B. $x = 2$

C. $x = 3$

D. $x = 4$

24. Consider the function f defined by $f(x) = x^6 + 5x^4 + 2$, for all $x \geq 0$. The derivative of its inverse function evaluated at $f(x) = 8$, that is, $f(^{-1})'(8)$ equals

A. 1/7

B. 1/15

C. 1/26

D. 1/20

25. Consider the following functions. $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $f(x, y) = (x + 2y, x - y, -2x + 3y)$. And $g : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $g(x, y) = (x + 1, y + 2)$. Then

A. Both f and g are linear transformations

B. f is a linear transformation, but g is not a linear transformation

C. f is not a linear transformation, but g is a linear transformation

D. Neither f nor g is a linear transformation

26. A six meter long string is cut in two pieces. The first piece, with length equal to some x, is used to make a circle, the second, with length $(6 - x)$, to make a square. What value of x will minimize the sum of the areas of the circle and the square? (x is allowed to be 0 or 6 as well).

A. $x = 24\pi/(1 + 4\pi)$

B. $x = 6\pi/(4 + \pi)$

C. $x = 6$

D. $x = 1/2\pi$

27. The repeated nonterminating decimal $0.272727\ldots$

A. cannot be represented as a fraction.

B. equals 27/99

C. lies strictly between 27/99 and 27/100.

D. is an irrational number.

Suppose three players, 1, 2 and 3, use the following procedure to allocate 9 indivisible coins. Player 1 proposes an allocation (x_1, x_2, x_3) where x_i is the number of coins given to player i. Players 2 and 3 vote on the proposal, saying either Y (Yes) or N (No). If there are two Y votes, then the proposed allocation is implemented. If there are two N votes, the proposal is rejected. If there is one Y vote and one N vote, then player 1 gets to vote Y or N. Now, the proposal is accepted if there are two Y votes and rejected if there are two N votes.

If 1's proposal is rejected, then 2 makes a proposal. Now, only 3 votes Y or N. If 3 votes Y, then 2's proposal is accepted. If 3 votes N, then the proposal is rejected and the allocation (3, 3, 3) is implemented.

Assume that, if the expected allocation to be received by a particular player by voting Y or N is identical, then the player votes N.

28. If 1's proposal is rejected and 2 gets to make a proposal, her proposal will be

    A. $(0, 5, 4)$

    B. $(0, 4, 5)$

    C. $(0, 6, 3)$

    D. $(0, 3, 6)$

29. 1's proposal will be

    A. $(5, 0, 4)$

    B. $(4, 0, 5)$

    C. $(3, 6, 0)$

    D. $(6, 3, 0)$

30. Consider the following change of the above situation. If 2 makes a proposal and 3 votes Y, then 2's proposal is implemented. However, if 3 votes N, then 1 gets to choose between 2's proposal and the allocation $(3, 3, 3)$. If 1's proposal is rejected and 2 gets to make a proposal, her proposal will be

    A. $(4, 5, 0)$

    B. $(0, 5, 4)$

    C. either (a) or (b)

    D. neither (a) nor (b)

31. Utility of a consumer is given by $u(x_1, x_2) = \min\{x_1, x_2\}$. His income is M, and price of good 2 is 1. There are two available price schemes for good 1: (i) per unit price 2 and (ii) a reduced per unit price $2 - \theta$ along with a fixed fee T. A consumer would be indifferent between the above schemes if

A. $\theta = 2T/M$

B. $\theta = 3T/M$

C. $\theta = T/M$

D. $\theta = (T+1)/M$

Suppose, an individual lives for two periods. In each period she consumes only one good, which is rice. In period 2, she can costlessly produce 1 unit of rice, but in period 1 she produces nothing. However, in period 1 she can borrow rice at an interest rate $r > 0$. That is, if she borrows z units of rice in period 1, then in Period 2, she must return $z(1+r)$ units of rice. Let x_1 and x_2 denote her consumption of rice in period 1 and period 2, respectively; $x_1, x_2 \geq 0$. Her utility function is given by $U(x_1, x_2) = x_1 + \beta x_2$, where β is the discount factor, $0 < \beta < 1$. Note that there are only two sources through which rice can be available; own production and borrowing.

32. Find the interest rate r, at which the individual would borrow $\frac{1}{2}$ unit of rice in period 1.

A. 2

B. $\frac{1}{2}$

C. $\beta - 1$

D. $\frac{1}{\beta} - 1$

33. Now suppose that there are N agents in the above two period economy. The agents are identical (in terms of production and utility function) except that they have different discount factors. Suppose that β follows *uniform distribution* in the interval $\left[\frac{1}{2}, 1\right]$. Assuming $r \leq 1$, the demand function for rice in period 1 will be

A. $N\frac{(1-r)}{(1+r)}$

B. $N\frac{(1-r)}{(1+r)^2}$

C. $N\frac{r}{1+r}$

D. $N\frac{r}{2}$

E. $N\frac{(1-r)}{(1+r)^2}$

Answer 34, 35 based on the following situation:
Consider a two-person two-good exchange economy: persons/agents are A and B, and goods are 1 and 2. The agents have the following utility functions:

$$u_A(x_1, x_2) = \alpha x_1 + x_2 \quad u_B(y_1, y_2) = y_1 y_2$$

where x_1 and x_2 denote the allocation to A of good 1 and good 2, respectively. Similarly, y_1 and y_2 denote the allocation to B of good 1 and good 2, respectively. There are 5 units of each good; i.e., $x_1 + y_1 = 5$ and $x_2 + y_2 = 5$.

Now, consider the following allocation: Agent A gets 4 units of good 1 only, but agent B gets 1 unit of good 1 and 5 units of good 2.

34. Suppose an agent i is said to envy agent j, if i strictly prefers j's allocation over her own allocation. And, an allocation is called 'No-envy allocation' if none of the agents envies the other. In that case,

    A. the above allocation is always 'No-envy allocation'

    B. the above allocation is never 'No-envy allocation'

    C. the above allocation is 'No-envy allocation' if $\alpha \geq \frac{5}{3}$

    D. the above allocation is 'No-envy allocation' if $\alpha \leq \frac{3}{5}$

35. The above allocation is

    A. always Pareto optimal

    B. never Pareto optimal

    C. Pareto optimal if $\alpha \leq 5$

    D. Pareto optimal if $\alpha \geq 5$

Answer 36, 37 based on the following situation: There are two goods: a basic good, say a car, and a complementary good, say car audio. Suppose that the basic good is produced by a monopolist at no cost and the complementary good is produced by a competitive industry at cost c per unit. Let p be the price of the basic good. Each consumer has three choices:

(i) consume nothing, which gives 0 utility

(ii) consume one unit of the basic good, which gives $v - p$ utility

(iii) consume one unit of the basic good and one unit of the complementary good (**called bundle**), which gives $w - p - c$ utility.

Assume $w > v > 0$.

Next, suppose there are two types of consumers of cars:

Middle Class]: They have valuations v_1 and w_1 for the basic good and the bundle, respectively. [Rich]: They have valuations v_2 and w_2 for the basic good and the bundle, respectively. Suppose, $v_2 > v_1$ and $w_2 - v_2 > c > w_1 - v_1$.

36. Find the socially efficient consumption.

    A. Rich choose (ii) and Middle class choose (i)

    B. Rich choose (iii) and Middle class choose (i)

    C. Rich choose (iii) and Middle class choose (ii)

    D. Both Rich and Middle class choose (ii)

37. Suppose that the monopolist can distinguish between two types of consumers. What prices would she charge?

    A. v_2 from Rich, and v_1 from Middle Class

    B. $w_2 - c$ from Rich, and $w_1 - c$ from Middle Class

C. $w_1 - c$ from Rich as well as Middle Class

D. $w_2 - c$ from Rich, and v_1 from Middle Class

38. Suppose that a city can be described by an interval [0, 1]. Only two citizens, A and B, live in this city at different locations; A at 0.2 and B at 0.7. Government has decided to set up a nuclear power plant in this city but is yet to choose its location. Each citizen wants the plant as far as possible from her home and hence both of them have the same utility function, $u(d) = d$, where d denotes the distance between the plant and home. Find the set of Pareto optimal locations for the plant.

    A. All locations in the interval [0, 1] are Pareto optimal

    B. All locations in the interval [0.2, 0.7] are Pareto optimal

    C. 0.5 is the only Pareto optimal location

    D. 0 and 1 are the only Pareto optimal locations

Answer 39, 40 based on the following situation: Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1 lexicographically prefers the good x: when offered two non-identical bundles of x and y, she strictly prefers the bundle with more of good x, but if the bundles have the same amount of good x, then she strictly prefers the bundle with more of good y. However, Agent 2 lexicographically prefers good y.

39. Suppose 1's endowment is $(10, 0)$ and 2's endowment is $(0, 10)$. The vector (p_x, p_y) is a competitive equilibrium vector of prices *if and only if*

    A. $p_x = 1 = p_y$

    B. $p_x > 0$ and $p_y > 0$

    C. $p_x > p_y > 0$

    D. $p_y > p_x > 0$

40. Suppose we make only one change in the above situation: Person 1 lexicographically prefers y and 2 lexicographically prefers x. The vector (p_x, p_y) is a competitive equilibrium vector of prices *if and only if*

    A. $p_x = 1 = p_y$

    B. $p_x > 0$ and $p_y > 0$

    C. $p_x > p_y > 0$

    D. $p_y = p_x > 0$

41. Consider two disjoint events A and B in a sample space S. Which of the following is correct?

    A. A and B are always independent

    B. A and B cannot be independent

    C. A and B are independent if exactly one of them has positive probability

    D. A and B are independent if both of them have positive probability

42. A bowl contains 5 chips, 3 marked \$1 and 2 marked \$4. A player draws 2 chips at random and is paid the sum of the values of the chips. The player's expected gain (in \$) is

    A. less than 2

    B. 3

    C. above 3 and less than 4

    D. above 4 and less than 5

43. Consider the following two income distributions in a 10 person society. A : (1000, 1000, 1000, 1000, 1000, 1000, 1000, 2000, 2000, 2000) and B : (1000, 1000, 1000, 1000, 1000, 2000, 2000, 2000, 2000, 2000). Which of the following statements most accurately describes the relationship between the two distributions?

    A. The Lorenz curve for distribution A lies to the right of that for distribution B

    B. The Lorenz curve for distribution B lies to the right of that for distribution A

    C. The Lorenz curves for the two distributions cross each other

    D. The Lorenz curves for the two distributions are identical for the bottom half of the population

44. A certain club consists of 5 men and 5 women. A 5-member committee consisting of 2 men and 3 women has to be constituted. Also, suppose that Mrs. F refuses to work with Mr. M. How many ways are there of constituting a 5-member committee that ensures that both of them do not work together?

    A. 50

    B. 76

    C. 108

    D. None of the above

45. Suppose, you are an editor of a magazine. Everyday you get two letters from your correspondents. Each letter is as likely to be from a male as from a female correspondent. The letters are delivered by a postman, who brings one letter at a time. Moreover, he has a 'ladies first' policy; he delivers letter from a female first, if there is such a letter. Suppose you have already received the first letter for today and it is from a female correspondent. What is the probability that the second letter will also be from a female?

    A. 1/2

    B. 1/4

    C. 1/3

    D. 2/3

46. On an average, a waiter gets no tip from two of his customers on Saturdays. What is the probability that on next Saturday, he will get no tip from three of his customers?

A. $\frac{9}{2}e^{-3}$

B. $2e^{-3}$

C. $\frac{4}{3}e^{-2}$

D. $3e^{-2}$

47. A linear regression model $y = \alpha + \beta x + \epsilon$ is estimated using OLS. It turns out that the estimated R^2 equals zero. This implies that:

A. All x's are necessarily zero

B. $\hat{\beta} = 1$ and $y = \hat{\alpha} + x$

C. $\hat{\beta} = 0$ or all x's are constant

D. There are no implications for $\hat{\beta}$

48. Using ordinary least squares, a market analyst estimates the following demand function

$$\log X = \alpha + \beta \log P + \epsilon$$

where X is the output and P is the price. In another formulation, she estimates the above function after dividing all prices by 1000. Comparing the two sets of estimates she would find that

A. $\hat{\alpha}$ and $\hat{\beta}$ will be the same in both formulations

B. $\hat{\alpha}$ and $\hat{\beta}$ will differ across both formulations

C. $\hat{\alpha}$ will change but $\hat{\beta}$ will not

D. $\hat{\beta}$ will change but $\hat{\alpha}$ will not

49. A linear regression model is estimated using ordinary least squares $y = \alpha + \beta x + \epsilon$. But the variance of the error term is not constant, and in fact varies directly with another variable z, which is not included in the model. Which of the following statements is true?

A. The OLS estimated coefficients will be biased because of the correlation between x and the error term

B. The OLS estimated coefficients will be unbiased but their estimated standard errors will be biased

C. The OLS estimated coefficients will be unbiased and so will their estimated standard errors because the error variance is not related to x, but to z which is not included in the model

D. Both the OLS estimated coefficients and their estimated standard errors will be biased

50. Suppose the distribution function $F(x)$ of a random variable X is rising in the interval $[a, b)$ and horizontal in the interval $[b, c]$. Which of the following statements CAN be correct?

A. $F(x)$ is the distribution function of a continuous random variable X.

B. c is not the largest value that X can take.

C. The probability that $X = b$ is strictly positive.

D. Any of the above

51. The equilibrium level of income is given by:

A. 8,000

B. 16,000

C. 10,000

D. None of the above

52. At this equilibrium level of income

A. there is trade surplus

B. there is a trade deficit

C. trade is balanced

D. one cannot comment on the trade account without further information

53. Now suppose the government decides to maintain a balanced trade by appropriately adjusting the tax rate τ (thereby affecting domestic absorption) - without changing the exchange rate or the amount of government expenditure. Values of other parameters remain the same. The government

A. can attain this by decreasing the tax rate to 1/5

B. can attain this by increasing the tax rate to 4/5

C. can attain this by simply keeping that tax rate unchanged at 2/5

D. can never attain this objective by adjusting only the tax rate

54. Suppose in an economy banks maintain a cash reserve ratio of 20%. People hold 25% of their money in currency form and the rest in the form of demand deposits. If government increases the high-powered money by 2000 units, the corresponding increase in the money supply would be

A. 5000 units

B. 2000 units

C. 7200 units

D. None of the above

55. Consider the standard IS-LM framework with exogenous money supply. Now suppose the government introduces an endogenous money supply rule such that the money supply becomes an increasing function of the interest rate. As compared to the standard IS-LM case, now

A. the IS curve will be flatter and fiscal policy would be more effective

B. the IS curve will be steeper and fiscal policy would be less effective

C. the LM curve will be flatter and fiscal policy would be more effective

D. the LM curve will be steeper and fiscal policy would be less effective

56. This production function violates

A. the neoclassical property of constant returns to scale

B. the neoclassical property of diminishing returns to each factor

C. the neoclassical property of factor returns being equal to the respective marginal products

D. the neoclassical property of substitutability between capital and labour

57. Let $k_t = K_t/L_t$. The corresponding *per capita* output, y_t, is given by which of the following equations?

A. $y_t = \alpha + \beta k_t$

B. $y_t = \beta(k_t)^{\alpha}$

C. $y_t = \alpha k_t + \beta$

D. $y_t = \alpha(k_t)^{\beta}$

58. The dynamic equation for capital accumulation per worker is given by

A. $\frac{dk}{dt} = s\beta k_t^{\alpha} - nk_t$

B. $\frac{dk}{dt} = s\alpha k_t + s\beta - nk_t$

C. $\frac{dk}{dt} = s\alpha k_t - nk_t$

D. $\frac{dk}{dt} = s\alpha k_t^{\beta} - nk_t$

59. Let $\alpha = \frac{1}{2};\ \beta = 12;\ s = \frac{1}{4};\ n = \frac{1}{2}$. The corresponding steady state value of capital per worker is given by

A. 8

B. 36

C. $(4)^{\frac{1}{11}}$

D. There does not exist any well-defined steady state value

60. Consider the same set of parameter values as above. An increase in the savings ratio

A. unambiguously increases the steady state value of capital per worker

B. unambiguously decreases the steady state value of capital per worker

C. leaves the steady state value of capital per worker unchanged

D. has an ambiguous effect; a steady state may not exist if the savings ratio increases sufficiently

9 DSE 2014

1. Suppose that we classify all households into one of two states, rich and poor. The probability of a particular generation being in either of these states depends only on the state in which their parents were. If a parent is poor today, their child is likely to be poor with probability 0.7. If a parent is rich today, their child is likely to be poor with probability 0.6. What is the probability that the great grandson of a poor man will be poor?

   A. 0.72

   B. 0.67

   C. 0.62

   D. 0.78

2. Consider the experiment of tossing two fair coins. Let the event A be a head on the first coin, the event C be a head on the second coin, the event D be that both coins match and the event G be two heads. Which of the following is *false*?

   A. C and D are statistically independent

   B. A and G are statistically independent

   C. A and D are statistically independent

   D. A and C are statistically independent

3. Let Y denote the number of heads obtained when 3 fair coins are tossed. Then the expectation of $Z = 4 + 5Y^2$ is

   A. 17

   B. 18

   C. 19

   D. None of the above

4. Let Y denote the number of heads obtained when 3 fair coins are tossed. Then the variance of $Z = 4 + 5Y^2$ is

   A. 185.5

   B. 178.5

   C. 187.5

   D. None of the above

5. Let events E, F and G be pairwise independent with $\Pr(G) > 0$ and $\Pr(E \cap F \cap G) = 0$. Let X^c denote the complement of event X. Then $\Pr(E^c \cap F^c | G) =$

   A. $\Pr(E^c) + \Pr(F^c)$

   B. $\Pr(E^c) - \Pr(F^c)$

   C. $\Pr(E^c) - \Pr(F)$

D. None of the above

6. Let $a_n = \left(1 + \frac{1}{n}\right)^{n+1}$, $n = 1, 2, \ldots$ Then the sequence $(a_n)_{n=1}^{\infty}$
   A. is an increasing sequence
   B. first increases, then decreases
   C. is a decreasing sequence
   D. first decreases, then increases

7. Let M, A, B, C be respectively the four matrices below:

$$\begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}$$

   Then $M = xA + yB + zY$,
   A. but x, y, z are not unique.
   B. $z = -1$
   C. $z = -1$ and $z = -2$ both can hold
   D. x, y, z are unique but $z = 2$

8. Let f be a continuous function from $[a, b]$ to $[a, b]$, and is differentiable on (a, b). We will say that point $y \in [a, b]$ is a *fixed point* of f if $y = f(y)$. If the derivative $f'(x) \neq 1$ for any $x \in (a, b)$, then f has
   A. multiple, and an odd number of, fixed points
   B. no fixed points in $[a, b]$
   C. multiple, but an even number of, fixed points
   D. exactly one fixed point in $[a, b]$

9. Which of the following statements is true for all real numbers a, b with $a < b$?
   A. $\sin b - \sin a \leq b - a$
   B. $\sin b - \sin a \geq b - a$
   C. $|\sin b - \sin a| \geq b - a$
   D. $|\sin b - \sin a| \leq |b - a|$

10. Let $O(0, 0)$, $P(3, 4)$ and $Q(6, 0)$ be the vertices of a triangle OPQ. If a point S in the interior of the OPQ is such that triangles OPS, PQS and OQS have equal area, then the coordinates of S are:
    A. $(4/3, 3)$
    B. $(3, 2/3)$
    C. $(3, 4/3)$
    D. $(4/3, 2/3)$

11. 5 men and 5 women are seated randomly in a single row of chairs. The expected number of women sitting next to at least one man equals

    A. 11/3

    B. 13/3

    C. 35/9

    D. 37/9

12. Let M be a 3×3 matrix such that $M^2 = M$. Which of the following is necessarily true?

    A. M is invertible

    B. $\det(M) = 0$

    C. $\det(M^5) = \det(M)$

    D. None of the above

13. Suppose a straight line in $\mathbb{R}^3$ passes through the point $(-1, 3, 3)$ in the direction of the vector $(1, 2, 3)$. The line will intersect with the xy-plane at point

    A. $(2, -1, 0)$

    B. $(1, 3, 0)$

    C. $(3, 1, 0)$

    D. None of the above

14. X is a random variable. Which of the following statements is always true?

    A. The expectation of X exists.

    B. The distribution function of X is strictly increasing

    C. X has a median

    D. None of the above

15. Consider two disjoint events A and B in a sample space S. Which of the following is correct?

    A. A and B are always independent

    B. A and B cannot be independent

    C. A and B are independent if both of them have positive probability

    D. None of the above

16. This exchange economy has

    A. one competitive equilibrium allocation

    B. two competitive equilibrium allocations

    C. an infinite number of competitive equilibrium allocations

    D. no competitive equilibrium allocations

17. Which of the following changes makes $(p_x, p_y) = (1, 0)$ a competitive equilibrium price vector?

    A. agent 2's endowment changes to $(9, 0)$

    B. agent 2's endowment changes to $(10, 0)$

    C. agent 1's endowment changes to $(0, 12)$

    D. none of the above

18. Suppose only agent 2's preferences are changed. The changed preferences of agent 2's become identical to those of agent 1. Then,

    A. there is no equilibrium price ratio

    B. both of the following are true

    C. $p_x/p_y = 0$ is an equilibrium price ratio

    D. $p_y/p_x = 0$ is an equilibrium price ratio

19. Suppose only agent 2's preferences are changed. The changed preferences is such that agent 2 strictly prefers bundle (a, b) to bundle (c, d) if, either $b > d$, or $b = d$ and $a > c$. Then,

    A. there is no equilibrium price ratio

    B. both of the following are true

    C. $p_x/p_y = 0$ is an equilibrium price ratio

    D. $p_x/p_y > 0$ is an equilibrium price ratio

    E. $p_x/p_y = 10/11$ is an equilibrium price ratio

20. Suppose only agent 1's preferences are changed. The changed preferences of agent 1's become identical to those of agent 2. Then,

    A. there is no equilibrium price ratio

    B. both of the following are true

    C. $p_x/p_y = 0$ is an equilibrium price ratio

    D. $p_y/p_x = 0$ is an equilibrium price ratio

21. 5 men and 5 women are seated randomly in a single circle of chairs. The expected number of women sitting next to at least 1 man equals

    A. 23/6

    B. 25/6

    C. 4

    D. 17/4

22. Ms. A selects a number X randomly from the uniform distribution on $[0, 1]$. Then Mr. B repeatedly, and independently, draws numbers $Y_1, Y_2, \ldots$ from the uniform distribution on $[0, 1]$, until he gets a number larger than $X/2$, then stops. The expected number of draws that Mr. B makes equals

A. $2\ln 2$

B. $\ln 2$

C. $2/e$

D. $6/e$

23. Ms. A selects a number X randomly from the uniform distribution on $[0, 1]$. Then Mr. B repeatedly, and independently, draws numbers $Y_1, Y_2, \ldots$ from the uniform distribution on $[0, 1]$, until he gets a number larger than $X/2$, then stops. The expected sum of the number Mr. B draws, given $X = x$, equals

A. $\ln 2$

B. $1/(1 - \frac{1}{2})$

C. $1/(2 - x)$

D. $3/(1 - \frac{1}{2})$

24. There are two fair coins (i.e. Heads and Tails are equally likely for tosses of both). Coin 1 is tossed 3 times. Let X be the number of Heads that occur. After this, Coin 2 is tossed X times. Let Y be the number of Heads we get with Coin 2. The probability $\Pr(X \geq 2|Y = 1)$ equals

A. 1/2

B. 4/7

C. 2/3

D. 11/18

E. 10/18

25. Two independent random variables X and Y have the same probability density functions:

$$f(x) = \begin{cases} c(1+x) & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Then the variance of their sum, $\mathbb{V}(X + Y)$ equals

A. 2/9

B. 13/81

C. 4/45

D. 5/18

26. Suppose two restaurants are going to be located at a street that is ten kilometres long. The location of each restaurant will be chosen randomly. What is the probability that they will be located less than five kilometres apart?

A. 1/4

B. 1/2

C. 3/4

D. 1/3

27. Consider the linear regression model: $y_i = \beta_1 D1_i + \beta_2 D2_i + \epsilon_i$, where $D1_i = 1$ if $1 \leq i \leq N$ and $D1_i = 0$ if $N + 1 \leq i \leq n$ for some $1 < N < n$ and $D2_i = 1 - D1_i$. Can this model be estimated using least squares?

    A. No, because $D1$ and $D2$ are perfectly collinear

    B. Yes, and it is equivalent to running two separate regressions of y on $D1$ and y on $D2$, respectively

    C. No, because there is no variability in $D1$ and $D2$

    D. Yes, provided an intercept term is included.

28. Consider the least squares regression of y on a single variable x. Which of the following statements is true about such a regression?

    A. The coefficient of determination R^2 is always equal to the squared correlation coefficient between y and x

    B. The coefficient of determination R^2 is equal to the squared correlation coefficient between y and x only if there is no intercept in the equation

    C. The coefficient of determination R^2 is equal to the squared correlation coefficient between y and x only if there is an intercept in the equation

    D. There is no relationship between the coefficient of determination R^2 and the squared correlation coefficient between y and x

29. An analyst runs two least squares regressions: first, of y on a single variable x, and second of x on y. In both cases, she decides to include an intercept term. Which of the following is true of what she finds?

    A. The slope coefficient of the first regression will be the inverse of the slope coefficient of the second regression; this will also be true of the associated t-ratios

    B. The slope coefficients will be different, the associated t-ratios will also be different, but the R^2 from the two regressions will be the same

    C. The slope coefficients will be different, but the associated t-ratios and the R^2 from the two regressions will be the same

    D. The slope coefficient will be the inverse of each other, the associated t-ratios will also be the inverse of each other, but the R^2 from the two regressions will be the same

30. Consider the two regression models

    (i) $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$

    (ii) $y = \gamma_0 + \gamma_1 Z_1 + \gamma_2 Z_2 + v$

where variables Z_1 and Z_2 are distinct from X_1 and X_2. Assume $u \sim \mathcal{N}(0, \sigma_u^2)$ and $v \sim \mathcal{N}(0, \sigma_v^2)$ and the models are estimated using ordinary least squares. If the true model is (i) then which of the following is true?

A. $\mathbb{E}[\hat{\beta}_1] = \mathbb{E}[\hat{\gamma}_1] = \beta_1$ and $\mathbb{E}[\hat{\sigma}_v^2] = \sigma_u^2$

B. $\mathbb{E}[\hat{\sigma}_v^2] \geq \sigma_u^2$

C. $\mathbb{E}[\hat{\sigma}_v^2] \leq \sigma_u^2$

D. None of the above as the two models cannot be compared

The following information is the starting point for Q 31 - 35. Please read it carefully before you proceed to answer.

Consider an economy consisting of N identical firms producing a single final commodity to be used for consumption as well as investment purposes. Each firm is endowed with a Cobb-Douglas production technology, such that

$$Y_t^i = \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha}; \quad 0 < \alpha < 1$$

where K_t^i and L_t^i denote the amounts of capital and labor employed by the i-th firm at time period t. The final commodity is the numeraire; wage rate for labour (w_t) and the rental rate for capital (r_t) are measured in terms of the final commodity. The firms are perfectly competitive and employ labor and capital so as to maximise their profits - taking the factor prices as given. The aggregate output produced is thus given by:

$$Y_t = \sum_{i=1}^{N} \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha} = \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha},$$

where $K_t = \sum_{i=1}^{N} K_t^i$ and $L_t = \sum_{i=1}^{N} L_t^i$ are the total capital and labor employed in the aggregate economy in period t.

Labor and Capital on the other hand are provided by the households. There are H identical households, each endowed with k_t^h units of capital and 1 unit of labor at the beginning of period t. Capital stock of the households gets augmented over time due to the savings and investment made by the households. In particular, each household saves and invests exactly half of its total income y_t^h - (which includes its labour as well as capital income) in every period and consumes the rest, such that $\frac{dk_t^h}{dt} = \frac{1}{2} y_t^h$ (There is no depreciation of capital).

The entire capital endowment at the beginning of every period is supplied inelastically to the market at the given rental rate (r_t). Labor supply however is endogenous and responds to the market wage rate. Out of the total endowment of 1 unit of labour, a household optimally supplies l_t^h units so as to maximise its utility:

$$U_t^h = w_t l_t^h - \left(l_t^h\right)^{\delta}; \quad \delta > 1$$

where the first term captures the (indirect) utility derived from labour earnings while the second term captures the dis-utility of labour.

31. The labor demand schedule for the aggregate economy is given by the following function:

A. $L_t = \left[\frac{1}{w_t}\right]^{1/\alpha} K_t$

B. $L_t = N\left[\frac{1-\alpha}{w_t}\right]^{1/\alpha} K_t$

C. $L_t = \left[\frac{1-\alpha}{w_t}\right]^{1/\alpha} K_t$

D. None of the above

32. The aggregate labour supply schedule by the households is given by the following function:

A. $L_t^S = \begin{cases} H\left[\frac{w_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \bar{w} \equiv (\delta)^{1/(\delta-1)} \\ H & \text{for } w_t \geq \bar{w} \end{cases}$

B. $L_t^S = \begin{cases} H\left[\frac{w_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \hat{w} \equiv \delta \\ H & \text{for } w_t \geq \hat{w} \end{cases}$

C. $L_t^S = \begin{cases} \left[\frac{Hw_t}{\delta}\right]^{1/(\delta-1)} & \text{for } w_t < \hat{w} \equiv \delta \\ 1 & \text{for } w_t \geq \hat{w} \end{cases}$

D. None of the above

33. The market clearing wage rate in the short run (period t) is given by:

A. $w_t^* = \begin{cases} \left[\frac{K_t(1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < \frac{H}{\delta} \equiv \hat{K} \\ \left[\frac{K_t(1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \hat{K} \end{cases}$

B. $w_t^* = \begin{cases} \left[\frac{K_t(1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K} \\ \left[\frac{K_t(1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \bar{K} \end{cases}$

C. $w_t^* = \begin{cases} \left[\frac{K_t(1-\alpha)^{1/\alpha}(\delta)^{1/(\delta-1)}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K} \\ \left[\frac{K_t(1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \bar{K} \end{cases}$

D. None of the above

34. Equilibrium output in the short run (period t):

A. is a strictly convex function of K_t for $K_t < H\left(\frac{\delta}{1-\alpha}\right)^{1/\alpha} \equiv \bar{K}$; and is a strictly concave function of K_t for $K_t \geq \bar{K}$

B. is a strictly concave function of K_t for all values of K_t

C. is a strictly convex function of K_t for all values of K_t

D. is a linear function of K_t for all values of K_t

35. Over time the aggregate output in this economy

A. initially increases until $K_t < \bar{K}$, and then reaches a constant value within finite time when $K_t \geq \bar{K}$

B. initially decreases until $K_t < \bar{K}$, and then reaches a constant value within finite time when $K_t \geq \bar{K}$

C. keep increasing at a decreasing rate and approaches a constant value only in the very long run (when $t \to \infty$)

D. increases at a constant rate until $K_t < \bar{K}$; increases at a decreasing rate when $K_t \geq \bar{K}$ and approaches a constant value only in the very long run (when $t \to \infty$)

The following information is the starting point for Q 36 - 40. Please read it carefully before you proceed to answer.

Consider an economy consisting of N identical firms producing a single final commodity to be used for consumption as well as investment purposes. Each firm is endowed with a Cobb-Douglas production technology, such that

$$Y_t^i = \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha}; \quad 0 < \alpha < 1$$

where K_t^i and L_t^i denote the amounts of capital and labor employed by the i-th firm at time period t. The final commodity is the numeraire; wage rate for labour (w_t) and the rental rate for capital (r_t) are measured in terms of the final commodity. The firms are perfectly competitive and employ labor and capital so as to maximise their profits - taking the factor prices as given. The aggregate output produced is thus given by:

$$Y_t = \sum_{i=1}^{N} \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha} = \left(K_t\right)^{\alpha} \left(L_t\right)^{1-\alpha},$$

where $K_t = \sum_{i=1}^{N} K_t^i$ and $L_t = \sum_{i=1}^{N} L_t^i$ are the total capital and labor employed in the aggregate economy in period t.

Labor and Capital on the other hand are provided by the households. There are H identical households, each endowed with k_t^h units of capital and 1 unit of labor at the beginning of period t. Capital stock of the households gets augmented over time due to the savings and investment made by the households. In particular, each household saves and invests exactly half of its total income y_t^h - (which includes its labour as well as capital income) in every period and consumes the rest, such that $\frac{dk_t^h}{dt} = \frac{1}{2} y_t^h$ (There

is no depreciation of capital).
The entire capital endowment at the beginning of every period is supplied inelastically to the market at the given rental rate (r_t). Labor supply however is endogenous and responds to the market wage rate. Out of the total endowment of 1 unit of labour, a household optimally supplies l_t^h units so as to maximise its utility:

$$U_t^h = \begin{cases} w_t l_t^h - D \ (D > 0) & \text{for } l_t^h > 0 \\ 0 & \text{for } l_t^h = 0 \end{cases}$$

For the case $l_t^h > 0$, the first term captures the (indirect) utility derived from labour earnings while the constant term D captures the dis-utility of labour - which is independent of quantity of labour supplied.

36. The new aggregate labour supply schedule by the households is given by the following function:

A. $L_t^S = \begin{cases} 0 & \text{for } w_t < \underline{w} \equiv \dfrac{D}{H} \\ H & \text{for } w_t \geq \underline{w} \end{cases}$

B. $L_t^S = H$ for all values of w_t

C. $L_t^S = \begin{cases} 0 & \text{for } w_t < \tilde{w} \equiv D \\ H & \text{for } w_t \geq \tilde{w} \end{cases}$

D. None of the above

37. The new market clearing wage rate in the short run (period t) is given by:

A. $w_t^* = \begin{cases} D & \text{for } K_t < H\left(\dfrac{D}{1-\alpha}\right)^{1/\alpha} \equiv \tilde{K} \\ \left[\dfrac{K_t (1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \tilde{K} \end{cases}$

B. $w_t^* = D$ for all values of K_t

C. $w_t^* = \begin{cases} \left[\dfrac{K_t (1-\alpha)^{1/\alpha}}{H}\right]^{\frac{\alpha(\delta-1)}{\alpha+\delta-1}} & \text{for } K_t < \dfrac{H}{D} \equiv \hat{K} \\ \left[\dfrac{K_t (1-\alpha)^{1/\alpha}}{H}\right]^{\alpha} & \text{for } K_t \geq \hat{K} \end{cases}$

D. None of the above

38. An increase in the number of firms (N)

A. leaves the wage rate unchanged in the short run (until $K_t < \tilde{K}$) and increases it thereafter

B. increases the wage rate in the short run (until $K_t < \hat{K}$) and leaves it unchanged thereafter

C. leaves the wage rate unchanged irrespective of K_t

D. None of the above

39. The new equilibrium output in the short run (period t)

A. is a linear function of K_t for $K_t < H\left(\frac{D}{1-\alpha}\right)^{1/\alpha} \equiv \tilde{K}$, and is strictly concave function of K_t for $K_t \geq \tilde{K}$

B. is a strictly concave function of K_t for all values of K_t

C. is a strictly convex function of K_t for all values of K_t

D. is a linear function of K_t for all values of K_t

40. Over time the aggregate output in this economy

A. initially increases until $K_t < \tilde{K}$, and then reaches a constant value within finite time when $K_t \geq \tilde{K}$

B. initially decreases until $K_t < \tilde{K}$, and then reaches a constant value within finite time when $K_t \geq \tilde{K}$

C. increases at a constant rate until $K_t < \tilde{K}$; increases at a decreasing rate when $K_t \geq \tilde{K}$ and approaches a constant value only in the very long run (when $t \to \infty$)

D. None of the above

41. $\lim_{n\to\infty} \sqrt[n]{n} =$

A. 0

B. 0.5

C. 1

D. 2

42. $\lim\limits_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) =$

A. -1

B. 0

C. 1

D. The limit does not exist

43. Suppose $A_1, A_2, \ldots$ is a countably infinite family of subsets of a vector space. Suppose all of these sets are linearly independent, and that $A_1 \subseteq A_2 \subseteq \cdots$ Then $\cup_{i=1}^{\infty} A_i$ is

A. a linearly independent set of vectors

B. a linearly dependent set of vectors

C. linearly independent provided the vectors are orthogonal

D. not necessarily either dependent or independent

44. If u and v are distinct vectors and k and t are distinct scalars, then the vectors $u+k(u-v)$ and $u+t(u-v)$

A. are linearly independent

B. may be identical

C. are linearly dependent

D. are distinct

45. Let $d((x_1,x_2),(y_1,y_2)) = \max\{|x_1-y_1|,|x_2-y_2|\}$ be the distance between two points (x_1,x_2) and (y_1,y_2) on the plane. Then the locus of points at distance 1 from the origin is

A. a square with side length $= 1$

B. a square with side length $= \sqrt{2}$

C. a square with side length $= 2$

D. a circle with radius $= 1$

46. The set of all pairs of positive integers a, b with $a < b$ such that $a^b = b^a$

A. is an empty set

B. consists of a single pair

C. consists of multiple, but finite number of pairs

D. is countably infinite

47. Suppose c is a given positive real number. The equation $\ln x = cx^2$ must have a solution if

A. $c < 1/(2e)$

B. $c < 1/e$

C. $c > 1/(2e)$

D. $c > (1/e)$

48. Sania's boat is at point A on the sea. The closest point on land, point B, is 2 km away. Point C on land is 6 km from point B, such that triangle (ABC) is right-angled at point B. Sania wishes to reach point C, by rowing to some point P on the line $\bar{BC}$, and jog the remaining distance to C. If she rows 2 km per hour and jogs 5 km per hour, at what distance from point B should she choose her landing point P, in order to minimise her time to reach point C?

A. $21/\sqrt{4}$

B. $4/\sqrt{21}$

C. $4/\sqrt{12}$

D. $21/\sqrt{21}$

49. Suppose $A_j, j = 1, 2, \ldots$ are non-empty sets of real numbers. Define the sets $C_n = \cap_{k=n}^{\infty} \cup_{j=k}^{\infty} A_j, n = 1, 2, \ldots$. Which of the choices below must then hold for a given n? (where the symbol $\subset$ stands for 'strict subset').

    A. $C_n \subset C_{n+1}$

    B. $C_{n+1} \subset C_n$

    C. $C_n = C_{n+1}$

    D. None of the above need hold

50. Suppose x and y are given integers. Consider the following statements:

    A.] If $2x + 3y$ is divisible by 17, then $9x + 5y$ is divisible by 17.

    B.] If $9x + 5y$ is divisible by 17, then $2x + 3y$ is divisible by 17.

    Which of the following is true?

    A. A is true and B is false

    B. B is true and A is false

    C. Both A and B are true

    D. Neither A nor B is true

    The following information is the starting point for Q 51 - 52. Consider an exchange economy with two goods. Suppose agent i and j have the same preferences. Moreover, suppose their preferences have the following property: if (a, b) and (c, d) are distinct bundles that are indifferent to each other, then the bundle $((a + c)/2, (b + d)/2)$ is strictly preferred to (a, b) and (c, d).

51. In a Pareto efficient allocation, i and j

    A. will get the same bundle

    B. may get different bundles

    C. will get the same bundle, provided their endowments are identical

    D. will get the same bundle, provided their endowments are identical and the preferences are monotonically increasing

52. In a competitive equilibrium allocation, i and j

    A. will get the same bundle

    B. may get different bundles

    C. will get the same bundle, if their endowments are identical

    D. will get the same bundle, only if their endowments are identical and the preferences are monotonically increasing

    The following information is the starting point for Q 53 - 55. Two firms produce the same commodity. Let x_1 and x_2 be the quantity choices of firms 1 and 2 respectively. The total quantity is $X = x_1 + x_2$. The inverse demand function is $P = a - bX$, where P is the market price, and a and b are the intercept and the slope parameters respectively.

Firms 1 and 2 have constant average costs equal to c_1 and c_2 respectively. Suppose $b > 0$, $0 < c_1 < c_2 < a$ and $a + c_1 > 2c_2$.

53. In a Cournot equilibrium,
    A. firm 1 has the larger market share and the larger profit
    B. firm 2 has the larger market share and the larger profit
    C. firm 1 has the larger market share and the smaller profit
    D. firm 2 has the larger market share and the smaller profit

54. If a increases, then
    A. the market share of firm 1 increases and price increases
    B. the market share of firm 1 decreases and price increases
    C. the market share of firm 1 increases and price decreases
    D. the market share of firm 1 decreases and price decreases

55. If b decreases, then
    A. the price and market share of firm 1 increase
    B. the price and market share of firm 1 decrease
    C. the market shares are unchanged and price increases
    D. neither price, nor market shares, change

56. Suppose that an economy has endowment of K units of capital and L units of labour. Two final goods X_1 and X_2 can be produced by the following technologies,

$$X_1 = \sqrt{kl}, \quad X_2 = \sqrt{l}$$

where k is quantity of capital and l is quantity of labour. Find the production possibility frontier.
    A. $X_1^2 + KX_2^2 = KL$
    B. $X_1^2 + X_2^2 = KL$
    C. $X_1 + \sqrt{K}X_2 = \sqrt{KL}$
    D. $X_1 + X_2^2 = KL$

57. A two-person two commodity economy has social endowment of $x = 1$ unit of food and $y = 1$ unit of wine. Agents preferences are increasing in own consumption but decreasing in wine consumption of the other person. Preferences of agents A and B are as follows,

$$u_A(x_A, y_A, y_B) = x_A[1 + \max(y_A - y_B, 0)], \quad u_B(x_B, y_B, y_A) = x_B[1 + \max(y_B - y_A, 0)],$$

where A consumes x_A and y_A units of x and y respectively, similarly B's consumption is x_B and y_B. Which of the following is a Pareto optimum allocation.
    A. $x_A = \frac{1}{4}, x_B = \frac{3}{4}, y_A = y_B = \frac{1}{2}$

B. $x_A = x_B = \frac{1}{2}, y_A = \frac{1}{4}, y_B = \frac{3}{4}$

C. $x_A = \frac{1}{4}, x_B = \frac{3}{4}, y_A = 1, y_B = 0$

D. $x_A = \frac{1}{4}, x_B = \frac{3}{4}, y_A = 0, y_B = 1$

58. A two-person two-goods pure exchange economy. The initial endowment vectors are $e^1 = (1, 0)$ and $e^2 = (0, 1)$. The two individuals have identical preferences represented by the utility functions:

$$u^1(x, y) = u^2(x, y) = \begin{cases} 1 & \text{when } x + y < 1 \\ x + y, & \text{when } x + y \geq 1 \end{cases}$$

where x is the quantity of the first good and y is the quantity of the second good. For this economy the set of Pareto optimum allocations

A. consists of the entire Edgeworth box

B. is just the equal division of goods

C. is a null set

D. is $\{((0, 0), (1, 1)), ((1, 1), (0, 0))\}$

59. A monopolist seller produces a good with constant marginal cost $c \geq 0$. The monopolist sells the entire output to a consumer whose utility from consuming x units of the product is given by $\theta\sqrt{x} - t$, where t is the payment made by the consumer to the monopolist. Suppose, consumer's outside option is 0, i.e., if she does not buy the good from the monopolist, she gets 0 utility. Then, the monopolist's profit is

A. $\theta/(4c)$

B. $\theta^2/(4c)$

C. $c\theta^2$

D. $c\theta/2$

60. Consider an economy consisting of $n \geq 2$ individuals with preference relations defined over the set of alternatives X. Let $S = \{a, b, c, d, e\}$ and $T = \{a, b, c, d\}$ be two subsets of X. Now consider the following statements:

- A. If a is Pareto optimal (PO) with respect to set S, then a is PO with respect to set T.
- B. If a is PO with respect to set T, then a is PO with respect to set S.
- C. If a is PO with respect to set S and b is *not* PO with respect to set T, then a is Pareto superior to b.
- D. If a is the only PO alternative in set S and b is *not* PO with respect to set S, then a is Pareto superior to b.

How many of the above statements are necessarily correct?

A. 1

B. 2

C. 3

D. 4

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1. There are two individuals, 1 and 2. Suppose, they are offered a lottery that gives Rs 160 or Rs 80 each with probability equal to 1/2. The alternative to the lottery is a fixed amount of money given to the individual. Assume that individuals are expected utility maximizers. Suppose, individual 1 will prefer to get Rs 110 with certainty over the lottery. However, Individual 2 is happy receiving a sure sum of Rs 90 rather than facing the lottery. Which of the following statements is correct? Given these statements, which of the following is true?

    A. both individuals are risk averse

    B. 2 is risk averse but 1 loves risk

    C. 1 is risk averse but 2 loves risk

    D. none of the above

2. Consider an exchange economy with agents 1 and 2 and goods x and y. The agents' preferences over x and y are given. If it rains, 1's endowment is (10, 0) and 2's endowment is (0, 10). If it shines, 1's endowment is (0, 10) and 2's endowment is (10, 0)

    A. the set of Pareto efficient allocations is independent of whether it rains or shines

    B. the set of Pareto efficient allocations will depend on the weather

    C. the set of Pareto efficient allocations may depend on the weather

    D. whether the set of Pareto efficient allocations varies with the weather depends on the preference of the agents

3. Deadweight loss is a measure of

    A. change in consumer welfare

    B. change in producer welfare

    C. change in social welfare

    D. change in social inequality

4. To regulate a natural monopolist with cost function $C(q) = a + bq$, the government has to subsidize the monopolist under

    A. average cost pricing

    B. marginal cost pricing

    C. non-linear pricing

    D. all of the above

5. Suppose an economic agent lexicographically prefers x to y, then her indifference curves are

    A. straight lines are parallel to the x axis

B. straight lines are parallel to the y axis

C. convex sets

D. L shaped curves

The following Two questions are based on the function $f : \Re^2 \to \Re$ given by

$$f(x,y) = \begin{cases} 0 & \text{for } (x,y) = (0,0) \\ \dfrac{xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \end{cases}$$

for $(x,y) \in \Re^2$

6. Which of the following statements is correct?

   A. f is continuous and has partial derivatives at all points

   B. f is discontinuous but has partial derivatives at all points

   C. f is continuous but does not have partial derivatives at all points

   D. f is discontinuous and does not have partial derivatives at all points

7. Which of the following statements is correct?

   A. f is continuous and differentiable

   B. f is discontinuous and differentiable

   C. f is continuous but not differentiable

   D. f is discontinuous and non-differentiable

8. Consider the following system of equations:

$$\begin{gathered} x + 2y + 2z - s + 2t = 0 \\ x + 2y + 3z + s + t = 0 \\ 3x + 6y + 8z + s + 4t = 0 \end{gathered}$$

   The dimension of the solution space of this system of equations is

   A. 1

   B. 2

   C. 3

   D. 4

9. The vectors $v_0, v_1, ..., v_n$ in $\Re^m$ are said to be affinely independent if with scalars $c_0, c_1, ..., c_n$, $\sum_{i=0}^{n} c_i v_i$ and $\sum_{i=0}^{n} c_i = 0$ implies $c_i = 0$ for $i = 0, 1, ..., n$. For such an affinely independent set of vectors, which of the following is an implication:

   (I) $v_0, v_1, ..., v_n$ are linearly independent.

(II) $(v_1 - v_0), (v_2 - v_0), ..., (v_n - v_0)$ are linearly independent.

(III) $n \leq m$.

A. Only I and II are true

B. Only I and III are true

C. Only II is true

D. Only II and III are true

10. $\Re^2 \to \Re^2$ be a linear mapping (i.e., for every pair of vectors $(x_1, x_2), (y_1, y_2)$ and scalars $c_1, c_2, F(c_1(x_1, x_2) + c_2(y_1, y_2)) = c_1F(x_1, x_2) + c_2F(y_1, y_2)$. Suppose $F(1, 2) = (2, 3)$ and $F(0, 1) = (1, 4)$. Then in general, $F(x_1, x_2)$ equals

A. $(x_2, 4x_2)$

B. $(x_2, x_1 + x_2)$

C. $(1 + x_1, 4x_2)$

D. $(x_2, -5x_1 + 4x_2)$

11. A correlation coefficient of 0.2 between Savings and Investment implies that:

A. A unit change in Income leads to a less than 20 percent increase in Savings

B. A unit change in Income leads to a 20 percent increase in Savings

C. A unit change in Income may cause Savings to increase by less than or more than 20

D. If we plot Savings against Income, the points would lie more or less on a straight line

12. In a simple regression model estimated using OLS, the covariance between the estimated errors and the regressors is zero by construction. This statement is:

A. True only if the regression model contains an intercept term

B. True only if the regression model does not contain an intercept term

C. True irrespective of whether the regression model contains an intercept term

D. False

13. Consider the uniform distribution over the interval $[a, b]$.

A. The mean of this distribution depends on the length of the interval, but the variance does not

B. The mean of this distribution does not depend on the length of the interval, but the variance does

C. Neither the mean, nor the variance, of this distribution depends on the length of the interval

D. The mean and the variance of this distribution depend on the length of the interval

The next Two questions are based on the following information. Let $F : \Re \to \Re$ be a (cumulative) distribution function. Define $b : [0, 1] \to \Re$ by

$$b(c) = \begin{cases} 0 & \text{if } c = 0 \\ inf F^{-1}([c, 1]), & \text{if } c \in (0, 1] \end{cases}$$

14. If F has a jump at x, say $c = F(x) > a \geq F(x-)$, then
    A. b has a jump at c
    B. b has a jump at a
    C. b is strictly increasing over (a, c)
    D. b is constant over (a, c)

15. If F is constant over (x, y) with $F(z) < F(x)$ for every $z < x$, then
    A. b has a jump at y
    B. b has a jump at x
    C. b is continuous at $F(x)$
    D. b is decreasing over $[0, F(x)]$

The following set of information is relevant for the next Four questions. Consider a closed economy where at any period t the actual output (Y_t) is demand-determined. Aggregate demand on the other hand has two components: consumption demand (C_t) and investment demand (I_t). Both consumption and investment demands depend on agents' expectation about period t output (Y_t^e) in the following way:

$$C_t = \alpha Y_t^e \ ; 0 < \alpha < 1 \ ,$$
$$I_t = \gamma (Y_t^e)^2 \ ; \gamma > 0$$

16. Suppose agents have static expectations. Static expectation implies that
    A. in every period agents expect the previous period's actual value to prevail
    B. in every period agents adjust their expected value by a constant positive fraction of the expectational error made in the previous period
    C. in every period agents use all the information available in that period so that the expected value can differ from the actual value if and only if there is a stochastic element present
    D. none of the above

17. Under static expectations, starting from any given initial level of actual output $Y_0 \neq \frac{1-\alpha}{\gamma}$, in the long run the actual output in this economy
    A. will always go to zero

B. will always go to infinity

C. will always go to a finite positive value given by $\frac{1-\alpha}{\gamma}$

D. will go to zero or infinity depending on whether $Y_0 >$ or $< \frac{1-\alpha}{\gamma}$

18. Suppose now agents have rational expectations. Rational expectation implies that

A. in every period agents expect the previous period's actual value to prevail

B. in every period agents adjust their expected value by a constant positive fraction of the expectational error made in the previous period

C. in every period agents use all the information available in that period so that the expected value can differ from the actual value if and only if there is a stochastic element present

D. none of the above

19. Under rational expectations, in the long run the actual output in this economy

A. will always go to zero

B. will always go to infinity

C. will always go to a finite positive value given by $\frac{1-\alpha}{\gamma}$

D. will go to zero or infinity depending on agent's expectations

20. Suppose we conduct n independent Bernoulli trials, each with probability of success p. If k is such that the probability of k successes is equal to the probability of $k+1$ successes, then

A. $(n+1)p = n(1+p)$

B. $np = (n-1)(1+p)$

C. np is a positive integer

D. $(n+1)p$ is a positive integer

The next Two questions are based on the following. Consider a pure exchange economy with three persons: 1 , 2, 3 and two goods x and y.
The utilities are given by $u^1(.) = xy, u^2(.) = x^3y$ and $u^3(.) = xy^2$, respectively.

21. If the endowments are (2,0), (0,12) and (12,0), respectively, then

A. an equilibrium price ratio does not exist

B. $p_x/p_y = 1$ is an equilibrium price ratio

C. $p_x/p_y > 1$ is an equilibrium price ratio

D. $p_x/p_y < 1$ is an equilibrium price ratio

22. If the endowments are (0,2), (12,0) and (0,12), respectively, then

A. an equilibrium price ratio does not exist

B. equilibrium price ratio is the same as in the above question

C. $p_x/p_y < 1$ is an equilibrium price ratio

D. $p_x/p_y > 1$ is an equilibrium price ratio

The next Two questions are based on the following information. A city has a single electricity supplier. Electricity production cost is Rs. c per unit. There are two types of customers. Utility function for type i is given by $u_i(q,t) = \theta_i \; ln(1+q) - t$, where q is electricity consumption and t is electricity tariff. High type customers are more energy efficient, that is, $\theta_H > \theta_L$; moreover $\theta_L > c$.

23. Suppose the supplier can observe type of the consumer, i.e., whether $\theta = \theta_H$ or $\theta = \theta_L$. If the supplier decides to sell package (q_H, t_H) to those for whom $\theta = \theta_H$ and (q_L, t_L) to those for whom $\theta = \theta_L$, then profit maximizing tariffs will be

A. $t_H = c \; ln(\frac{\theta_H}{c})$ and $t_L = c \; ln(\frac{\theta_L}{c})$

B. $t_H = \theta_H \; ln(\frac{\theta_H}{c})$ and $t_L = 0$

C. $t_H = \theta_H \; ln(\frac{\theta_H}{c})$ and $t_L = \theta_L \; ln(\frac{\theta_L}{c})$

D. $t_H = t_L = c \; ln(\frac{\theta_H + \theta_L}{c})$

24. Now, assume that the supplier cannot observe type of the consumer. Suppose, he puts on offer both of the packages that he would offer in the above question. If consumers are free to choose any of the offered packages, then

A. Both types will earn zero utility

B. Only low type can earn positive utility

C. Only high type can earn positive utility

D. Both types can earn positive utility

25. Suppose buyers of ice-cream are uniformly distributed on the interval $[0, 1]$. Ice-cream sellers 1 and 2 simultaneously locate on the interval, each locating so to maximize her market share given the location of the rival. Each seller's market share corresponds to the proportion of buyers who are located closer to her location than to the rival's location

A. Both will locate at 1/2

B. One will locate at 1/4 and the other at 3/4

C. One will locate at 0 and the other at 1

D. One will locate at 1/3 and the other at 2/3

26. In the context of previous question, suppose it is understood by all players that seller 3 will locate on [0, 1] after observing the simultaneous location choices of sellers 1 and 2. Seller 3 aims to maximize market share given the locations of 1 and 2. The locations of sellers 1 and 2 are as follows:

    A. Both will locate at 1/2

    B. One will locate at 1/4 and the other at 3/4

    C. One will locate at 0 and the other at 1

    D. One will locate at 1/3 and the other at 2/3

27. Consider a government and two citizens. The government has to decide whether to create a public good, say a park, at cost Rs 100. The value of the park is Rs 30 to the citizen 1 and Rs 60 to citizen 2; each valuation is private information for the relevant citizen and not known to the government. The government asks the citizens to report their valuations, say r_1 and r_2. It cannot verify the truthfulness of the reports. It decides to build the park if $r_1 + r_2 \geq 100$, in which case, citizen 1 will pay the tax 100 - r_2 and citizen 2 will pay the tax 100 - r_1. If the park is not built, then no taxes are imposed. The reported valuations will be

    A. $r_1 < 30$ and $r_2 > 60$

    B. $r_1 > 30$ and $r_2 < 60$

    C. $r_1 = 60$ and $r_2 = 30$

    D. $r_1 = 30$ and $r_2 = 60$

28. In the context of the previous question, suppose the only change is that citizen 1's valuation rises to 50 and the same procedure is followed, then

    A. The park will be built and result in a government budget surplus of Rs 10

    B. The park will be built and result in a government budget deficit of Rs 10

    C. The park will be built and result in a government balanced budget

    D. The park will not be built

29. Consider the following two games in which player 1 chooses a row and player 2 chooses a column.

$$\begin{array}{c} \\ \textit{Enter} \\ \textit{Not enter} \end{array} \begin{array}{c} \textit{Hawk} \\ \begin{pmatrix} -1,1 \\ 0,6 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \textit{Enter} \\ \textit{Not enter} \end{array} \begin{array}{cc} \textit{Hawk} & \textit{Dove} \\ \begin{pmatrix} -1,1 & 3,3 \\ 0,6 & 0,7 \end{pmatrix} \end{array}$$

    Analysis of these games shows

    A. Having an extra option cannot hurt

    B. Having an extra option cannot hurt as long as it dominates other options

    C. Having an extra option can hurt if the other player is irrational

D. Having an extra option can hurt if the other player is rational

30. Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1 lexicographically prefers x to y. Agent 2's utility function is $\min\{x, y\}$. Agent 1's endowment is (0, 10) and agent 2's endowment is (10, 0). The competitive equilibrium price ratio, p_x/p_y, for this economy

A. can be any positive number

B. is greater than 1

C. is less than 1

D. does not exist

31. Consider a strictly increasing , differentiable function $u : \Re^2 \to \Re$ and the equations:

$$\frac{D_1 u(x_1, x_2)}{D_2 u(x_1, x_2)} = \frac{p_1}{p_2} \text{ and}$$
$$p_1 x_1 + p_2 x_2 = w,$$

where p_1, p_2, w are strictly positive. What additional assumptions will guarantee the existence of continuously differentiable functions $x_1(p_1, p_2, w)$ and $x_2(p_1, p_2, w)$ that will solve these equations for all strictly positive p_1, p_2, w?

A. u is injective

B. u is bijective

C. u is twice continuously differentiable

D. u is twice continuously differentiable and $\begin{bmatrix} D_{11}u(x_1,x_2)p_2 - D_{12}u(x_1,x_2)p_1 & D_{12}u(x_1,x_2)p_2 - D_2 2u(x_1,x_2)p_1 \\ p_1 & p_2 \end{bmatrix}$ is nonsingular

32. As $n \uparrow \infty$, the sequence $(-1)^n(1 + n^{-1})$

A. converges to 1

B. converges to -1

C. converges to both -1 and 1

D. does not converge

33. The set $(0, 1)$ can be expressed as

A. the union of a finite family of closed intervals

B. the intersection of a finite family of closed intervals

C. the union of an infinite family of closed intervals

D. the intersection of an infinite family of closed intervals

34. The set $[0, 1]$ can be expressed as

A. the union of a finite family of open intervals

B. the intersection of a finite family of open intervals

C. the union of an infinite family of open intervals

D. the intersection of an infinite family of open intervals

The following information is used in the next Two questions. Consider a linear transformation
$P : \Re^n \to \Re^n$. Let $\mathcal{R}(\mathcal{P}) = \{Px \mid x \in \Re^n\}$ and $\mathcal{N}(\mathcal{P}) = \{x \in \Re^n \mid Px = 0\}$.
P is said to be a projector if
(a) every $x \in \Re^n$ can be uniquely written as $x = y + z$ for some $y \in \mathcal{R}(\mathcal{P})$ and $z \in \mathcal{N}(\mathcal{P})$, and
(b) $P(y + z) = y$ for all $y \in \mathcal{R}(\mathcal{P})$ and $z \in \mathcal{N}(\mathcal{P})$

35. If P is a projector, then

A. $P^2 = I$, where I is the identity mapping

B. $P = P^{-1}$

C. $P^2 = P$

D. Both (a) and (b)

36. If P is a projector and $Q : \Re^m \to \Re^n$ is a linear transformation such that $\mathcal{R}(\mathcal{P}) = \mathcal{R}(\mathcal{Q})$, then

A. $QP = P$

B. $PQ = Q$

C. $QP = I$

D. $PQ = I$

37. Suppose that a and b are two consecutive roots of a polynomial function f, with $a < b$. Suppose a and b are non-repeated roots. Consequently, $f(x) = (x - a)(x - b)g(x)$ for some polynomial function g. Consider the statements:

(I) $g(a)$ and $g(b)$ have opposite signs

(II) $f'(x) = 0$ for some $x \in (a, b)$

Of these statements,

A. Both I and II are true

B. Only I is true

C. Only II is true

D. Both I and II are false

38. Suppose $f : [0, 1] \to \Re$ is a twice differentiable function that satisfies $D^2 f(x) + Df(x) = 1$ for every $x \in (0, 1)$ and $f(0) = 0 = f(1)$. Then,

A. f does not attain positive values over (0,1)

B. f does not attain negative values over (0,1)

C. f attains positive and negative values over (0,1)

D. f is constant over (0,1)

39. Suppose $x_1, \ldots, x_n$ are positive and $\lambda_1, \ldots, \lambda_n$ are non-negative with $\sum_{i=1}^{n} \lambda_i = 1$. Then

A. $\sum_{i=1}^{n} \lambda_i x_i \geq x_1^{\lambda_1} \ldots x_n^{\lambda_n}$

B. $\sum_{i=1}^{n} \lambda_i x_i < x_1^{\lambda_1} \ldots x_n^{\lambda_n}$

C. $\sum_{i=1}^{n} \lambda_i x_i \leq \sqrt{x_1^{\lambda_1} \ldots x_n^{\lambda_n}}$

D. None of the above is necessarily true

40. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$. Suppose there is a bijection, i.e., a one-to-one correspondence (an "into" and "onto" mapping), between $\mathcal{N}$ and a set X. Suppose there is also a bijection between $\mathcal{N}$ and a set Y . Then,

A. there is a bijection between $\mathcal{N}$ and $X \cup Y$

B. there is a bijection between $\mathcal{N}$ and $X \cap Y$

C. there is no bijection between $\mathcal{N}$ and $X \cap Y$

D. there is no bijection between $\mathcal{N}$ and $X \cup Y$

The next Three questions pertain to the following: A simple linear regression of wages on gender, run on a sample of 200 individuals, 150 of whom are men, yields the following

$$\underset{(20)}{W_i = 300} \underset{(10)}{- 50 D_i} + u_i$$

where W_i is the wage in Rs per day of the i^{th} individual, $D_i = 1$ if individual i is male, and 0 otherwise, u_i is a classical error term , and the figures in parentheses are standard errors.

41. What is the average wage in the sample?

A. Rs. 250 per day

B. Rs. 275 per day

C. Rs. 262.50 per day

D. Rs. 267.50 per day

42. The most precise estimate of the difference in wages between men and women would have been obtained if, among these 200 individuals,

A. There were an equal number (100) of men and women in the sample

B. The ratio of the number of men and women in the sample was the same as the ratio of their average wages

C. There were at least 30 men and 30 women; this is sufficient for estimation: precision does not depend on the distribution of the sample across men and women

D. None of the above

43. The explained (regression) sum of squares in this case is:

A. 93750

B. 1406.25

C. 15000

D. This cannot be calculated from the information given

44. A researcher estimate the following two models using OLS
Model A: $y_i = \beta_0 + \beta_1 S_i + \beta_2 A_i + \epsilon_i$
Model B: $y_i = \beta_0 + \beta_1 S_i + \epsilon_i$
where y_i refers to the marks (out of 100) that a student i gets on an exam, S_i refers to the number of hours spent studying for the exam by the student, and A_i is an index of innate ability (varying continuously from a low ability score of 1 to a high ability score of 10). ϵ_i the usual classical error term.
The estimated β_1 coefficient is 7.1 for Model A, but 2.1 for Model B; both are statistically significant. The estimated β_2 coefficient is 1.9 and is also significantly different from zero. This suggests that:

A. Students with lower ability also spend fewer hours studying

B. Students with lower ability spend more time studying

C. There is no way that students of even high ability can get more than 40 marks

D. None of the above

45. An analyst estimates the model $Y = \beta_1 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ using OLS.
But the true $\beta_3 = 0$. In this case, by including X_3

A. there is no harm done as all the estimates would be unbiased and efficient

B. there is a problem because all the estimates would be biased and inconsistent

C. the estimates would be unbiased but would have larger standard errors

D. the estimates may be biased but they would still be efficient

46. Let $\hat{\beta}$ be the OLS estimator of the slope coefficient in a regression of Y on X_1. Let $\tilde{\beta}$ be the OLS estimator of the coefficient on X_1 on a regression of Y on X_1 and X_2. Which of the following is true

A. $\text{Var}(\hat{\beta}) < \text{Var}(\tilde{\beta})$

B. $\text{Var}(\hat{\beta}) > \text{Var}(\tilde{\beta})$

C. $\text{Var}(\hat{\beta}) <$ or $> \text{Var}(\tilde{\beta})$

D. $\text{Var}(\hat{\beta}) = \text{Var}(\tilde{\beta})$

47. You estimate the multiple regression $Y = a + b_1X_1 + b_2X_2 + u$ with a large sample. Let t_1 be the test statistic for testing the null hypothesis $b_1 = 0$ and t_2 be the test statistic for testing the null hypothesis $b_2 = 0$. Suppose you test the joint null hypothesis that $b_1 = b_2 = 0$ using the principle 'reject the null if either t_1 or t_2 exceeds 1.96 in absolute value', taking t_1 and t_2 to be independently distributed.
    A. The probability of error Type 1 is 5 percent in this case
    B. The probability of error Type 1 is less than 5 percent in this case
    C. The probability of error Type 1 is more than 5 percent in this case
    D. The probability of error Type 1 is either 5 percent or less than 5 percent in this case

48. Four taste testers are asked to independently rank three different brands of chocolate (A, B, C). The chocolate each tester likes best is given the rank 1, the next 2 and then 3. After this, the assigned ranks for each of the chocolates are summed across the testers. Assume that the testers cannot really discriminate between the chocolates, so that each is assigning her ranks at random. The probability that chocolate A receives a total score of 4 is given by:
    A. 1/4
    B. 1/3
    C. 1/27
    D. 1/81

49. Suppose 0.1 percent of all people in a town have tuberculosis (TB). A TB test is available but it is not completely accurate. If a person has TB, the test will indicate it with probability 0.999. If the person does not have TB, the test will erroneously indicate that s/he does with probability 0.002. For a randomly selected individual, the test shows that s/he has TB. What is the probability that this person actually has TB?
    A. $\frac{0.002}{0.999}$
    B. $\frac{1}{1000}$
    C. $\frac{1}{3}$
    D. $\frac{2}{3}$

50. There exists a random variable X with mean μ_X and variance σ_X^2 for which $P[\mu_X - 2\sigma_X \leq X \leq \mu_X + 2\sigma_X] = 0.6$. This statement is:
    A. True for any distribution for appropriate choices of μ_X and σ_X^2 .
    B. True only for the uniform distribution defined over an appropriate interval
    C. True only for the normal distribution for appropriate choices of μ_X and σ_X^2
    D. False

51. Consider a sample size of 2 drawn without replacement from an urn containing three balls numbered 1, 2, and 3. Let X be the smaller of the two numbers drawn and Y the larger. The covariance between X and Y is given by

A. $\frac{1}{9}$

B. $\frac{3}{11}$

C. $\frac{11}{3}$

D. $\frac{3}{4}$

52. Consider the square with vertices (0, 0), (0, 2), (2, 0) and (2, 2). Five points are independently and randomly chosen from the square. If a point (x, y) satisfies $x+2y \leq 2$, then a pair of dice are rolled. Otherwise, a single die is rolled. Let N be the total number of dice rolled. For $5 \leq n \leq 10$, the probability that $N = n$ is

A. $\binom{5}{n-5}(1/2)^{n-5}(1/2)^{5-(n-5)}$

B. $\binom{10}{n-10}(1/4)^{n-10}(3/4)^{n}$

C. $\binom{5}{n-5}(1/4)^{n-5}(3/4)^{10-n}$

D. $\binom{10}{n-10}(1/2)^{n-10}(1/2)^{n}$

53. Suppose S is a set with $n > 1$ elements and $A_1, ..., A_m$ are subsets of S with the following property: if $x, y \in S$ and $x \neq y$, then there exists $i \in \{1, ..., m\}$ such that, either $x \in A_i$ and $y \notin A_i$, or $y \in A_i$ and $x \notin A_i$. Then the following necessarily holds.

A. $n = 2^m$

B. $n \leq 2^m$

C. $n > 2^m$

D. None of the above

The following set of information is relevant for the next Six questions. Consider the following version of the Solow growth model where the aggregate output at time t depends on the aggregate capital stock (K_t) and aggregate labour force (L_t) in the following way:

$$Y_t = (K_t)^{\alpha}(L_t)^{1-\alpha}; 0 < \alpha < 1.$$

At every point of time there is full employment of both the factors and each factor is paid its marginal product. Total output is distributed equally to all the households in the form of wage earnings and interest earnings. Households' propensity to save from the two types of earnings differ. In particular, they save s_w proportion of their wage earnings and s_r proportion of their interest earnings in every period. All savings are invested which augments the capital stock over time $\frac{dK}{dt}$. There is no depreciation of capital. The aggregate labour force grows at a constant rate n

54. Let $s_w = 0$ and $s_r = 1$. An increase in the parameter value α

A. unambiguously increases the long run steady state value of the capital-labour ratio

B. unambiguously decreases the long run steady state value of the capital-labour ratio

C. increases the long run steady state value of the capital-labour ratio if $\alpha > n$

D. leaves the long run steady state value of the capital-labour ratio unchanged

55. Now suppose $s_r = 0$ and $0 < s_w < 1$. An increase in the parameter value n

A. unambiguously increases the long run steady state value of the capital-labour ratio

B. unambiguously decreases the long run steady state value of the capital-labour ratio

C. increases the long run steady state value of the capital-labour ratio if $\alpha > n$

D. leaves the long run steady state value of the capital-labour ratio unchanged

56. Now let both s_w and s_r be positive fractions such that $s_w < s_r$. In the long run, the capital-labour ratio in this economy

A. approaches zero

B. approaches infinity

C. approaches a constant value given by $\left[\frac{(1-\alpha)s_w+\alpha s_r}{n}\right]^{\frac{1}{1-\alpha}}$

D. approaches a constant value given by $\left[\frac{\alpha s_w+(1-\alpha)s_r}{n}\right]^{\frac{1}{\alpha}}$

57. Suppose now the government imposes a proportional tax on wage earnings at the rate τ and redistributes the tax revenue in the form of transfers to the capital-owners. People still save s_w proportion of their net (post-tax) wage earnings and s_r proportion of their net (post-transfer) interest earnings. In the new equilibrium, an increase in the tax rate τ

A. unambiguously increases the long run steady state value of the capital-labour ratio

B. unambiguously decreases the long run steady state value of the capital-labour ratio

C. increases the long run steady state value of the capital-labour ratio if $\alpha > n$

D. leaves the long run steady state value of the capital-labour ratio unchanged

58. Let us now go back to case where both s_w and s_r are positive fractions such that $s_w < s_r$ but without the tax-transfer scheme. However, now let the growth rate of labour force be endogenous such that it depends on the economy's capital-labour ratio in the following way:

$$\frac{1}{L}\frac{dL}{dt} = \begin{cases} Ak_t & \text{for } k_t < \overline{k}; \\ 0 & \text{for } k_t >> k, \end{cases}$$

where $\overline{k} > [\frac{(1-\alpha)s_w+\alpha s_r}{A}]^{\frac{1}{2-\alpha}}$ is a given constant. In the long run, the capital- labour ratio in this economy

A. approaches zero

B. approaches infinity

C. approaches a constant value given by $[\frac{\alpha s_w+(1-\alpha)s_r}{A}]^{\frac{1}{\alpha}}$

D. approaches a constant value given by $[\frac{(1-\alpha)s_w+\alpha s_r}{A}]^{\frac{1}{1-\alpha}}$ depending on the initial $k_0 > or < \overline{k}$

59. In the above question, an increase in the parameter value A

A. unambiguously increases the long run steady state value of the capital-labour ratio

B. unambiguously decreases the long run steady state value of the capital-labour ratio

C. decreases the long run steady state value of the capital-labour ratio only when the initial $k_0 < \overline{k}$

D. leaves the long run steady state value of the capital-labour ratio unchanged

60. A profit maximizing firm owns two production plants with cost functions $c_1(q) = \frac{q^2}{2}$ and $c_2(q) = q^2$, respectively. The firm is free is use either just one or both of the plants to achieve any given level of output. For this firm, the marginal cost curve

A. lies above the 45 degree line through the origin, for all positive output levels

B. lies below the 45 degree line through the origin, for all positive output levels

C. is the 45 degree line through the origin

D. None of the above

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1. If the following equation is estimated using OLS, and a 95% confidence interval for β_1 is constructed, then which of the following is true?

   A. The OLS estimate of β_1 lies in the given interval with 95% probability

   B. The true value of β_1 lies in the given interval with 95% probability

   C. In repeated sampling 95% of the times the confidence interval will contain the true value of β_1

   D. Both (B) and (C)

2. What term would best describe the shape of the given boxplot?

   A. Right-skewed

   B. Left-skewed

   C. Uniform

   D. Normal

3. The vitamin content of a particular brand of vitamin supplement pills is normally distributed with mean 490 mg and standard deviation 12 mg. What is the probability (approximately) that a randomly selected pill contains at least 500 mg of Vitamin C?

   A. 0.8

   B. 0.2

   C. 0.025

   D. 0.55

4. A sample of 100 cows is drawn to estimate the mean weight of a large herd of cattle. If the standard deviation of the sample is 100 kg, what is the approximate maximum error in a 95% confidence interval estimate?

   A. 10

   B. 20

   C. 30

   D. 40

5. Recent studies suggest that the migration to Indian cities from rural regions(MIG) can be explained be Quality of life(QL), state income as a ratio of Aggregate Indian income(Y) and the ratio of state employment to overall employment in India(E). Using data for the 29 Indian states, the following model is estimated:

   $\hat{MIG} = \underset{(0.9)}{-4.2} + \underset{(0.8)}{1.2}QL - \underset{0.05}{(0.6)}Y - \underset{(0.02)}{0.8}E$ The figures in the parentheses are standard errors. The t-statistic for the null hypothesis that the quality of life(QL) index does not impact migration is

   A. 0.8

B. 0.9

C. 1.5

D. Insufficient information to calculate

6. Consider the following estimated regression relating expenditures on Food(Y_i) to Income (X_i) $\hat{Y}_i = 145 + 0.3X_i - 0.1(X_i - X_i^*)D_i$

   where i denotes the individual; $X_i^* = 500$, this threshold distinguishes low-income from high-income individuals, and D_i takes value 1 if $X_i \geq X^*$ and 0 otherwise. All the estimated coefficients are significant at 5% level. Which of the following statements is false?

   A. The marginal propensity to consume for people with low income is 0.3, and is lower for those with higher incomes; this makes sense as it is in accordance with Engel's Law

   B. This is a differential slope, common intercept dummy variable formulation with an additional restriction that leads to kinked Engel curve

   C. This is a standard differential slope common-intercept dummy variable formulation

   D. Using this formulation yields predicted expenditure on food for people at income level 490 that is not very different from those people at income level 510 (estimate lie within 5 percent of each other)

7. The parameters of the following multiple regression model has been estimated using OLS. $Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$ Denote the estimated residuals from this regression as e. Which of the following statements is false?

   A. The R-squared from the regression of e on a constant, X_2 and X_3 is zero

   B. The R-squared from the regression of Y on a constant and $\hat{Y}$ is the same as in the original regression ($\hat{Y}$ stands for estimated value of Y)

   C. The slope coefficient from the regression of Y on a constant and $\hat{Y}$ is 1 and the intercept is 0

   D. The R-squared from the regression of Y on a constant and e is same as in the original regression

8. In the multiple regression model with 4 explanatory variables, with standard assumptions, estimated using ordinary least squares, all the coefficients turn out to be insignificant although the overall R-squared is high and associated F-statistic is significant. Also, pair-wise correlations amongst the four explanatory variables are all low, and range between 0.1 and 0.2, but are not statistically different from 0. Which of the following statements is false?

   A. Even though the pair-wise correlations between the explanatory variables are low, since they are (individually) statistically different from zero, OLS coefficients are likely biased

B. This is likely a case of multicollinearity even though the pairwise correlations are low

C. If the analyst is only interested in making forecasts then the insignificance of coefficients is per se not a problem since the F-statistic is significant and R-squared is high

D. Dropping a variable may improve significance of remaining coefficients but they may be biased

9. Instead of estimating a true cost function, which is described as a quadratic, where costs are regressed on an intercept, output and a quadratic term in output, a researcher estimates a linear function by regressing costs on an intercept and output. The estimates from linear cost function are likely to

A. have autocorrelated residuals

B. be a biased estimate of marginal cost, even though there is no exact linear relationship between the linear and quadratic terms in output

C. both (A) and (B)

D. neither (A) nor (B)

10. Using data on class size (CS) and average test percentage(TP) from 101 classes, the following OLS regression is estimated: $\hat{TP} = 96.4 - 1.12CS$ R-squared is 0.1 and SER(Standard error of regression) is 5. What is the sample variance of test percentages across the 101 classes?

A. 27.5

B. 22.5

C. 5.0

D. 2.5

11. In the OLS regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, suppose the coefficient of determination is estimated to be 0.6. We now transform the variables such that $Y_i^* = 0.5Y$ and $X_i^* = 0.75X_i$ and re-run the regression. The coefficient of determination is now:

A. 0.6

B. 0.4

C. 0.9

D. 0.3

12. The OLS regression of infant birth weight(BWT) on the mother's age(AGE) and years of mother's education(EDU) is: $\hat{BWT} = \underset{(97)}{2600} + \underset{(3.5)}{2.3}AGE + \underset{8}{26}EDU$ Where the standard errors are reported below the estimated coefficients. Sample size is 1000 and R-squared si 0.015. Sample information is provided in the table below:

	BWT	AGE	EDU
Mean	3000	25	10
Standard deviation	500	5	2

A one standard deviation change in AGE is associated with an x standard deviation change in birth weight, where x is

A. 0.046

B. 0.007

C. 0.023

D. 0.035

13. If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function ($\mathbb{R}$ is the set of real numbers) and E is an open subset of $\mathbb{R}$, then the set $\{x \in \mathbb{R} | f(x) \in E\}$ is necessarily

A. open

B. closed

C. neither open, nor closed

D. open and closed

14. If X is a $n \times n$ non singular matrix, such that $XX^T = X^TX$ (X^T denotes the transpose of X). Let $Y = X^{-1}X^T$ where X^{-1} denotes the inverse of X. Then YY^T is equal to

A. $I + Y$ (I is the identity matrix)

B. I

C. Y^{-1}

D. Y^T

15. Suppose that $g(x)$ is a twice differential function and $g(1) = 1; g(2) = 4; g(3) = 9$. Which of the following is necessarily true? First and second derivatives of g are represented as g' and g'' respectively

A. $g''(x) = 3$ for some $x \in [1, 2]$

B. $g''(x) = 5$ for some $x \in [2, 3]$

C. $g''(x) = 2$ for some $x \in [1, 3]$

D. $g''(x) = 2$ for some $x \in [1.5, 2.5]$

16. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sum_{i=1}^{11} |x - a_i|$ for all $x \in R$, where $a_1 < \ldots, a_{11}$. This function has a minimum when x equals :

A. a_2

B. a_6

C. a_7

D. a_{11}

17. Consider the set $\{x, y \in \mathbb{R}_+ \times \mathbb{R} | y \leq \ln(x) - e^x\}$. This set is

    A. a linear subspace of $\mathbb{R}^2$

    B. Convex

    C. convex and linear subspace of $\mathbb{R}^2$

    D. neither convex nor linear subspace of $\mathbb{R}^2$

18. A school with n students has m clubs to which they can belong, and students are allowed to be members of multiple clubs. It is known that (I) Each club has an odd number of members (II) Every pair of clubs has an even number of common members (including 0) Then it must be that:

    A. $m \geq n$

    B. $m \leq n$

    C. m and n can each be larger than the other, but $m \neq n$

    D. $m = n$

19. Let $n > 1$, and let S be the set of all $n \times n$ matrices whose entries are all chosen from the set $0, 1$. Then the sum of the determinants of all these matrices

    A. is equal to 0

    B. is positive but less than $n^{\frac{n}{2}}$

    C. can be positive or negative, but is bounded by n^n

    D. is either -1 or 1

20. $g : \mathbb{R} \to [0, 1]$ is a non-decreasing and right continuous step function such that $g(x) = 0$ for all $x \leq 0$ and $g(x) = 1$ for all $x \geq 1$. Let us define g^{-1} as follows, $g^{-1}(y) = \inf\{x \geq 0 | g(x) \geq y\}$. Which of the following is true about g^{-1}:

    A. is a continuous function

    B. is right-continuous but not left continuous

    C. is left-continuous but not right continuous

    D. neither left-continuous nor right-continuous

    The next two questions pertain to the following: Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. There are 6 units of X and 4 units of Y available. Agent 1 has the utility function $u_1 = \min\{x_1, y_1\}$ and agent 2 has the utility function $u_2 = x_2 + y_2$.

21. Which of the following allocations is not Pareto efficient?

    A. $(2, 0), (4, 4)$

    B. $(4, 4), (2, 0)$

    C. $(2, 2), (4, 2)$

    D. $(0, 0), (6, 4)$

22. The set of equilibrium prices for this exchange economy is given by?

    A. $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} \leq 1\}$

    B. $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} = 1\}$

    C. $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} \geq 1\}$

    D. Insufficient information to conclude

23. The market for widgets has a demand function $Q = 100 - 10p$, where Q is the quantity demanded and P is the price in rupees. There are 10 price taking firms in the market, each having a cost function $c(q) = \frac{q^2}{2}$, where q is the firm's own output. There is no new entry. Suppose the government imposes an excise tax of Rs. 2 per unit of widgets, to be paid by sellers, the equilibrium market price is

    A. 7

    B. 6

    C. 5

    D. 2

24. A consumer has utility function $u(x_1, x_2) = \min\{3x_1 + x_2, x_1 + 2x_2\}$. Prices of the two goods are p_1 and p_2 respectively. The consumer will buy positive quantities of both the goods if and only if the price ratio $\frac{p_1}{p_2}$ is

    A. greater than 3

    B. between 1/2 and 3

    C. between 2 and 3

    D. less than

25. Utility function of a consumer over three goods X, Y, Z is $U = y\dot{\min}\{x, z\}$. Prices of all the three goods are the same in the market. Three discount deals are available, which are as follows: Deal I: Get a unit of Z free with a unit of X Deal II: Get a unit of Z free with a unit of Y Deal III: Get $\frac{1}{2}$ unit of X and $\frac{1}{2}$ unit of Z free with a unit of Y Which of the deals should the consumer choose?

    A. Deal I

    B. Deal II

    C. Deal III

    D. All the deals are equally good

26. Two widget producers, A and B, operate in perfectly competitive input and output markets. Firm A uses capital (k) and labour (l) to produce widget; its production function is $f_1(k_1, l_1) = (k_1 l_1)^{\frac{1}{3}}$. Firm B uses only labour; its production function is $f_2(l_2) = \frac{\sqrt{l_2}}{1+k_1}$. Efficiency of input allocation can be improved by:

    A. imposing a tax on capital use

    B. merging firms A and B

C. providing a subsidy on labour use to firm A

D. all of the above

The next two questions pertain to the following: A student has the opportunity to take a test at most thrice. The student knows that each time she takes the test, her score is an independent random draw from the uniform distribution $[0, 100]$. Each time the student takes the test and learns her score, she can either stop and accept it as her official score, or she can discard the result and retake the test. However, after the third attempt, the student has no more opportunities to retake the test. In that case,her score on the last(i.e., third) try will be her official score. The student's objective is to maximize her expected official score

27. If the student follows an optimal plan, her final expected score before taking any of the tests is approximately:

    A. 30

    B. 50

    C. 70

    D. 90

28. Now consider the case when the university allows the student to take the final test only when her score is below 40 in the second test. The student retains the choice to stop or retake after the first attempt. The student will decide to be retested after the first test if and only if her score is less than:

    A. 37.5

    B. 50

    C. 62

    D. 67.5

The next two questions pertain to the following: Consider a homogeneous goods market with two firms. Let x_1 and x_2 be the quantity choices of Firms 1 and 2 respectively. The total quantity is $X = x_1 + x_2$. The inverse demand function is $P = a - bX$, where P is the market price and A and B are the intercept and slope parameters respectively. Both firms have the same marginal costs denoted by c. Suppose $b > 0$, and $0 < 3c < a$.

29. Suppose firm 1 has an objective of maximizing revenue and firm 2 has an objective of maximizing profit. Both firms choose quantities simultaneously. Then:

    A. Firm 1 has larger market share and larger profits

    B. Firm 2 has smaller market share but larger profits

    C. Firm 1 has smaller market share but larger profits

    D. Firm 2 has larger market share and larger profits

30. Suppose both firms become revenue maximizers. Then:

A. Both will produce more than Cournot output

B. Both will produce less than Cournot output

C. Both will produce the perfectly competitive output

D. Both will produce more than the perfectly competitive output.

The next two questions pertain to the following: Four cities A,B,C,D are located as vertices of a square ABCD, and are connected by roads that form the four sides of the square. Mr. Rand Walker travels thus: if he is at city i ($i \in \{A, B, C, D\}$) in period t, then he randomly, with probability $\frac{1}{2}$ each, moves to one of the two vertices/cities that are adjacent to i in period $t + 1$.

31. If Mr. Walker is at city A at time $t = 0$ then the respective probabilities with which he is at cities A, B, C, D in period $t = 10$ are:

A. $1/4, 1/4, 1/4, 1/4$

B. $0, 1/2, 0, 1/2$

C. $1/2, 0, 0, 1/2$

D. $1/2, 0, 1/2, 0$

32. If Mr. Walker is at city A at time $t = 0$, then what is the probability that he never visits city A again till(including) period $t = 10$?

A. $(1/2)^5$

B. $(1/2)^{10}$

C. $(3/4)^5$

D. $(3/4)^{10}$

33. Consider a railway signalling system: a signal is received by the station A from the traffic control office and then transmit it to station B. Suppose that at the origin (traffic control office) signal can be yellow or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively. The probability of each station receiving the signal correctly from its predecessor is $\frac{3}{4}$. If the signal received at station B is yellow, then the probability that the signal was yellow is

A. 22/23

B. 11/20

C. 20/23

D. 9/20

34. Suppose X has a normal distribution with mean 0 and variance σ^2 . Let Y be an independent random variable taking values -1 and 1 with equal probability. Define $Z = XY + \frac{X}{Y}$. Which of the following is true?

A. $Var(Z) > \sigma^2$

B. $Var(Z) < \sigma^2$

C. $Var(Z) = \sigma^2$

D. $Var(Z)$ can be greater than or smaller than σ^2

35. According to Ricardian equivalence proposition, a reduction in the current (lump sum) taxation of household income

    A. would reduce current consumption, but leave future consumption unaffected

    B. would reduce future consumption, but leave current consumption unaffected

    C. would reduce both current and future consumption

    D. would leave both current and future consumption unaffected

36. Tobin's q theory suggests that firms will find it profitable to invest when the value of Tobin's q is:

    A. greater than zero

    B. less than zero

    C. greater than unity

    D. less than unity

The next 4 questions pertain to the following: Consider an agrarian economy with two single membered households. The households are engaged in own cultivation using their family land, labour and capital. Each household is endowed with 1 acre of land and 1 unit of labour. However the two households differ in terms of their initial capital endowment (K_0^R and K_0^P), where R denotes the relatively richer household and P denotes the relatively poorer household. Assume $2 < K_0^R < 4$ and $0 < K_0^P < 1$. The household have access to two technologies which are specified by the following production functions: Technology A: $Y_t = (N_t L_t)^{\frac{1}{2}}(K_t)^2$ Technology B: $Y_t = (N_t L_t)^{\frac{1}{2}}(K_t)^{\frac{1}{2}}$ where N_t represents land(in acres), L_t represents labour, and K_t represents capital respectively. The households choose the technology which gives them higher output (given their land, labour and capital stock) in any period t. In every period they consume half of their total income and save the rest, which adds to the next period's capital stock. Land and labour stock remain constant over time. Existing capital stock depreciates fully upon production.

37. Given their initial factor endowments, the technology choices of the rich and the poor households respectively are:

    A. A,B

    B. B,A

    C. A,A

    D. B,B

38. In the short run, the average capital stock in the economy ($\bar{K}$) evolves according to the following dynamic path

    A. $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[(K_t^R)^{\frac{1}{2}} + (K_t^P)^2 - 2(K_t^R + K_t^P)\right]$

    B. $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[(K_t^R)^2 + (K_t^P)^2 - 2(K_t^R + K_t^P)\right]$

C. $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[(K_t^R)^{\frac{1}{2}} + (K_t^P)^{\frac{1}{2}} - 2(K_t^R + K_t^P)\right]$

D. $\frac{d\bar{K}}{dt} = \frac{1}{4}\left[(K_t^R)^2 + (K_t^P)^{\frac{1}{2}} - 2(K_t^R + K_t^P)\right]$

39. In the long run:

    A. income of both households grow perpetually

    B. income of household R grows perpetually but that of household P approaches a constant

    C. income of household P grows perpetually but that of household R approaches a constant

    D. income of household R grows perpetually but that of household P falls perpetually

40. If, at the end of the initial time period, the households were given a choice to spend their savings in buying more land instead of investing in capital stock

    A. both households would have bought more land

    B. both households would have still invested in capital

    C. Household R would have still invested in capital but household P would have bought more land

    D. Household P would have still invested in capital but household R would have bought more land

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1. Person A lexicographically prefers good x to good y, i.e., when comparing two bundles of x and y, she strictly prefers the bundle that has more of good x; if the bundles have equal amounts of good x, then she strictly prefers the bundle that has more of good y. A's indifference curves in the (x, y)-space are

   A. vertical lines

   B. horizontal lines

   C. diagonal lines

   D. none of the above

2. Consider person A as described above. Consider the bundle $(2, 1)$ and a sequence of bundles (s^n) such that each bundle s^n is strictly preferred to $(2, 1)$. If this sequence of bundles converges to a bundle s, then

   A. s is strictly preferred to $(2, 1)$

   B. s is indifferent to $(2, 1)$

   C. $(2, 1)$ is strictly preferred to s

   D. any of the above

3. Consider an economy with two agents, A and B, and two goods, x and y. Both agents treat the two goods as perfect complements. Suppose the total endowment of this economy is $(4, 2)$. Which of the following allocations is **not** Pareto optimal?

   A. A gets $(1, 1)$ and B gets $(1, 1)$.

   B. A gets $(2, 1)$ and B gets $(3/2, 1)$.

   C. A gets $(1/2, 3/2)$ and B gets $(3, 1/2)$.

   D. A gets $(3, 2)$ and B gets $(0, 0)$.

   The next four questions are based on the following information.

   Consider an exchange economy with agents A and B and goods x and y. A's endowment is $(0, 1)$ (i.e., no good x and 1 unit of y) and B's endowment is $(2, 0)$ (i.e., 2 units of good x and no good y. The agents can consume only nonnegative amounts of x and y.

4. Suppose A lexicographically prefers x to y and B considers x and y to be perfect substitutes, i.e., between bundles (x, y) and (x', y'), she strictly prefers (x, y) if and only if $x + y > x' + y'$. The competitive equilibrium allocation for this economy is

   A. A gets $(0, 1)$ and B gets $(2, 0)$.

   B. A gets $(2, 0)$ and B gets $(0, 1)$.

   C. A gets $(3/2, 0)$ and B gets $(1/2, 1)$.

   D. A gets $(1, 0)$ and B gets $(1, 1)$.

5. Refer to the previous question. The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

A. $p_x/p_y = 1$

B. $p_x/p_y \geq 1$

C. $p_x/p_y \leq 1$

D. $p_x/p_y > 0$

6. Now suppose A lexicographically prefers y to x and B considers x and y to be perfect substitutes. The set of all possible competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

A. $p_x/p_y = 1$

B. $p_x/p_y \geq 1$

C. $p_x/p_y \leq 1$

D. $p_x/p_y > 0$

7. Now suppose A lexicographically prefers y to x and B considers x and y to be perfect complements. The set of competitive equilibrium allocations

A. includes the allocation $(1, 0)$ for A and $(1, 1)$ for B

B. includes the allocation $(1, 0)$ for A and $(1, 1)$ for B

C. is empty

D. includes all allocations $(x, 1)$ for A and $(2 - x, 0)$ for B, where $x \in [0, 2]$

The next two questions are based on the following information.

Suppose A is selling the Taj Mahal by the following auction procedure. There are two bidders, 1 and 2. Each bidder has a valuation v_i of the Taj and submits a bid b_i in a sealed envelope. The Taj is given to the bidder who submits the highest bid; if both bidders submit the same bid, then each gets the Taj with equal probability.

If bidder i wins, then she pays the price $\min(b_1, b_2)$ and gets payoff $v_i - \min(b_1, b_2)$. If bidder i loses, then she pays nothing and her payoff is 0.

Bidder i's valuation v_i is known only to bidder i and her bid b_i may or may not match v_i.

8. In order to maximize her payoff, Bidder i must bid

A. $b_i = v_i$

B. $b_i < v_i$

C. $b_i \leq v_i$

D. $b_i \geq v_i$

9. If b_i is the optimal bid for bidder i, then

A. it varies with bidder i's belief about the other bidder's valuation

B. it varies with bidder i's belief about the other bidder's bid

C. both (A) and (B)

D. neither (A) nor (B)

10. The Nash equilibrium of the Cournot duopoly model is a pair of quantities (x_1, x_2) such that

    A. the best response curves of firms' 1 and 2 are tangential at (x_1, x_2)

    B. isoprofit curves of firms' 1 and 2 are tangential at (x_1, x_2)

    C. an isoprofit curve of each firm is tangential to the best response curve of the other firm at (x_1, x_2)

    D. none of the above

11. Consider the Stackelberg duopoly model with firm 1 choosing quantity x_1 first. Firm 2 observes x_1 and sets quantity x_2 thereafter. The equilibrium outcome of this game is a pair of quantities (x_1, x_2) such that

    A. isoprofit curves of firms' 1 and 2 are tangential at (x_1, x_2)

    B. an isoprofit curve of firm 1 is tangential to the best response curve of firm 2 at (x_1, x_2)

    C. an isoprofit curve of firm 2 is tangential to the best response curve of firm 1 at (x_1, x_2)

    D. none of the above

12. Consider the following game:

	L	R
T	(x, x)	(b, y)
B	(y, b)	(a, a)

    Which of the following statements is true when $y > x > a > b$?

    A. The strategy profile (B, R) is the unique Nash equilibrium.

    B. (B, R) is one of many Nash equilibria.

    C. Both (B, R) and (B, R) are Nash equilibria.

    D. none of the above

13. Consider a consumer with utility function $u(x, y, z) = y \min\{x, z\}$. The prices of all three goods are the same. The consumer has Rs.100 to spend on these three goods. The demands will be such that

    A. $y < x = z$

    B. $y > x = z$

    C. $x = y = z$

    D. none of the above

14. A firm uses two inputs to produce its output. For all positive input prices, the firm employs an input combination of the form $(x, \alpha x)$ where $\alpha > 0$ is a constant. Which of the following production functions could represent this firm's technology?

    A. $f(x, y) = \min\{x^\alpha, y\}$

    B. $f(x, y) = \min\{\alpha x, y\}$

    C. $f(x, y) = \min\{x, \alpha y\}$

    D. $f(x, y) = \min\{x, y^\alpha\}$

15. Suppose there are two telecom firms, 1 and 2, who have paid fees $k_1 > 0$ and $k_2 > 0$ for telecom spectrum. Let $k_1 > k_2$. They produce an identical good (telecom service) at an identical average cost of production $c > 0$. If they engage in Bertrand competition, then the Nash equilibrium prices (p_1, p_2) are such that

    A. $p_1 > p_2 > c$

    B. $p_1 = p_2 > c$

    C. $p_1 = p_2 = c$

    D. are indeterminate

16. Consider the following game with players 1 and 2; payoffs are denoted by (a, b) where a is 1's payoff and b is 2's payoff. First, player 1 chooses either U or D. If she plays D, then the game ends and the payoff are $(1, 0)$. If she plays U, then player 2 chooses either U or D. If he plays D, then the game ends and the payoffs are $(0, 2)$. If he plays U, then player 1 again chooses either U or D. The game ends in both cases. If player 1 chooses D, then the payoffs are $(4, 0)$. If player 1 chooses U, then the payoffs are $(3, 3)$.

    A. This game has a unique Nash equilibrium

    B. This game has a unique subgame perfect equilibrium

    C. This game has no subgame perfect equilibrium

    D. This game has multiple subgame perfect equilibria

17. The interval $(0, 1)$ can be expressed as

    A. the union of a countable collection of closed intervals

    B. the intersection of a countable collection of closed intervals

    C. both (A) and (B)

    D. neither (A) nor (B)

18. The interval $[0, 1]$ can be expressed as

    A. the union of a countable collection of open intervals

    B. the intersection of a countable collection of open intervals

    C. both (A) and (B)

    D. neither (A) nor (B)

19. A function $f : \mathcal{R}^n \to \mathcal{R}$ is a convex function if and only if
    A. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \geq f(x)\}$ is a convex set
    B. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \leq f(x)\}$ is a convex set
    C. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \geq f(x)\}$ is a concave set
    D. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \leq f(x)\}$ is a concave set

20. A function $f : \mathcal{R}^n \to \mathcal{R}$ is a quasi-convex function if and only if
    A. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \geq f(x)\}$ is a convex set
    B. $\{x \in \mathcal{R}^n \mid y \geq f(x)\}$ is a convex set for every $y \in \mathcal{R}$
    C. $\{(x, y) \in \mathcal{R}^n \times \mathcal{R} \mid y \leq f(x)\}$ is a convex set
    D. $\{x \in \mathcal{R}^n \mid y \leq f(x)\}$ is a convex set for every $y \in \mathcal{R}$

21. If

$$\begin{aligned} A &= \{(x, y) \in \mathcal{R}^2 \mid x \geq 0, y \geq 0, xy \geq 1\}, \\ B &= \{(x, y) \in \mathcal{R}^2 \mid x \leq 0, y \geq 0, xy \leq -1\} \text{ and} \\ C &= \{a + b \mid a \in A, b \in B\} \end{aligned}$$

    then
    A. $\{(x, y) \in \mathcal{R}^2 \mid x = 0, y \geq 0\}$ is a subset of C
    B. $\{(x, y) \in \mathcal{R}^2 \mid x = 0, y > 0\}$ is a subset of C
    C. $\{(x, y) \in \mathcal{R}^2 \mid x \geq 0, y = 0\}$ is a subset of C
    D. $\{(x, y) \in \mathcal{R}^2 \mid x > 0, y = 0\}$ is a subset of C

22. $\lim_{n\to\infty} \left(1 + \frac{1}{2n}\right)^{5n}$ equals
    A. e^2
    B. $e^{2.5}$
    C. ∞
    D. $e^{1.5}$

23. Consider the function

$$f(x) = \begin{cases} \ln\ x, & \text{if } 0 < x < 1 \\ ax^2 + b, & \text{if } 1 \leq x < \infty \end{cases}$$

    such that $f(2) = 3$. Function f is continuous if
    A. $a = 2$ and $b = -1$
    B. $a = -1$ and $b = 2$
    C. $a = -1$ and $b = 1$

D. $a = 1$ and $b = -1$

24. If A and B are $n \times n$ matrices such that $A + B = AB$, then

A. $AB = BA$

B. $AB \neq BA$

C. $B = A^{-1}$

D. $B = A^T$, where A^T is the transpose of A

25. If B is an $n \times n$ real matrix and B^T is the transpose of B, then

A. $B^T B$ is negative definite

B. $B^T B$ is positive definite

C. $B^T B$ is negative semidefinite

D. $B^T B$ is positive semidefinite

26. Consider a twice differentiable function $f : \mathcal{R} \to \mathcal{R}$ and $a, b \in \mathcal{R}$ such that $a < b$, $f(a) = 0 = f(b)$ and $D^2 f(x) + Df(x) - 1 = 0$ for every $x \in [a, b]$. Then,

A. $f(x) \leq 0$ for every $x \in [a, b]$

B. $f(x) \geq 0$ for every $x \in [a, b]$

C. $f(x) = 0$ for every $x \in [a, b]$

D. f must take positive and negative values on the interval $[a, b]$

27. Suppose $f : \mathcal{R}_+ \to \mathcal{R}$ is decreasing and differentiable. If $F : \mathcal{R}_+ \to \mathcal{R}$ satisfies $F(x) = \int_0^x f(t)dt$, then F is

A. convex

B. concave

C. increasing

D. decreasing

28. If real numbers p and q satisfy $0 < q < p$, then the following is true for the numbers $p, q, p + q$ and $q - p$:

A. their mean equals their median

B. their mean is greater than their median

C. their mean is less than their median

D. there is insufficient information to compare their mean and their median

29. If a binomial random variable X has expectation 7 and variance 2.1, then the probability that $X = 11$ is

A. $462(0.7)^5(0.3)^6$

B. 0

C. $11(0.7)^{11}$

D. $462(0.7)^6(0.3)^5$

30. A machine starts operating at time 0 and fails at a random time T. The distribution of T has density $f(t) = (1/3)e^{-t/3}$ for $t > 0$. The machine will not be monitored until time $t = 2$.
The expected time of discovery of the machine's failure is

A. $2 + e^{-6}/3$

B. $2 - 2e^{-2/3} + 5e^{-4/3}$

C. $2 + 3e^{-2/3}$

D. 3

31. An insuree has an insurance policy against a random loss $X \in [0, 1]$. If loss X occurs, then the insurer pays $X - C$ to the insuree, who bears the remaining loss $C \in (0, 1)$. The loss X is a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

If the probability of the insurance payment being less than $1/2$ is equal to 0.64, then C is

A. 0.1

B. 0.3

C. 0.4

D. 0.6

32. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 to that tourist. The expected revenue of the tour operator is

A. 950

B. 967

C. 976

D. 985

33. An insurance policy-holder can submit up to 5 claims. The probability that the policy-holder submits exactly n claims is p_n, for $n = 0, 1, 2, 3, 4, 5$. It is known that

- The difference between p_n and p_{n+1} is constant for $n = 0, 1, 2, 3, 4$, and
- 40% of the policyholders submit 0 or 1 claim.

What is the probability that a policyholder submits 4 or 5 claims?

A. 0.06

B. 0.19

C. 0.26

D. 0.34

34. Suppose n students are asked to solve a problem at time 0. The random time to completion for student i is $T_i \geq 0$. Each T_i is uniformly distributed on $[0, 1]$. If $Y = \max\{T_1, \ldots, T_n\}$, then the mean of Y is

A. $\left[\dfrac{n}{n+1}\right]^2$

B. $\dfrac{n}{2(n+1)}$

C. $\dfrac{n}{(n+1)}$

D. $\dfrac{2n}{(n+1)}$

35. A hospital determines that N, the number of patients in a week, is a random variable with $\Pr[N = n] = 2^{-n-1}$, where $n \geq 0$. The hospital also determines that the number of patients in a given week is independent of the number of patients in any other week. The probability that there are exactly seven patients during a two-week period is

A. 1/2

B. 1/64

C. 1/128

D. 1/256

The next two questions are based on the following information.
X and Y are random variables and their joint probability distribution is as follows:

	$Y=1$	$Y=2$	$Y=3$	$Y=4$	$Y=5$	$Y=6$
$X=-1$	0.1	α	0.3	0	0	0
$X=1$	0	0	β	0.1	0.1	0.1

It is known that the expectations of the two random variables are $\mathbb{E}(X) = -0.2$ and $\mathbb{E}(Y) = 3.2$. Then

36. The value of α is

A. 0

B. 0.1

C. 0.2

D. 0.3

37. The value of β is

    A. 0

    B. 0.1

    C. 0.2

    D. 0.3

38. There are 3 red and 5 black balls in an urn. You draw two balls in succession without replacing the first ball. The probability that the second ball is red equals

    A. 2/7

    B. 3/8

    C. 5/7

    D. 1/4

39. Suppose X and Y are independent random variables with standard Normal distributions. The probability of $X > 1$ is p. The probability of the event $X^2 > 1$ and $Y^3 < 1$ is

    A. $2p(1-p)$

    B. $4p$

    C. $p(1-p)$

    D. $2p^2$

40. Suppose 1/10 of the population has a disease. If a person has the disease, then a test detects it with probability 8/10. If a person does not have the disease, then the test incorrectly shows the presence of the disease with probability 2/10. What is the probability that the person tested has the disease if the test indicates the presence of the disease?

    A. 1

    B. 9/13

    C. 4/13

    D. 7/13

41. Two patients share a hospital room for two days. Suppose that, on any given day, a person independently picks up an airborne infection with probability 1/4. An individual who is infected on the first day will certainly pass it to the other patient on the second day. Once contracted, the infection stays for at least two days. What is the probability that fewer than two patients have the infection by the end of the second day?

    A. 135/256

    B. 121/256

    C. 131/256

    D. 125/256

Answer the next question using the following information.
Let $w = W/P$ be the real wage rate, where W is the nominal wage rate and P is the aggregate price level. The demand for labour is given by $D(w) = 1 - w$ and the supply of labour is described by the equation $S(w) = w$. If N is the employment level, then $f(N)$ is the aggregate output.

42. If nominal wage is always such that the labour market clears, then the aggregate supply curve is given by the equation

    A. $Y = Pf(N)$

    B. $Y = f(N)$

    C. $Y = Pf(1/2)$

    D. $Y = f(1/2)$

Answer the next five questions using the following information.
Consider the above-described labour market with the following change: the nominal wage rate W minimizes $|D(W/P) - S(W/P)|$ subject to the constraint $W \geq W_0$, where $W_0 > 0$ is an exogenously given minimum nominal wage.

43. Given the price level P, the nominal wage W is

    A. $\max\{W_0, P/2\}$

    B. $\min\{W_0, P/2\}$

    C. $1/2$

    D. $W_0/2$

44. Given the price level P, the employment level is

    A. $\min\{1/2, 1 - W_0/P\}$

    B. $\max\{1/2, 1 - W_0/P\}$

    C. $1 - W_0/2P$

    D. $1 - 1/2P$

45. Given the price level P such that $1/2 \leq 1 - W_0/P$, the aggregate supply is

    A. $f(1/2)$

    B. $f(1 - 1/2P)$

    C. $f(1 - W_0/P)$

    D. $f(1 - W_0/2P)$

46. If the marginal productivity of capital is positive and the stock of capital increases, then the aggregate supply schedule will

    A. shift up

    B. shift down

    C. shift to the left

D. shift to the right

47. If the fixed nominal wage W_0 increases, then the aggregate supply schedule will

A. shift up

B. shift down

C. shift to the left

D. shift to the right

48. Consider a closed economy. If the nominal wage is flexible and nominal money supply is increased, then which of the following will be true in equilibrium?

A. Real wage decreases and real money supply decreases

B. Real wage decreases and real money supply increases

C. Real wage is unchanged and real money supply is unchanged

D. Real wage decreases and real money supply is unchanged

49. Which of the following would make the LM curve flatter in the (Y, r) space?

A. An increase in income sensitivity of money demand

B. An increase in interest sensitivity of planned investment

C. An increase in the marginal propensity to consume

D. An increase in the interest sensitivity of money demand

50. In an IS-LM model with fixed exchange rates and perfect capital mobility, an increase in government spending will lead to

A. a deterioration in the trade balance

B. an improvement in the trade balance

C. no change in the trade balance

D. an increase in export without affecting imports

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1. Let $\mathbb{R}$ be the set of real numbers and $f : \mathbb{R} \to \mathbb{R}$ be a continuous and concave function. Which of the following statements is correct?

   A. $|f|$ must be concave

   B. $-f$ must be concave

   C. $f + f$ must be concave

   D. $f \circ f$ must be concave

2. The maximum value of $f(x, y) = (rxy)^{\frac{1}{2}}$, subject to $|x| \geq |y|$ and $|x| + |y| \leq 1$ is

   A. 1

   B. 2

   C. 0.5

   D. 0.25

3. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Each agent's consumption set is $\mathbb{R}^2_+$. The endowments of agents 1 and 2 are $(10, 1)$

   and $(0, 9)$ respectively. (In any commodity bundle, the first entry is a quantity of X and the second one is a quantity of Y.)

   If $a > c$, or $a = c$ and $b > d$, then Agent 1 strictly prefers bundle (a, b) to (c, d).

   If $b > d$, or $b = d$ and $a > c$, then Agent 2 strictly prefers bundle (a, b) to (c, d). Which of the following allocations is a competitive equilibrium allocation?

   A. 1 gets $(10, 1)$ and 2 gets $(0, 9)$

   B. 1 gets $(10, 10)$ and 2 gets $(0, 0)$

   C. 1 gets $(5, 5)$ and 2 gets $(5, 5)$

   D. None of the above

   E. Incorrect options provided for this question.

4. . Let $\mathbb{R}$ be the set of real numbers and let $\mathcal{D}$ be the set of functions $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that satisfy the following properties for all $x, y, z \in \mathbb{R}$:

   - $d(x, y) \geq 0$
   - $d(x, y) = 0$ if and only if $x = y$
   - $d(x, y) = d(y, x)$
   - $d(x, z) \leq d(x, y) + d(y, z)$

   Which of the following is not a function in $\mathcal{D}$?

   A. $d(x, y) = \min\{|x - y|, 1\}$

   B.
   $$d(x, y) = \left\{ \begin{array}{ll} 0, & \text{if } x = y \\ 1, & \text{if } x \neq y \end{array} \right\}$$

C.
$$d(x,y) = \left\{ \begin{array}{ll} 0, & \text{if } |x-y| \leq 1 \\ 1, & \text{if otherwise} \end{array} \right\}$$

D. $d(x,y) = |x-y|$

5. Let $f : [0,1] \to \mathbb{R}$ be twice differentiable. Suppose that the line segment joining the points $(0, f(0))$ and $(1, f(1))$ intersects the graph of f at a point $(a, f(a))$, where $0 < a < 1$. Then,

   A. there exists $z \in [0,1]$ such that $f'(z) = 0$

   B. there exists $z \in [0,1]$ such that $f''(z) = |f(1) - f(0)|$

   C. there exists $z \in [0,1]$ such that $f''(z) = f(1) - f(0)$

   D. there exists $z \in [0,1]$ such that $f''(z) = 0$

6. Consider the matrix $A = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ where $\theta \in (0, 2\pi)$. The inner product of vectors $v = (v_1, v_2)$ and $w = (w_1, w_2)$ in $\mathbb{R}^2$ is defined by $< v, w >= v_1 w_1 + v_2 w_2$. So, for the vectors v and w in $\mathbb{R}^2$

   A. $< Av, Aw >>< v, w >$

   B. The comparison of $< Av, Aw >$ and $< v, w >$ depends on θ

   C. $< Av, Aw >=< v, w >$

   D. $< Av, Aw >< < v, w >$

7. The set $[0,1]$

   A. cannot be the intersection of a countable collection of sets of the form (a,b).

   B. is the intersection of a countable collection of sets of the form (a,b).

   C. is the union of a countable collection of sets of the form (a,b).

   D. is the union of a countable collection of sets of the form $[a,a]$

8. A sequence of real numbers (x_n) converges to x. Consider the following claims:

   (I) The sequence $(\frac{x_{n+1}}{x_n})$ converges to 1

   (II) The sequence $x_n + x_{n+1}$ converges to $2x$.

   A. Only statement II is correct.

   B. Ouly Statement I is correct.

   C. None of the statements is correct.

   D. Both the statements are correct.

Persons 1, 2 and 3 have to divide 12 indivisible chocolates among themselves. Each person's preference is strictly increasing in chocolates. The procedure for dividing the chocolates is as follows:

Person 1 proposes a division. Each person votes either Y (Yes) or N (No). If at least two persons vote Y, then the proposal is implemented. If not, then Person 1 is eliminated

from the voting and Person 2 makes a proposal. Now, only persons 2 and 3 can vote Y or N. If at least one of them votes Y, then Person 2's proposal is implemented, Otherwise, Person 3 makes a proposal, which will be implemented.

9. What division of chocolates will occur from a subgame perfect.equilibrium of this game? (Assume that a person votes N if voting Y and N are expected to result in the same number of chocolates for that person.)

   A. 1 gets 11, 2 gets 1. 3 gets 0

   B. l gets 11, 2 gets 0, 3 gets 1

   C. 1 gets 12, 2 gets 0, 3 gets 0

   D. 1 gets 4, 2 gets 4, 3 gets 4

10. Suppose the above procedure for dividing the chocolates is changed as follows: if Person 1's proposal is rejected and Person 2 makes a proposal, if both the remaining voters, 2 and 3, vote Y, then Person 2's proposal is implemented. Otherwise, Person 3 makes a proposal, which will be implemented.

    What division of chocolates will occur from a subgame perfect equilibrium of this game? (Assume that a person votes N if voting Y and N are expected to result in the same number of chocolates for that person.)

    A. 1 gets 11, 2 gets 1. 3 gets 0

    B. l gets 11, 2 gets 0, 3 gets 1

    C. 1 gets 12, 2 gets 0, 3 gets 0

    D. 1 gets 4, 2 gets 4, 3 gets 4

11. The set $\{f_1, f_2, \ldots, f_n\}$ where each f_k is a real-valued function defined on $\mathbb{R}$, is said to be linearly independent if $c_1, c_2, \ldots, c_n \in \mathbb{R}$ and $\sum c_k f_k(x) = 0$ for every $x \in \mathbb{R}$ implies $c_1 = c_2 = \cdots = c_n = 0$

    A. the set $\{f1, \ldots, f_n\}$ is linearly independent.

    B. each pair of these functions is linearly independent, but larger n-tuples are not.

    C. only the subset of odd-numbered functions and the subset of even-numbered functions are linearly independent.

    D. every "proper subset of this set of functions is linearly independent, but the whole set is not.

12. Let $f : [0, 1] \to \mathbb{R}$ be differentiable and suppose that $|f(x)| < 1$ for every $x \in [0, 1]$. Then. there

    A. is at least one $c \in [0, 1]$ such that $f(c) = c$.

    B. are two numbers c_1 and c_2 such that $f(c_i) = c_i$ for $i = 1, 2$.

    C. is exactly one $c \in [0, 1]$ such that $f(c) = c$.

    D. is at most one $c \in [0, 1]$ such that $f(c) = c$.

13. Consider a closed macroeconomy whose demand side is represented by $Y = C_0 + c(Y - \tau Y) - \alpha r + G_0$ $M_0 = KPY - \beta r$

    where $C_0, G_0, M_0, K, c, \tau, \alpha, \beta$ are all positive constants and $c, \tau \in (0, 1)$.

    The AD curve for this economy is given by

    A. $Y = \frac{M_0 + \beta r}{KP}$

    B. $Y = \frac{C_0 + \frac{\alpha}{\beta} M_0 + G_0}{1 - c(1-\tau) + \frac{\alpha}{\beta} KP}$

    C. $Y = \frac{C_0 + M_0 + G_0}{1 - c(1-\tau) + KP}$

    D. $Y = \frac{C_0 + M_0 + G_0}{1 - c(1-\tau) + \frac{KP}{\beta}}$

14. Consider a closed macroeconomy whose demand side is represented by $Y = C_0 + c(Y - \tau Y) - \alpha r + G_0$ and

    $M_0 = KPY - \beta r$

    where $C_0, G_0, M_0, K, c, \tau, \alpha, \beta$ are all positive constants and $c, \tau \in (0, 1)$.

    Now suppose the government, instead of following a given money supphy rule, follows an interest rate targeting policy such that the quantity of money demanded is always supplied so as to keep the interest rate fixed at given level r_0. The AD curve for this economy is

    A. $Y = \frac{C_0 - ar_0 + G_0}{1 - c(1-\tau)}$

    B. $Y = \frac{C_0 + M_0 + G_0}{1 - c(1-\tau) + KP}$

    C. $Y = \frac{C_0 + M_0 + G_0}{1 - c(1-\tau) + \frac{KP}{\beta}}$

    D. $Y = \frac{C_0 + \frac{\alpha}{\beta} M_0 + G_0}{1 - c(1-\tau) + \frac{\alpha}{\beta} KP}$

15. Consider a closed macroeconomy whose demand side is represented by $Y = C_0 + c(Y - \tau Y) - \alpha r + G_0$ $M_0 = KPY - \beta r$

    where $C_0, G_0, M_0, K, c, \tau, \alpha, \beta$ are all positive constants and $c, \tau \in (0, 1)$.

    Suppose there an increase in the interest sensitivity of the IS curve (parameter α), which is accompanied by an increase in the (interest sensitivity of the LM curve (parameter β) by exactly the same proportion. As a result,

    A. the AD curve will be steeper but there will be no shift of the entire curve

    B. the entire AD curve will shift to the right and it will also be steeper

    C. the entire AD curve will shift to the right with no change in its slope

    D. there will be no change in the AD curve (neither any change in its slope, nor a shift)

16. In a multiple regression model involving three right-hand-side variables with 105 observations estimated using OLS, the researcher needs to decide whether to include a fourth right-had-side variable or not. The residual sum of squares is 250 when four variables are

included and is 300 when three variables are included. Some critical values of the F-table (with $\alpha = 0.05$) are: $F(1, 100) = 3.89, F(2, 100) = 3.09, F(3, 100) = 2.70, F(4, 100) = 2.46$ and $F(5, 100) = 2.31$.

This means when the fourth variable is included, the fit of the regression

A. there is insufficient information to make a determination about fit

B. worsens significantly

C. has no significant change

D. improves significantly

17. For a variable x the standard error of the sample mean is calculated as 20 when samples of size 25 are taken and as 10 when samples of size 100 are taken. A quadrupling of sample size has only halved the error. What must be the value of the standard error of x?

A. 1000

B. 900

C. 100

D. 377.5

18. Suppose a consumer lives for two periods and chooses consumptions C_1 and C_2 to maximise utility $U(C_T) = \frac{\sigma}{\sigma-1}\left(C_t^{\frac{\sigma-1}{\sigma}} - 1\right)$

Future consumption is discounted by ρ. The intertemporal elasticity of substitution in consumption between the two periods is

A. ρ

B. 1

C. $\frac{\sigma-1}{\sigma}$

D. $\frac{\sigma}{\sigma-1}$

19. Consider the game

$$\begin{array}{c} \\ U \\ M \\ D \end{array} \begin{array}{c} \begin{array}{ccc} L & M & R \end{array} \\ \begin{pmatrix} 2,0 & 3,3 & 0,0 \\ -1,1 & 0,0 & 1,0 \\ 4,-4 & 2,2 & 1,1 \end{pmatrix} \end{array}$$

where the row player's payoff is given first, followed by the column player's payoff. Which of the following statements is false?

A. There is a Nash equilibrium of this game in which the column player plays a strictly dominated strategy.

B. Column player has a strictly dominated strategy.

C. Row player has a weakly dominated strategy.

D. There is a Nash equilibrium of this game in which both players play weakly dominated strategies.

20. In a multiple regression model, the Durbir Watson test statistic is 1.3, while the critical lower and upper values are 1.5 and 1.7 respectively. This implies that

A. There is positive autocorrelation,

B. There is no positive autocorrelation:

C. The test is inconclusive about autocorrelation.

D. There is heteroscedasticity but no autocorrelation.

21. Consider the model $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ estimated using OLS. Which of the following will lead to a higher $Var(\hat{\beta}_2)$?

- Smaller sample size
- Less variation in X_2
- More variation in ϵ
- High correlation between X_1 and X_2

The correct answer is:

A. (i), (ii), (iii) and (iv)

B. (ii), (iii) and (iv)

C. (ii) and (iii)

D. (iii) and (iv)

22. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Each agent's consumption set is $\mathbb{R}^2$ The endowments of agents 1 and 2 are $(10, 1)$

and $(0, 9)$ respectively. (In any commodity bundle, the first entry is a quantity of X and the second one is a quantity of Y.)

If $a > c$, or $a = c$ and $b > d$, then Agent 1 strictly prefers bundle (a, b) to (b, d). If $b > d$, or $b = d$ and $a > c$, then Agent 1 strictly prefers bundle (a, b) to (b, d).

Which of the following allocations is an efficient allocation?

A. 1 gets $(5, 5)$ and 2 gets $(5, 5)$

B. 1 gets $(10, 1)$ and 2 gets $(0, 9)$

C. 1 gets $(10, 10)$ and 2 gets (0.0)

D. All of the above

E. Incorrect options provided for this question.

23. Consider a country with two citizens, 1 and 2. The government is considering a scheme that will cost 100. The government does not know the true benefits of the scheme to the citizens, say B_1, and B_2, and must decide whether to implement the scheme on the basis of their reported benefits. say R_1 and R_2. It will implement the scheme if and only if $R_1 + R_2 \geq 100$. If it is implemented, the government will impose tax $100 - R_2$ on person 1 and tax $100 - R_1$, on person 2. Each citizen's reported benefit seeks to maximize the difference between her true benefit (known only to her) and the tax that must be paid if and only if the scheme is implemented. The optimal choices of R_1 and R_2 must be such that

    A. $R_1 < B_1$ and $R_2 < B_2$

    B. Nothing systematic can be said about R_1 and R_2

    C. $R_1 = B_1$ and $R_2 = B_2$

    D. $R_1 > B_1$ and $R_2 > B_2$

    Consider a Solovian economy with the aggregate production function $Y_t = K_t^{\frac{1}{2}} N_t^{\frac{1}{2}}$. The initial size of the population is 100 and the initial capital stock is given by 9 units. The entire output produced in each period is distributed to the households as factor incomes (since households are the owners of the capital stock and Jabour at any time t), who consume half of their income and save the rest. All savings are automatically invested which augment the capital stock available for production over time. Population does not grow and there is 100% depreciation of capital stock within one period.

24. The corresponding steady state value of aggregate output is

    A. 30

    B. 50

    C. 5

    D. 10

25. The corresponding steady state value of aggregate consumption is

    A. 50

    B. 10

    C. 30

    D. 25

26. The steady state value of aggregate capital stock is

    A. 9

    B. 25

    C. 2.5

    D. 10

27. Suppose households were free to choose their savings rate. If they wanted to maximize the steady state level of consumption, the savings rate would be

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{5}$

D. $\frac{1}{10}$

28. In the previous problem, where households choose their savings rate, at that savings rate, the steady state value of the aggregate capital stock is

A. 25

B. 2.5

C. 10

D. 9

E. Incorrect options provided for this question.

29. Suppose $X_1, \ldots, X_n$ are observed completion times of an experiment with values in $[0, 1]$. Each of these random variables is uniformly distributed on $(0, 1)$. If Y is the maximum observed completion time, then the mean of Y is

A. $\frac{n}{2(n+1)}$

B. $\frac{2n}{n+1}$

C. $(\frac{n}{(n+1)})^2$

D. $\frac{n}{n+1}$

30. If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x) = \left\{ \begin{array}{ll} \frac{xy}{x^2+y^2}, & \text{for } xy \neq 0 \\ 0, & \text{for } xy = 0 \end{array} \right\}$$

then

A. f is differentiable at $(0, 0)$ and both partial derivatives at $(0, 0)$ are 0

B. f is non-differentiable at $(0, 0)$ and both partial derivatives at $(0, 0)$ are 0

C. f is differentiable at $(0, 0)$ and neither partial derivatives at $(0, 0)$ is 0

D. f is non-differentiable at $(0, 0)$ and neither partial derivatives at $(0, 0)$ exist

31. Suppose we have estimated $y = 10 + 1.5x + 4D$, where y is earnings, $\ll$ is experience and D is 0 for females and 1 for males. If we had coded the dummy as -1 for females and 1 for males, the new estimated coefficients (in the same order) would have been:

A. $12, 1.5, -2$

B. $12, 4.5, 2$

C. $14, 1.5, -4$

D. $10, 1.5.4$

32. Which of the following is/are the consequence(s) of including an irrelevant variable in a multiple linear regression model?

    (i) The variances of the OLS coefficients may be larger

    (ii) The OLS coefficients will be biased unless the irrelevant variable is orthogonal to the other included variables

    (iii) The variances of the OLS coefficients will be unaffected.

    The correct answer is:

    A. (i) only

    B. (ii) and (iii) only

    C. (iii) only

    D. (i) and (ii)

33. Suppose Y is a random variable with uniform distribution of the interval $[-\pi/2, \pi/2]$. The value of the (cumulative) distribution function of thé random variable $X = sinY$ at $x \in [--1, 1]$ is

    A. $\frac{sin^{-1}(x)}{\pi} + \frac{1}{2}$

    B. $sin^{-1}(x) + \frac{\pi}{2}$

    C. $sin^{-1}(x) + \pi$

    D. $\frac{sin^{-1}(x)}{\pi} + \frac{\pi}{2}$

34. A student is answering a multiple-choice examination. Suppose a question has m possible answers. The student knows the correct answer with probability p. If the student knows the correct answer, then she picks that answer; otherwise, she picks randomly from the choices with probability $1/m$ each. Given that the student picked the correct answer, the probability that she knew the correct answer is

    A. $\frac{p}{1+(1-p)m}$

    B. $\frac{p}{1+(m-1)p}$

    C. $\frac{mp}{1+(1-p)m}$

    D. $\frac{mp}{1+(m-1)p}$

35. Consider the game

$$\begin{array}{c} \\ U \\ D \end{array} \begin{array}{c} \begin{array}{cc} L & R \end{array} \\ \begin{pmatrix} x, x & z, y \\ y, z & y, y \end{pmatrix} \end{array}$$

    where the row player's payoff is given first, followed by the column player's payoff. This game has only one Nash equilibrium when

    A. $x > y > z$

B. $x < y < z$

C. $x = y = z$

D. $y > z > x$

36. Consider an economy where there is no capital. Production of the final good is carried out in each period using a linear production function that uses only labour. The production technology is specified as follows:

$Y_t = A_t(1 - \lambda_t)L_0$

where L_0 is the constant labour force in the economy, $1 - \lambda_t$, is the proportion of labour force engaged in final goods production and A_t is the state of technology in period t. The index of technology A_t changes over time depending on the proportion of the labour force engaged in $R\&D$, which is λ_t. The evolution of technology in the $R\&D$ sector is determined by the equation

$\frac{dA}{dt} = \lambda_t L_0 A_t$ where the initial A_0 is a positive constant.

Now suppose that the relative price of the new technologies generated in the $R\&D$ sector in terms of the final commodity is unity. Also suppose the labour across two sectors are employed in such a way that the value of marginal product of labour across the two sectors are identical. In this case, the equilibrium value of λ_t will be

A. λ_0

B. $\frac{1}{2}$

C. 1

D. $1 - \lambda_0$

37. Consider an economy where there is no capital. Production of the final good is carried out in each period using a linear production function that uses only labour. The production technology is specified as follows:

$Y_t = A_t(1 - \lambda_t)L_0$

where L_0 is the constant labour force in the economy, $1 - \lambda_t$, is the proportion of labour force engaged in final goods production and A_t is the state of technology in period t. The index of technology A_t changes over time depending on the proportion of the labour force engaged in $R\&D$, which is λ_t. The evolution of technology in the $R\&D$ sector is determined by the equation

$\frac{dA}{dt} = \lambda_t L_0 A_t$ where the initial A_0 is a positive constant.

For any exogenously given value of $\lambda_t = \lambda_0$, the long run balanced growth rate in this economy is

A. $(1 - \lambda_0)L_0$

B. $(1 - \lambda_0)$

C. λ_0

D. $\lambda_0 L_0$

38. Duopolist firms 1 and 2 sell a homogeneous good in a market with demand function $Q = 100 - 2P$, where Q is the quantity demanded at price P. Firms 1 and 2 have constant marginal costs of 0 and 30 respectively. The firms simultaneously announce prices and consumers buy from the firm whose price is lower. If the firms choose the same price, all the consumers buy from firm 1. Firm 1's equilibrium price is

    A. 30

    B. 0

    C. 25

    D. 20

39. Voters arrive at a social ranking of alternatives by consulting a "holy book": the social ranking is the ranking found in this book. Which of Arrow's axioms defining an attractive preference aggregation method is violated by this method?

    A. Independence of irrelevant alternatives

    B. Non-dictatorship

    C. Unrestricted domain

    D. The Pareto principle

40. A consumer weakly prefers a basket (a_1, a_2, a_3) to a basket (b_1, b_2, b_3) if $a_1 \geq b_1, a_1 + a_2 \geq b_1 + b_2, a_1 + a_2 + a_3 \geq b_1 + b_2 + b_3$

    Which of the following statements about this preference is false?

    A. It is transitive.

    B. It is monotonic.

    C. It is continuous.

    D. It is complete,

41. Suppose the economy-wide union sets wage for employed workers by $W = P^e(Z - \alpha u)$ with unemployment rate u and labour force of the economy L. The producer levies price over wage W with mark-up m as $P = (1+m)W$. If each employed worker produces one unit of output Y, then the aggregate supply function is

    A. $P = -P^e(1+m)(Z - 1 + \frac{\alpha Y}{L})$

    B. $P = P^e(1+m)(Z - 1 + \frac{\alpha Y}{L})$

    C. $P = -P^e(1+m)(Z - \alpha + \frac{\alpha Y}{L})$

    D. $P = P^e(1+m)(Z - \alpha + \frac{\alpha Y}{L})$

42. Consider the following regression model

    $y_i = \alpha_0 + \alpha_1 x_{1i} + \alpha_2 x_{2i} + u_i$ Suppose a researcher is interested in conducting White's heteroscedasticity test using the residuals from an estimation of the above equation. What would be the most appropriate form for the auxiliary regression?

    A. $u_i^2 = \gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}^2 + \gamma_4 x_{2i}^2 + \gamma_5 x_{1i} x_{2i} + \nu_i$

B. $u_i^2 = \gamma_0 + \gamma_1 u_{i-1} + \nu_i$

C. $u_i^2 = \gamma_0 + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}^2 + \gamma_4 x_{2i}^2 + \nu_i$

D. $u_i = \gamma_0 + \gamma_1 u_{i-1} + \nu_i$

43. Suppose you have run the following regression:

$y = \alpha + \beta_x + \gamma \text{Urban} + \theta \text{Immigration} + \delta \text{UrbanImmigration} + \epsilon$ where Urban is a dummy indicating that the person lives in a city rather than a rural area, Immigrant is a dummy indicating that the person is an immigrant rather than a native. The coefficient θ is interpreted as the ceteris paribus difference in y between

A. A rural immigrant and a rural native

B. An immigrant and a native

C. An urban immigrant and an urban native

D. None of the above

44. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Each agent's consumption set is $\mathbb{R}^2_+$. Given bundles $(a, b), (c, d) \in \mathbb{R}^2$ such that $(a, b) \geq (c, d)$ and $(a, b) \neq (c, d)$, agent 1 strictly prefers (a, b). (In any commodity bundle, the first entry is a quantity of X and the second one is a quantity of Y.) Consider the following claims: In a competitive equilibrium for this economy,

(I) both prices must be positive, and (II) the sum of the allocations to 1 and 2 must equal the sum of their endowments. Which of the following statements is correct?

A. i and ii are true

B. i and ii are false

C. i is true and ii is false

D. i is false and ii is true

45. A monopolist faces a demand function $D(p) = \alpha - p$ and cost function $C(q) = cq$. She can advertise her product to increase demand. Advertisement level θ costs $\frac{\theta^2}{2}$ and it shifts the demand function by θ, i.e., the new demand function is $D(p) = \alpha + \theta - p$. The monopolist's profit is

A. $\frac{(\alpha-c)^2}{4}$

B. $\frac{(\alpha-c)^2}{2}$

C. $\frac{(\alpha-c-\theta)^2}{2}$

D. $(\alpha - \theta)^2$

46. Consider the following game. Player 1 moves first and chooses L or R. If she plays R, the game ends and the payoffs are $(10, 0)$. If she plays L, then player 2 moves and chooses cither L or R. If he plays R, the game ends and the payoffs are $(0, 20)$. If he plays L, then player 1 moves and chooses either L or R. The game ends in both cases. If player 1 chooses L, then the payoffs are (30.30). If she chooses L, then the payoffs are $(40, 0)$. This game

A. bas three subgame perfect equilibria

B. has a unique Nash equilibrium

C. has a subgame perfect equilibrium in which 2 plays L

D. has a unique Nash equilibrium outcome

47. If the function $f : \mathbb{R}^2 \to \mathbb{R}$ is concave, then

A. $\{(x, r) \in \mathbb{R}^2 \times \mathbb{R} | f(x) < r\}$ is convex

B. $\{(x, r) \in \mathbb{R}^2 \times \mathbb{R} | f(x) = r\}$ is convex

C. $\{(x, r) \in \mathbb{R}^2 \times \mathbb{R} | f(x) \geq r\}$ is convex

D. $\{(x, r) \in \mathbb{R}^2 \times \mathbb{R} | f(x) \leq r\}$ is convex

48. Let $\mathbb{R}$ be the set of real numbers. A subset of $\mathbb{R}$, say E, is said to be open-if for every $x \in E$, there exists $r > 0$, such that $(x - r, x + r)$ is a subset of E. Then,

A. $E_1 \cap \cdots \cap E_n$ is open, for every collection of open sets $\{E_1, \ldots, E_n\}$

B. $E_1 \cup \cdots \cup E_n$ is open, for every collection of open sets $\{E_1, \ldots, E_n\}$

C. ϕ is open

D. all of the above are true

49. You have 100 observations on y with average value 15, and on x, with average value 8. From an OLS regression, you have estimated the slope on x to be 2. Your estimate of the mean of y conditioned on x is

A. 15

B. 16

C. 17

D. None of the above

50. Suppose your data produces the regression result $y = 10 + 3x$. Scale y by multiplying observations by 0.9 and do not scale x. The new intercept and slope estimates will be

A. 10 and 3

B. 9 and 3

C. 9 and 2.7

D. 10 and 2.7

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1. The range of the function $f : \Re \to \Re$ defined by $f(x) = \frac{x^2+x+2}{x^2+x+1}$ is

   A. $(1, \infty)$

   B. $[1, \frac{4}{3}]$

   C. $[1, \frac{7}{3}]$

   D. $[\frac{1}{3}, \frac{8}{3}]$

   The next six questions pertain to the following

   Data from a random sample of 107 home sales in 2003 yielded the regression

   $\hat{P} = \underset{(23.9)}{119.2} + \underset{(2.61)}{0.485}{}^{*}BD + \underset{(10.76)}{23.4}{}^{*}BA + \underset{(0.011)}{0.156}{}^{*}HS + \underset{(0.00048)}{0.002}{}^{*}PS + \underset{(0.311)}{0.090}{}^{*}A - \underset{(10.5)}{35.6}{}^{*}PC$
   $R^2 = 0.72; \quad SER = 41.5, P$ is price or value (Rs. 1000) , BD is number of bedrooms, BA is number of baths, HS is house size (sq. ft.), PS is plot size (sq. ft.), A is age (years), PC is a dummy variable = 1 if the house is in poor condition and = 0 otherwise; and the parentheses contain standard errors of the corresponding coefficients. SER is the standard error of the regression.

2. If a homeowner adds a new bathroom to her house which increases the house size by 100 sq. ft., what is the expected increase in the value of the house?

   A. $Rs.37,000$

   B. $Rs.39,450$

   C. $Rs.37,200$

   D. $Rs.39,000$

3. If a homeowner converts a bedroom into a bathroom, what is the expected increase in the value of house?

   A. $Rs.22,800$

   B. $Rs.22,915$

   C. $Rs.21,800$

   D. $Rs.23,915$

4. Are the coefficients of BA and PC individually statistically significant at the 5% level.

   A. Coefficient of BA is significant, but that of PC is not

   B. Coefficient of PC is significant, but that of BA is not

   C. Both are significant

   D. Neither is significant

5. What is the loss in value if a homeowner allows his house to get into poor condition?

   A. $Rs.34,300$

   B. $Rs.36,000$

C. $Rs.35,600$

D. $Rs.35,100$

6. If variable age were measured in decades, what would be its coefficient?

   A. 0.090

   B. 0.900

   C. 0.009

   D. 9.000

7. What is the adjusted coefficient of determination?

   A. 0.7032

   B. 0.7042

   C. 0.7022

   D. 0.7052

The next two questions pertain to the following

Consider utility functions

$$u_1(x,y) = \begin{cases} 2x, & \text{if } y/x > 2 \\ \max\{x,y\}, & \text{if } y/x \in [1/2, 2] \\ 2y, & \text{if } y/x < 1/2 \end{cases}$$

and

$$u_2(x,y) = \begin{cases} 2x, & \text{if } y/x > 2 \\ x+y, & \text{if } y/x \in [1/2, 2] \\ 2y, & \text{if } y/x < 1/2 \end{cases}$$

Let $p_x > 0$ and $p_y > 0$ be the prices of goods x and y respectively. Let $jp > 0$ denote wealth (or income).

8. For $i = 1, 2$, let $h_i(p_x, p_y, U)$ denote the set of solutions of the problem: choose $x > 0$ and $y > 0$ to minimise $p_x x + p_y y$ subject to $u_i(x,y) \geq U$. Let $e_i(p_x, p_y, U) = p_x X + p_y Y$, where $(X, Y) \in h_i(p_x, p_y, U)$

   A. $h_1(p_x, p_y, U) \subset h_2(p_x, p_y, U)$

   B. $h_1(p_x, p_y, U) \supset h_2(p_x, p_y, U)$

   C. $h_1(p_x, p_{yr}, U) = h_2(p_x, p_{yr}, U)$

   D. None of the above hold necessarily

9. Let $m_i(p_x, p_y, w)$ denote the set of Marshallian demands for utility u_i and $v_1(p_x, p_y, w) = u_i \circ m_i(p_x, p_y, w)$

   A. $m_1(p_x, p_y, w) \subset m_2(p_x, p_y, w)$ and $v_1(p_x, p_y, w) = v_2(p_x, p_y, w)$

   B. $m_1(p_x, p_y, w) \supset m_2(p_x, p_y, w)$ and $v_1(p_x, p_y, w) = v_2(p_x, p_y, w)$

   C. $m_1(p_x, p_y, w) \subset m_2(p_x, p_y, w)$ and $v_1(p_x, p_y, w) \leq v_2(p_x, p_y, w)$

D. $m_1(p_x, p_y, w) \supset m_2(p_x, p_y, w)$ and $v_1(p_x, p_y, w) \geq v_2(p_x, p_y, w)$

The next Three questions pertain to the following

Trader 1 is endowed with 100 identical Left shoes. Trader 2 is endowed with 99 identical Right shoes. Each trader's utility from her allocation of shoes is equal to the mumber of complete pairs of shoes in the allocation. Traders 1 and 2 trade shoes in competitive markets and arrive at a competitive equilibrium. Assume that shoes are infinitely divisible.

10. Given their endowments, an efficient allocation
    A. must give trader 1 at least 50 left shoes
    B. must give trader 1 at least 50 right shoes
    C. must give trader 1 at least 99 left shoes
    D. none of the above

11. An equilibrium allocation of shoes gives trader 2:
    A. at most 50 left shoes
    B. at least 99 left shoes
    C. at most 99 left shoes
    D. at most 50 right shoes

12. The equilibrium price of left shoe divided by the equilibrium price of right shoe is
    A. 1
    B. slightly less than 1
    C. slightly more than 1
    D. 0

13. Assume that the aggregate production of an economy is $Y_i = \sqrt{K_t L_t}$, where $K_{t+1} = (1-\delta)K_t + I_t, S_t = sY_i$ and $L_t = L$ (i.e., the notation and meanings correspond to the sctting for the Solow Model with constant population). Then, the savings rate s that maximizes the steady state rate of consumption equals
    A. $\frac{1}{2}$
    B. $\frac{\delta}{1+\delta}$
    C. $\frac{1}{1+\delta}$
    D. None of the above

14. Consider a function $f : \Re^2 \to \Re$. Suppose, for every $p \in \Re^2$, there exists $x(p) \in \Re^2$ such that $f(x(p)) \geq 1$ and $p \cdot x(p) \leq p.y$ for every $y \in \Re^2$ such that $f(u) \geq 1$. Define $g : \Re^2 \to \Re$ by $g(p) = p.x(p)$. Then, g is
    A. linear
    B. convex

C. quasi-convex

D. concave

15. Given nonempty-subsets of $\mathfrak{R}^2$, say $Y_1, \ldots, Y_n$, let $Y^* = \left\{ \sum_{j=1}^{n} y_j \mid y_1 \in Y_1, \ldots, y_n \in Y_n \right\}$. Given $p \in 3x^2$ and a nonempty set $Y \subset 3x^2$, let $v(p, Y) = \sup\{p \cdot y \mid y \in Y\}$ Then, for every p

A. $v(p, Y^*) < \sum_{j=1}^{n} v(p, Y_j)$ or $v(p, Y^*) > \sum_{j=1}^{n} v(p, Y_j)$

B. $v(p, Y^*) = \sum_{j=1}^{n} v(p, Y_j)$

C. $v(p, Y^*) \leq \sum_{j=1}^{n} v(p, Y_j)$

D. $v(p, Y^*) \geq \sum_{j=1}^{n} v(p, Y_j)$

16. A consumer lives for two periods 1 and 2. The lifetime utility function is $U = u(c_1) + \frac{u(c_2)}{(1+p)}$. The consumer earns w_1 and w_2 in the two periods, and her consumption c_1 and c_2 satjsfies a lifetime budget constraint $c_1 + \frac{c_2}{1+r} = w_1 + \frac{w_2}{1+r}$. Assume that $u(c_i) = \frac{c_i^{1-\alpha}}{t-\sigma}, \quad t = 1, 2.$ Then, if $r \geq \rho$, it follows that

A. $c_1 \geq c_2$

B. $c_1 \leq c_2$

C. $c_1 = c_2$

D. None of the above is necessarily true

17. The price-setting relation determines the real wage paid by firms depending on the level of technology (A) and mark-up m, and is represented by $\frac{W}{P} = \frac{A}{1+m}$. Under the wage-setting relation, the real wage is determined by the level of productivity (A) and the unemployment u. This is represented by $\frac{W}{P} = A(1-u)$. The effect of an increase in the level of technology on the unemployment is:

A. Positive

B. Negative

C. Zero

D. ambiguous

18. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{array} \right\}$$

and

$$g(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \text{ is irrational} \\ x, & \text{if } x \text{ is rational} \end{array} \right\}$$

Then, $h : \mathbb{R} \to \mathbb{R}$, defined by $h(x) = f(x) - g(x)$ is

A. injective but not surjective

B. surjective but not injective

C. neither injective nor surjective

D. bijective

19. Let $\|\cdot\|_n$ and $\|.\|_m$ be norms on $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. Let $\mathcal{L}$ be the space of linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$ and let $\|L\|_* = \sup\{\|L(x)\|_m \mid x \in \mathcal{R}^n\}$ and $\|L\|_{**} = \sup\{\|L(x)\|_m \mid x \in \mathcal{R}^n \text{ and } \|x\|_n \leq 1\}$ Then,

A. $\|\|_*$ defines a norm on $\mathcal{L}$

B. $\|\|_{**}$ defines a norm on $\mathcal{L}$

C. $\|\|_*$ and $\|\|_{**}$ are norms on $\mathcal{L}$

D. Neither $\|\|_*$ nor $\|\|_{**}$ is a norm on $\mathcal{L}$

20. Suppose that the mark-up over cost is 20% for a representative firm in an economy with labour being the single factor; and the wage-setting equation is: $W = P(1-u)$ (where, u = the unemploynent rate, P = Price and W = wage rate). Then the natural rate of unemployment is:

A. 20%

B. 17%

C. 13%

D. 10%

21. What is the money demand function when the utility of money for the represen- tative household is given by, $U(Y, M/P) = 0.5 \ln Y + 0.5 \ln(M/P)$ (i represents the oppottunity cost of holding money)?

A. $M^D/P = \frac{Y}{i}$

B. $M^D/P = \frac{2Y}{i}$

C. $M^D/P = \frac{Y}{2i}$

D. None of the above

22. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

and B_1, B_2, B_3 be three 3×1 column vectors, such that,

$$AB_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AB_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AB_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

Let B be the 3×3 matrix whose 3 columns are B_1, B_2 and B_3 respectively. Then the determinant det (B) equals

A. -3

B. 3

C. $\frac{-3}{2}$

D. $\frac{3}{2}$

23. A random number X, uniformly distributed on $[0, 1]$, divides [0,1] into 2 segments of lengths X and $(1 - X)$. Let R be the ratio of the smaller to the larger segment (i.e., $R = X/(1 - X)$, or $R = (1 - X)/X$, depending on whether, $X \leq 1/2$ or $X > 1/2$ The distribution of $R, F(r)$, that is the probability that $R \leq r$ equals

    A. $\frac{r}{r+1}$

    B. $\frac{2r}{r+1}$

    C. $\frac{1}{r+1}$

    D. $\frac{1-r}{r+1}$

24. If the marginal propensity to save is 0.3 and the marginal propensity to import is 0.1 , and the government increases expenditures by Rs. 10 billion, ignoring foreign-income repercussions, by how much will GDP rise?

    A. $Rs.20$ billion

    B. $Rs.10$ billion

    C. $Rs.25$ billion

    D. $Rs.15$ billion

25. The formula for the effective tariff rate is given by the following formula:

$$e = \frac{(n - ab)}{1 - a}$$

    where $e =$ the effective rate of protection, $n =$ the nominal tariff rate on the final product, $a =$ the ratio of the value of the imported input to the yalue of the final product, and $b =$ the nominal tariff rate on the imported input.

    Suppose that the tariff rate on the final product is 5 percent, If no imported inputs are used in the domestic production of the final product, the effective tariff rate is

    A. 3%

    B. 5%

    C. 8%

    D. 12%

26. Suppose that in the Solow Model of an economy with some positive savings rate, population growth rate, and rate of depreciation, k^* is the steady state capital-labour ratio. Suppose k_1 and k_2 are capital-labour ratios such that $k_1 < k_2 < k^*$, and let g_1, g_2 be the growth rates of per capita output at k_1 and k_2 respectively. Then

A. $g_1 > g_2$

B. $g_1 = g_2$

C. $g_1 > g_2$

D. None of the above

27. The maximum value attained by the function $f(x) = x^3 - x^2 - x - 1$ on the set $S = \{x \mid x^2 - x - 2 \leq 0\}$ occurs at $x =$

A. 1

B. $\frac{1}{3}$

C. 2

D. $\frac{5}{2}$

28. A random variable X has a standard normal distribution. What is the closest guess to the probability that X lies in the interval $[2, 3]$?

A. 0.001

B. 0.025

C. 0.25

D. 0.05

29. $\lim_{x\to\infty} \left(\frac{x^2-x+1}{x+1} - c_1 x - c_2\right) = -5$. So, it must be that (c_1, c_2) equals

A. $(2, -3)$

B. $(2, 3)$

C. $(1, 2)$

D. $(1, 3)$

30. The efficiency wage theory argues that firms choose to pay a _____ wage than the classical equilibrium wage, thus the real wage is _____ than the wage at which the labor market clears.

A. lower, lower

B. lower, higher

C. higher, lower

D. higher, higher

31. According to the theory of comparative advantage, countries gain from trade because

A. All firms can take advantage of cheap labour

B. Trade makes firms behave more competitively, reducing their market power

C. Output per worker in each firm increases

D. World output can rise when each country specializes in what it does relatively best

32. In the 2 factor, 2 good Heckscher-Ohlin model, the two countries differ in

    A. tastes

    B. relative availabilities of factors of production

    C. labour productivities

    D. technologies

33. The line $y = 2x + 5$ is tangent to a circle with equation $x^2 + y^2 + 16x + 12y + c = 0$, at point P. So, P equals

    A. $(-9, -7)$

    B. $(-10, -12)$

    C. $(-6, -7)$

    D. $(-11, -15)$

34. The random variable X denotes the number of successes in a sequence of independent trials, each with probability p of success. Let $\bar{X}$ denote the mean number of successes. We know that $\bar{X}$

    A. has a binomial distribution with mean p

    B. has a normal distribution with mean p

    C. approximates a normal distribution with mean p

    D. none of the above

35. A family has two children and it is known that at least one is a girl. What is the probability that both are girls given that at least one is a girl?

    A. $\frac{1}{2}$

    B. $\frac{1}{3}$

    C. $\frac{3}{4}$

    D. $\frac{2}{3}$

36. It is known that there is a rational number between any two distinct irrational numbers. Consider a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = \sin x$ for every rational number x. If x is an irrational number then,

    A. $f(x) = \sin(\frac{x}{2}) + \cos(\frac{x}{2})$

    B. $f(x) = \frac{\sin x}{2} + \frac{\cos x}{2}$

    C. $f(x) = \cos x$

    D. $f(x) = \sin x$

37. In a simple open economy framework, an increase in government spending leads to

    A. a rise in budget deficit and a fall in current account deficit

    B. a fall in budget deficit and a rise in current account deficit

C. a fall in both budget deficit and current account deficit

D. a rise in both budget deficit and current account deficit

38. The matrix $Q = PAP^T$ where P^T is the transpose of P, and $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

Then, $P^T Q^{12} P$ equals

A. $\begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 144 \\ 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 \\ 144 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{pmatrix}$

39. Nitin is a stamp collector and consumes only stamps and cheese sandwiches. His utility function is $u(s, c) = s + \log c$. If Nitin is at a point where he is consuming both goods, then the total amount that he is spending on cheese sandwiches depends

A. only on price of sandwiches

B. only on price of stamps

C. only on his income

D. on all three of the above

40. Consider the following set of 2 equations: $(2x)^{\ln 2} = (3y)^{\ln 3}$ $3^{\ln x} = 2^{\ln y}$ Suppose a pair (x, y) of numbers is a solution to this set of equations. Then x equals

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{6}$

41. Your budget is such that if you spend your entire income, you can afford either 4 units of good x and 6 units of good y or 12 units of good x and 2 units of y. What is the ratio of price of x to price of y?

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{3}$

D. $\frac{2}{3}$

42. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$ Then, $A^4 - 4A^3 + 2A^2 + A$ equals

    A. I

    B. $I + A$

    C. A

    D. A^{-1}

43. Consider a small open economy. If there is a positive productivity shock in the country, how will the domestic capital market be affected?

    A. There will be net capital inflow

    B. There will be net capital outflow

    C. Net capital inflow is zero

    D. The investment demand will fall

44. You have a single draw from a Bernoulli distribution. The maximum likelihood estimate of the probability of success p is

    A. 0

    B. 1

    C. either 0 or 1

    D. strictly between 0 and 1

45. The function $f(x)$ is twice differentiable, and $f(2) = 4, f(3) = 9, f(4) = 16$. Then, it must be that

    A. $f''(x) = 3$ for some $x \in (2, 3)$

    B. $f''(x) = 2$ for some $x \in (2, 4)$

    C. $f''(x) = 4$ for some $x \in (2, 3)$

    D. $f''(x) = 3$ for some $x \in (2, 4)$

46. Under a floating exchange rate regime, following an expansion in the money supply, monetary authorities will:

    A. buy domestic currency in the foreign exchange market

    B. sell domestic currency in the foreign exchange market

    C. do nothing in the foreign exchange market

    D. buy foreign currency in the foreign exchange market

47. In a roll of two fair dice, X is the number on the first die and Y is the number on the second die. Which of the following is true?

    A. $X - Y$ and $X + Y$ are dependent random variables

    B. X^2 and Y are independent random variable

C. X^2 and Y^2 are independent random variable

D. All of the above

48. A random variable has a uniform distribution on the interval $[-1, 1]$. The probability density function of X conditional on $X > 0.3$ is given by:

A. $\frac{7}{10}$

B. $\frac{10}{7}$

C. 1

D. $\frac{3}{10}$

49. The interval $(0, \infty)$ can be expressed as

A. $\cup_{n=1}^{\infty}[a_n, b_n]$ where each a_n, b_n is a rational number

B. $\cup_{n=1}^{\infty}(a_n, b_n)$ where a_n and b_n is a real number

C. $\cup_{n=1}^{\infty}[a_n, b_n]$ where each a_n and b_n is a real number

D. All of the above

50. What is the probability that at least one 6 appears when 6 fair dice are rolled?

A. $(\frac{5}{6})^6$

B. $\frac{1}{6}$

C. $1 - (\frac{5}{6})^6$

D. $\frac{5}{6}$

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1. Consider independently and identically distributed random variables $X_1, \ldots, X_n$ with values in $[0, 2]$. Each of these random variables is uniformly distributed on $[0, 2]$. If $Y = max\{X_1, \ldots, X_n\}$, then the mean of Y is

   A. $\left(\frac{n}{(n+1)}\right)^2$

   B. $\frac{n}{2n+1}$

   C. $\frac{2n}{n+1}$

   D. $\frac{n}{n+1}$

2. A coin toss has possible outcomes H and T with probabilities $\frac{3}{4}$ and $\frac{1}{4}$ respectively. A gambler observes a sequence of tosses of this coin until H occurs. Let the first H occur on the nth toss. If n is odd, then the gambler's prize is $-(2^n)$, and if n is even, then the gambler's prize is 2^n. What is the expected value of the gambler's prize?

   A. 1

   B. -1

   C. 3

   D. -3

3. Suppose two fair dice are tossed simultaneously. What is the probability that the total number of spots on the upper faces of the two dice is not divisible by 2, 3, or 5?

   A. 1/3

   B. 2/9

   C. 4/9

   D. 7/16

4. A student is answering a multiple-choice examination. Suppose a question has m possible answers. The student knows the correct answer with probability p. If the student knows the correct answer, then she picks that answer; otherwise, she picks randomly from the choices with probability $1/m$ each. Given that the student picked the correct answer, the probability that she knew the correct answer is

   A. $\frac{mp}{1+(m-1)p}$

   B. $\frac{mp}{1+(1-p)m}$

   C. $\frac{1-p}{1+(m-1)p}$

   D. $\frac{1-p}{1+(1-p)m}$

5. Suppose Y is a random variable with uniform distribution on $[0, 2]$. The value of the cumulative distribution function of the random variable $X = e^Y$ at $x \in [1, e^2]$ is

   A. $\frac{\log(x)}{2}$

B. $\frac{\log(x)}{4}$

C. $\log(x)$

D. $\log(x) - 1$

6. Consider an economy where the final commodity is produced by a single firm using labour only. The price-setting firm charges a 25% mark-up over its per unit nominal wage cost. The workers demand a real wage rate $\frac{W}{P} = (1 - u)$, where u is the unemployment rate, P is the price, and W is the nominal wage rate. The natural rate of unemployment in this economy is

   A. 20%

   B. 17%

   C. 13%

   D. 10%

7. The aggregate production function of an economy is $Y_t = (K_t L_t)^{\frac{1}{2}}$. Capital grows according to $K_{t+1} = (1 - \delta)K_t + S_t$, where $S_t = sY_t, L_t = \bar{L}$, s is the saving rate, δ is the depreciation rate and $\bar{L}$ is the total population. Then, the steady-state level of consumption per capita is

   A. $\frac{s}{\delta}$

   B. $\frac{s^2}{\delta^2}$

   C. $\delta^{\frac{1}{s}}$

   D. $\frac{s(1-s)}{\delta}$

8. Consider a production technology $Y = AL$, where Y is output, A is productivity, and L is labour input. A firm sets its price P at a constant mark-up μ over the effective wage cost per unit of production $\frac{W}{A}$. The expected real wage rate of workers is $\frac{W}{P^e} = A^{\alpha}(1-u)^{1-\alpha}$, where $0 < \alpha < 1$ and P^e is the expected price. If the price expected by workers matches the actual price level, then the effect of a rise in the level of productivity on unemployment is

   A. Positive

   B. Negative

   C. Zero

   D. Ambiguous

9. A household has an endowment of 1 unit of time .The household maximises its utility $u = \ln(c) + b\ln(1 - l)$, where c denotes consumption and $l \in [0, 1]$ denotes time spent working. It finances its consumption from labour income wl, where w is the market wage rate per unit of labour time. If the market wage rate goes up, then equilibrium labour supply of the household

   A. increases

   B. decreases

C. remains constant

D. changes in ambiguous manner

10. Consider the IS-LM model with a given price level P. Investment is a decreasing function of the interest rate and savings is an increasing function of aggregate income. The demand for real money balances $\frac{M}{P}$ is an increasing function of aggregate income and a decreasing function of the interest rate. The monetary authority chooses nominal money supply M to ensure that the resulting money market equilibrium keeps the interest rate fixed at some target level. In this setup, an increase in the target interest rate leads to

    A. a rise in equilibrium output

    B. a fall in equilibrium output

    C. no effect on equilibrium output

    D. an ambiguous effect on equilibrium output

11. Consider the Solow growth model with a given savings ratio, a constant population growth rate, zero rate of capital depreciation, and no technical progress. Let k^* be the steadystate capitallabour ratio in this economy. Suppose the economy is yet to reach the steadystate and has capitallabour ratio k_1 at time t_1 and capitallabour ratio k_2 at time t_2 such that $t_1 < t_2$ and $k_1 < k_2 < k^*$. Let the associated growth rates of per capita income at time t_1 and t_2 be g_1 and g_2 respectively. Then, by the properties of the Solow model,

    A. $g_1 < g_2$

    B. $g_1 > g_2$

    C. $g_1 = g_2$

    D. the relationship between g_1 and g_2 is ambiguous

12. A consumer lives for periods 1 and 2. Given consumptions c_1 and c_2 in these periods, her utility is $U = \ln(c_1) + (1+\rho)^{-1}\ln(c_2)$. She earns incomes w_1 and w_2 in the two periods and her lifetime budget constraint is $c_1 + (1+r)^{-1}c_2 = w_1 + (1+r)^{-1}w_2$, where r is the interest rate on savings. If $r > \rho$, then

    A. $c_1 > c_2$

    B. $c_1 < c_2$

    C. $c_1 = c_2$

    D. the relationship between c_1 and c_2 is ambiguous

13. A consumer lives for periods 1 and 2. Her lifetime utility function is $U(c_1, c_2) = \frac{c_1^\gamma + c_2^\gamma}{\gamma}$ where $0\gamma < 1$ and c_i is consumption in period i. The elasticity of substitution between consumption in period 1 and consumption in period 2 is

    A. $1 + \gamma$

    B. $1 - \gamma$

    C. $\frac{1}{1+\gamma}$

D. $\frac{1}{1-\gamma}$

14. A and B play a best-of-seven table-tennis match, i.e., the first to win four games will win the match. The two players are equally likely to win any of the games in the match. The probability that the match will end in 6 games is

    A. less than the probability that it will end in 7 games

    B. equal to the probability that it will end in 7 games

    C. greater than the probability that it will end in 7 games

    D. None of these

15. Let X and Y be jointly normally distributed, i.e. $(X, Y) \sim N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, If $\sigma_X^2 = \sigma_Y^2$ and $0 < \rho < 1$, then

    A. the OLS regression of Y on X will yield a slope that is less than unity, and that of X on Y will yield a slope greater than unity

    B. the OLS regression of Y on X will yield a slope that is less than unity, and that of X on Y will yield a slope less than unity

    C. the OLS regression of Y on X will yield a slope that is greater than unity, and that of X on Y will yield a slope less than unity

    D. it is not possible to draw conclusions from given information

16. Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. Relation $\succ$ is said to be irreflexive if $\neg(x \succ x)$ for every $x \in X$. Relation $\succ$ is said to be transitive if , for all $x, y, z \in X$,$x \succ y$ and $y \succ z$ implies $x \succ z$. If $\succ$ is asymmetric(i.e.,for all $x, y \in X$, $x \succ y$ implies $\neg(y \succ x)$) and negatively transitive(i.e.,for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$,or $z \succ y$, or both), then $\succ$ is

    A. irreflexive, but not transitive

    B. transitive, but not irreflexive

    C. irreflexive and transitive

    D. neither transitive nor irreflexive

17. Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \succeq y$ if and only if $\neg(y \succ x)$. Relation $\succeq$ is said to be total if , for all $x, y \in X$, $\neg(x \succeq y)$ implies $y \succeq x$. If $\succ$ is asymmetric(i.e.,for all $x, y \in X$, $x \succ y$ implies $\neg(y \succ x)$) and negatively transitive(i.e.,for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$,or $z \succ y$, or both), then $\succeq$ is

    A. is not total

    B. is total

    C. may not be total

    D. is not total over a nonempty subset of X

18. Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \succeq y$ if and only if $\neg(y \succ x)$. Relation $\succeq$ is said to be transitive if ,for all $x, y, z \in X$, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$. If $\succ$ is asymmetric(i.e.,for all $x, y \in X$, $x \succ y$ implies $\neg(y \succ x)$) and negatively transitive(i.e.,for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$,or $z \succ y$, or both), then $\succeq$ is

    A. is not transitive over a nonempty subset of X

    B. is not transitive

    C. may not be transitive

    D. is transitive

19. Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \sim y$ if and only if $\neg(x \succ y)$ and $\neg(y \succ x)$. Relation $\sim$ is said to be transitive if , for all $x, y, z \in X$, $x \sim y$ and $y \sim z$ implies $x \sim z$. If $\succ$ is asymmetric(i.e.,for all $x, y \in X$, $x \succ y$ implies $\neg(y \succ x)$) and negatively transitive(i.e.,for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$,or $z \succ y$, or both), then $\sim$ is

    A. is transitive

    B. is not transitive

    C. may not be transitive

    D. is not transitive over a nonempty subset of X

20. Let $\neg$ denote the negation of a statement. Consider a set X and a binary relation $\succ$ on X. For all $x, y \in X$, we say $x \sim y$ if and only if $\neg(x \succ y)$ and $\neg(y \succ x)$. Relation $\sim$ is said to be symmetric if , for all $x, y \in X$,$x \sim y$ implies $y \sim x$. If $\succ$ is asymmetric(i.e.,for all $x, y \in X$, $x \succ y$ implies $\neg(y \succ x)$) and negatively transitive(i.e.,for all $x, y, z \in X$, $x \succ y$ implies $x \succ z$,or $z \succ y$, or both), then $\sim$ is

    A. is symmetric

    B. is not symmetric

    C. may not be symmetric

    D. is not symmetric over a nonempty subset of X

21. Consider the following game for players 1 and 2. Player 1 moves first and chooses L or R. If she chooses L, then the game ends and the payoffs are $(1, 0)$, where the first entry is 1's payoff and the second entry is 2's payoff. If she chooses R, then 2 chooses U or D. If she chooses U, then the game ends and the payoffs are $(0, 2)$. If she chooses D, then 1 chooses L or R. If she chooses L, then the game ends and the payoffs are $(4, 0)$. If she chooses R, then the game ends and the payoffs are $(3, 3)$. This game has

    A. one Nash equilibrium in pure strategies

    B. two Nash equilibrium in pure strategies

    C. three Nash equilibrium in pure strategies

    D. no Nash equilibrium in pure strategies

22. Consider the following game for players 1 and 2. Player 1 moves first and chooses L or R. If she chooses L, then the game ends and the payoffs are $(1, 0)$, where the first entry is 1's payoff and the second entry is 2's payoff. If she chooses R, then 2 chooses U or D. If she chooses U, then the game ends and the payoffs are $(0, 2)$. If she chooses D, then 1 chooses L or R. If she chooses L, then the game ends and the payoffs are $(4, 0)$. If she chooses R, then the game ends and the payoffs are $(3, 3)$. This game has

    A. one subgame perfect Nash equilibrium

    B. two subgame perfect Nash equilibrium

    C. three subgame perfect Nash equilibrium

    D. no subgame perfect Nash equilibrium

23. In a non-cooperative game, if a profile of strategies

    A. is a Nash equilibrium, then it is an equilibrium in dominant strategies

    B. is a Nash equilibrium, then it is a subgame perfect equilibrium

    C. is a Nash equilibrium, then it is a sequential equilibrium

    D. is an equilibrium in dominant strategies, then it is a Nash equilibrium

24. If player 1 is the row player and player 2 is the column player in games

$$\text{G}= \begin{matrix} \\ U \\ D \end{matrix} \begin{matrix} L & R \\ \end{matrix} \begin{pmatrix} a,b & c,d \\ e,f & g,h \end{pmatrix} \quad \text{G'}= \begin{matrix} \\ U \\ D \end{matrix} \begin{matrix} L & M & R \\ \end{matrix} \begin{pmatrix} a,b & \alpha,\beta & c,d \\ e,f & \gamma,\delta & g,h \end{pmatrix}$$

    then

    A. 2s payoff in a Nash equilibrium of G' cannot be less than 2s payoff in a Nash equilibrium of G

    B. 2s payoff in a Nash equilibrium of G cannot be less than 2s payoff in a Nash equilibrium of G'

    C. 2s payoff in a Nash equilibrium of G must be equal to 2s payoff in a Nash equilibrium of G

    D. 2s payoff in a Nash equilibrium of G may be higher than 2s payoff in a Nash equilibrium of G'

25. Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1 lexicographically prefers x to y, i.e., between two non-identical bundles of x and y, she strictly prefers the bundle with more of x, but if the bundles have the same amount of x, then she strictly prefers the bundle with more of y. Agent 2's utility function is $u_2(x, y) = x + y$ Agent 1's endowment is $(\omega_x^1, \omega_y^1) = (0, 10)$ and Agent 2's endowment is $(\omega_x^2, \omega_y^2) = (10, 0)$ The set of competitive equilibrium price ratios $\frac{p_x}{p_y}$ for this economy is

    A. $\{1\}$

    B. $[0, 1]$

C. $(0, 1]$

D. ϕ

26. Consider an exchange economy with agents 1 and 2 and goods x and y. Agent 1 lexicographically prefers y to x, i.e., between two non-identical bundles of x and y, she strictly prefers the bundle with more of y, but if the bundles have the same amount of y, then she strictly prefers the bundle with more of x. Agent 2's utility function is $u_2(x, y) = x + y$ Agent 1's endowment is $(\omega_x^1, \omega_y^1) = (0, 10)$ and Agent 2's endowment is $(\omega_x^2, \omega_y^2) = (10, 0)$ The set of competitive equilibrium price ratios $\frac{p_x}{p_y}$ for this economy is

A. $\{1\}$

B. $[0, 1]$

C. $(0, 1]$

D. ϕ

27. Consider an exchange economy with goods x and y, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0, 9)$ and $(\omega_x^2, \omega_y^2) = (10, 0)$ respectively. The utility functions of 1 and 2 are $u_1(x, y) = \min\{x, y\}$ and $u_2(x, y) = \min\{x, y\}$ respectively. The competitive equilibrium price ratio $\frac{p_x}{p_y}$ for this economy is

A. $\frac{9}{10}$

B. $\frac{10}{9}$

C. 1

D. 0

28. Consider an exchange economy with goods x and y, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0, 9)$ and $(\omega_x^2, \omega_y^2) = (10, 0)$ respectively. The utility functions of 1 and 2 are $u_1(x, y) = \min\{x, y\}$ and $u_2(x, y) = \min\{x, y\}$ respectively. The competitive equilibrium allocations are

A. 1 gets $(10 - x, 9 - y)$ and 2 gets (x, y) where $x \in [9, 10]$ and $y = 9$

B. 1 gets (x, y) and 2 gets $(10 - x, 9 - y)$ where $x \in [9, 10]$ and $y = 9$

C. 1 gets (x, y) and 2 gets $(9 - x, 10 - y)$ where $x \in [8, 9]$ and $y = 10$

D. 1 gets (x, y) and 2 gets $(9 - x, 10 - y)$ where $x = 9$ and $y \in [9, 10]$

29. Consider an exchange economy with goods x and y, and agents 1 and 2, whose endowments are $(\omega_x^1, \omega_y^1) = (0, 9)$ and $(\omega_x^2, \omega_y^2) = (10, 0)$ respectively. The utility functions of 1 and 2 are $u_1(x, y) = \min\{x, y\}$ and $u_2(x, y) = \min\{x, y\}$ respectively. The allocation that gives $(10, 9)$ to 1 and $(0, 0)$ to 2 is

A. Pareto efficient but not a competitive equilibrium allocation

B. neither Pareto efficient nor a competitive equilibrium allocation

C. a competitive equilibrium allocation that is Pareto efficient

D. a competitive equilibrium allocation that is not Pareto efficient

30. Given a non-empty set $C \subset \mathbb{R}^n$, for every $p \in \mathbb{R}^n_+$, let $c(p) \in C$ be such that $p\dot{c}(p) \leq p\dot{c}$ for every $c \in C$. Then, the function $e : \mathbb{R}^n_+ \to \mathbb{R}$ given by $e(p) = p\dot{c}(p)$ is
    A. linear
    B. convex
    C. concave
    D. quasi-convex

31. Given a non-empty set $C \subset \mathbb{R}^n$, for every $p \in \mathbb{R}^n_+$, let $c(p) \in C$ be such that $p\dot{c}(p) \leq p\dot{c}$ for every $c \in C$. Then, the function $e : \mathbb{R}^n_+ \to \mathbb{R}$ given by $e(p) = p\dot{c}(p)$ is
    A. homogeneous of degree 0
    B. homogeneous of degree 1
    C. homogeneous of degree ∞
    D. non-homogeneous

32. Suppose $u : \mathbb{R} \to \mathbb{R}_+$ is strictly increasing and has the supremum (i.e., least upper bound) $\alpha \in \mathbb{R}$. Then the function $x \to \frac{u(x)}{\alpha - u(x)}$ is
    A. not well defined for some $x \in \mathbb{R}$
    B. bounded above
    C. unbounded above
    D. not strictly increasing

33. The interval $[0, \infty)$ can be expressed as
    A. $\cap_{n=1}^{\infty}(a_n, \infty)$ where each a_n is a rational number
    B. $\cup_{n=1}^{\infty}(a_n, b_n]$ where a_n and b_n is a real number
    C. $\cap_{n=1}^{\infty}[a_n, b_n]$ where each a_n and b_n is an irrational number
    D. All of these

34. If $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x) = \left\{ \begin{array}{ll} \frac{xy}{x^2+y^2}, & \text{for } xy \neq 0 \\ 0, & \text{for } xy = 0 \end{array} \right\}$$

    then
    A. f is differentiable at $(0, 0)$ and both partial derivatives at $(0, 0)$ are 0
    B. f is non-differentiable at $(0, 0)$ and both partial derivatives at $(0, 0)$ are 0
    C. f is differentiable at $(0, 0)$ and neither partial derivatives at $(0, 0)$ is 0
    D. f is non-differentiable at $(0, 0)$ and neither partial derivatives at $(0, 0)$ exist

35. Suppose is f a twice-differentiable function that solves the differential equation $D^2f - Df - f - 1 = 0$ over $\mathbb{R}$ and satisfies the condition $f(0) = 0 = fk)$ for some $k > 0$. Then,

A. f has positive and negative values over $(0, k)$

B. f has only positive values over $(0, k)$

C. f has only negative values over $(0, k)$

D. $f = -1$ on $(0, k)$

36. Let $\mathcal{B}$ be the collection of sets $E \subset \mathbb{R}$ satisfying: for every $x \in E$, there exist real numbers a and b such that $a < b$ and $x \in (a, b) \subset E$. Let $\mathcal{C}$ be the collection of sets $E \subset \mathbb{R}$ satisfying: for every $x \in E$, there exist rational numbers a and b such that $a < b$ and $x \in (a, b) \subset E$, then

A. $\mathcal{B} \subset \mathcal{C}$ and $\mathcal{B} \neq \mathcal{C}$

B. $\mathcal{C} \subset \mathcal{B}$ and $\mathcal{B} \neq \mathcal{C}$

C. $\mathcal{B} = \mathcal{C}$

D. Neither $\mathcal{B} \subset \mathcal{C}$ nor $\mathcal{C} \subset \mathcal{B}$

37. The set $\{(x, y) \in \mathbb{R}^2 | x > 0 \text{ and } y \leq \ln(x) - e^x\}$ is

A. a linear subspace of $\mathbb{R}^2$

B. convex

C. non-convex

D. convex polytope

38. Suppose the distance between $x, y \in \mathbb{R}$ is $|x - y|$ and $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. If E is an open subset of $\mathbb{R}$, then $\{x \in \mathbb{R} : f(x) \in E\}$ is

A. an open subset of $\mathbb{R}$

B. a closed subset of $\mathbb{R}$

C. neither an open nor a closed subset of $\mathbb{R}$

D. an open and closed subset of $\mathbb{R}$

39. Which of the following two numbers is larger for $k \neq 0$: $e^{k\pi}$ or π^{ke}

A. $e^{k\pi}$

B. π^{ke}

C. They are equal

D. It depends on the value of k

40. Consider the matrix $A = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$ where $\theta \in (0, 2\pi)$. The inner product of vectors $v = (v_1, v_2)$ and $w = (w_1, w_2)$ in $\mathbb{R}^2$ is defined by $< v, w >= v_1 w_1 + v_2 w_2$. So, for the vectors v and w in $\mathbb{R}^2$

A. $< Av, Aw >=< v, w >$

B. $< Av, Aw >> < v, w >$

C. $< Av, Aw >< < v, w >$

D. The comparison of $< Av, Aw >$ and $< v, w >$ depends on θ

41. Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to $x \in \mathbb{R}$. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x - \lfloor x \rfloor$ for $x \in \mathbb{R}$ is

    A. left-discontinuous at an integer

    B. right-discontinuous at an integer

    C. left discontinuous and right-discontinuous at an integer

    D. discontinuous everywhere

42. Let $\lceil x \rceil$ be the smallest integer that is greater than or equal to $x \in \mathbb{R}$. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \lceil x \rceil - x$ for $x \in \mathbb{R}$ is

    A. left-discontinuous at an integer

    B. right-discontinuous at an integer

    C. left discontinuous and right-discontinuous at an integer

    D. discontinuous everywhere

43. Let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to $x \in \mathbb{R}$. Let $\lceil x \rceil$ be the smallest integer that is greater than or equal to $x \in \mathbb{R}$. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \lceil x \rceil - \lfloor x \rfloor$ for $x \in \mathbb{R}$ is

    A. left-discontinuous at an integer

    B. right-discontinuous at an integer

    C. left discontinuous and right-discontinuous at an integer

    D. discontinuous everywhere

44. If $A = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, xy \geq 1\}$, $B = \{(x, y) \in \mathbb{R}^2 : x \leq 0, y \geq 0, xy \leq -1\}$ and $C = \{a + b : a \in A, b \in B\}$ Then,

    A. $\{(x, y) \in \mathbb{R}^2 : x = 0, y \geq 0\} \subset C$

    B. $\{(x, y) \in \mathbb{R}^2 : x = 0, y > 0\} \subset C$

    C. $\{(x, y) \in \mathbb{R}^2 : x \geq 0, y = 0\} \subset C$

    D. $\{(x, y) \in \mathbb{R}^2 : x > 0, y = 0\} \subset C$

45. Consider a 4×4 -matrix A. Obtain matrix B from matrix A by performing the following operations in sequence: (1) Interchange the first and fourth columns, and then (2) Interchange the second and fourth rows. Then,

    A. $detA = detB$

    B. $detA \neq detB$

    C. $detB \leq 0$

    D. $detB > 0$

46. The maximum value of $f(x, y) = xy$ subject to $|x| \geq |y|$ and $|x| + |y| \leq 1$ is

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 4

D. 2

47. Consider a decreasing differentiable function $g : \mathbb{R}_+ \to \mathbb{R}_+$ and an increasing continuous function $f : \mathbb{R}_+ \to \mathbb{R}_+$. If $F : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $F(x) = \int_0^{g(x)} f(t)dt$ for every $x \in \mathbb{R}_+$, then F is

    A. increasing over $[0, a]$ and decreasing over $[a, \infty)$ for some $a > 0$

    B. decreasing over $[0, a]$ and increasing over $[a, \infty)$ for some $a > 0$

    C. Increasing

    D. Decreasing

48. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{array} \right\}$$

    and

$$g(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \text{ is irrational} \\ x, & \text{if } x \text{ is rational} \end{array} \right\}$$

    Then, $h : \mathbb{R} \to \mathbb{R}$, defined by $h(x) = f(x) - g(x)$ is

    A. injective but not surjective

    B. surjective but not injective

    C. neither injective nor surjective

    D. bijective

49. Given nonempty subsets of $\mathbb{R}^2$, say $Y_1, \ldots, Y_n$, let $Y^* = \{\sum_{i=1}^n y_i | y_1 \in Y_1, \ldots, y_n \in Y_n\}$.

    Fix $p \in \mathbb{R}^2$. For a nonempty set $X \subset \mathbb{R}^2$, let $v(p, X) = \sup\{p\dot{x} | x \in X\}$.

    Suppose there exists $y^* \in Y^*$ such that $p\dot{y}^* = v(p, Y^*)$ and for every $i \in \{1, 2, \ldots, n\}$, there exists $y_i \in Y_i$ such that $p_i y_i = v(p, Y_i)$. Then,

    A. $v(p, Y^*) < \sum_{i=1}^n v(p, Y_i)$ or $v(p, Y^*) > \sum_{i=1}^n v(p, Y_i)$

    B. $v(p, Y^*) = \sum_{i=1}^n v(p, Y_i)$

    C. $v(p, Y^*) < \sum_{i=1}^n v(p, Y_i)$

    D. $v(p, Y^*) > \sum_{i=1}^n v(p, Y_i)$

50. If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 2 & -2 & 0 \\ 4 & 3 & 2 & -1 \end{bmatrix}$

and A^T is trannspose of A , then $det(A^T A)$ is

A. 16

B. -16

C. 4

D. -4