Indian Statistical Institute MSQE past year papers (2006-2020)

Econschool

Contents

1 ISI PEA 2006

1. If $f(x) = \log\left(\frac{1+x}{1-x}\right), \quad 0 < x < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ equals

   A. 2 f(x)

   B. $\frac{f(x)}{2}$

   C. $(f(x))^2$;

   D. none of these.

2. If $u = \phi(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ equals

   A. 0

   B. 1

   C. u

   D. none of these.

3. Let A and B be disjoint sets containing m and n elements, respectively, and let $C = A \cup B$. The number of subsets S of C that contain k elements and that also have the property that $S \cap A$ contains i elements is

   A. $\begin{pmatrix} m \\ i \end{pmatrix}$;

   B. $\begin{pmatrix} n \\ i \end{pmatrix}$

   C. $\begin{pmatrix} m \\ k-i \end{pmatrix}\begin{pmatrix} n \\ i \end{pmatrix}$

   D. $\begin{pmatrix} m \\ i \end{pmatrix}\begin{pmatrix} n \\ k-i \end{pmatrix}$.

4. The number of disjoint intervals over which the function $f(x) = |0.5x^2 - |x||$ is decreasing is

   A. one;

   B. two;

   C. three;

   D. none of these.

5. For a set of real numbers $x_1, x_2, \ldots\ldots, x_n$, the root mean square (RMS) defined as $\text{RMS} = \left\{\frac{1}{N}\sum_{i=1}^{n} x_i^2\right\}^{1/2}$ is a measure of central tendency. If AM denotes the arithmetic mean of the set of numbers, then which of the following statements is correct?

   A. RMS < AM always;

   B. RMS > AM always;

   C. RMS < AM when the numbers are not all equal;

D. RMS > AM when numbers are not all equal.

6. Let $f(x)$ be a function of real variable and let Δf be the function $\Delta f(\text{x}) = f(x+1) - f(x)$. For k > 1, put $\Delta^k f = \Delta\left(\Delta^{k-1} f\right)$. Then $\Delta^k f(x)$ equals

   A. $\sum_{j=0}^{k}(-1)^j \begin{pmatrix} k \\ j \end{pmatrix} f(x+j)$

   B. $\sum_{j=0}^{k}(-1)^{j+1} \begin{pmatrix} k \\ j \end{pmatrix} f(x+j)$

   C. $\sum_{j=0}^{k}(-1)^j \begin{pmatrix} k \\ j \end{pmatrix} f(x+k-j)$

   D. $\sum_{j=0}^{k}(-1)^{j+1} \begin{pmatrix} k \\ j \end{pmatrix} f(x+k-j)$

7. Let $I_n = \int_0^\infty x^n e^{-x} dx$, where n is some positive integer. Then I_n equals

   A. $n! - nI_{n-1}$

   B. $n! + nI_{n-1}$

   C. nI_{n-1}

   D. none of these.

8. If $x^3 = 1$, then $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ equals

   A. $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x & c & a \\ x^2 & a & b \end{vmatrix}$

   B. $(cx^2 + bx + a)\begin{vmatrix} x & b & c \\ 1 & c & a \\ x^2 & a & b \end{vmatrix}$

   C. $(cx^2 + bx + a)\begin{vmatrix} x^2 & b & c \\ x & c & a \\ 1 & a & b \end{vmatrix}$

   D. $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$

9. Consider any integer $I = m^2 + n^2$, where m and n are any two odd integers. Then

   A. I is never divisible by 2

   B. I is never divisible by 4

   C. I is never divisible by 6

   D. none of these.

10. A box has 10 red balls and 5 black balls. A ball is selected from the box. If the ball is red, it is returned to the box. If the ball is black, it and 2 additional black balls are added to the box. The probability that a second ball selected from the box will be red is

    A. $\frac{47}{72}$
    B. $\frac{25}{72}$
    C. $\frac{55}{153}$
    D. $\frac{98}{153}$.

11. Let $f(x) = \frac{\log\left(1+\frac{x}{p}\right)-\log\left(1-\frac{x}{q}\right)}{x}, x \neq 0$. If f is continuous at $x = 0$, then the value of $f(0)$ is

    A. $\frac{1}{p} - \frac{1}{q}$
    B. $p + q$
    C. $\frac{1}{p} + \frac{1}{q}$
    D. none of these.

12. Consider four positive numbers x_1, x_2, y_1, y_2 such that $y_1, y_2 > x_1x_2$ Consider the number $S = (x_1y_2 + x_2y_1) - 2x_1x_2$. The number S is

    A. always a negative integer;
    B. can be a negative fraction;
    C. always a positive number;
    D. none of these.

13. Given $x \geq y \geq z$, and $x + y + z = 12$, the maximum value of $x + 3y + 5z$ is

    A. 36
    B. 42
    C. 38
    D. 32 .

14. The number of positive pairs of integral values of (x, y) that solves $2xy - 4x^2 + 12x - 5y = 11$ is

    A. 4
    B. 1
    C. 2
    D. none of these.

15. Consider any continuous function $f : [0, 1] \to [0, 1]$. Which one of the following statements is incorrect?

    A. f always has at least one maximum in the interval [0,1]

B. f always has at least one minimum in the interval [0,1]

C. $\exists \text{x} \in [0, 1]$ such that $f(x) = x$

D. the function f must always have the property that $f(0) \in \{0, 1\}$ $f(1) \in \{0, 1\}$ and $f(0) + f(1) = I$

2 ISI PEA 2007

1. Let α and β be any two positive real numbers. Then $\lim_{x\to 0} \frac{(1+x)^a - 1}{(1+x)^{\frac{\beta}{\mu}} - 1}$ equals

   A. $\frac{\alpha}{\beta}$

   B. $\frac{\alpha+1}{\beta+1}$

   C. $\frac{\alpha-1}{\beta-1}$

   D. 1.

2. Suppose the number X is odd. Then $X^2 - 1$ is

   A. odd;

   B. not prime;

   C. necessarily positive;

   D. none of the above.

3. The value of k for which the function $f(x) = ke^{kx}$ is a probability density function on the interval [0,1] is

   A. $k = \log 2$;

   B. $k = 2\log 2$;

   C. $k = 3\log 3$;

   D. $k = 3\log 4$

4. p and q are positive integers such that $p^2 - q^2$ is a prime number. Then, $p - q$ is

   A. a prime number;

   B. an even number greater than 2

   C. an odd number greater than 1 but not prime;

   D. none of these.

5. Any non-decreasing function defined on the interval $[a, b]$

   A. is differentiable on (a, b)

   B. is continuous in $[a, b]$ but not differentiable;

   C. has a continuous inverse;

   D. none of these.

6. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 0$ is satisfied by

   A. $x = 1$;

   B. $x = 3$

C. $x = 4$

D. none of these.

7. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}}$, then $f'(x)$ is

A. $\frac{x}{2f(x)-1}$; (B) $\frac{1}{2f(x)-1}$; (

B. $\frac{1}{2f(x)-1}$.

C. $\frac{1}{x\sqrt{f(x)}}$;

D. $\frac{1}{2f(x)+1}$.

8. If $P = \log_x(xy)$ and $Q = \log_y(xy)$, then $P + Q$ equals

A. PQ

B. $\frac{P}{Q}$

C. $\frac{Q}{P}$

D. $\frac{PQ}{2}$

9. The solution to $\int \frac{2x^3+1}{x^4+2x}dx$ is

A. $\frac{x^4+2x}{4x^3+2}+$ constant;

B. $\log x^4 + \log 2x+$ constant;

C. $\frac{1}{2}\log|x^4 + 2x| +$ constant;

D. $\left|\frac{x^4+2x}{4x^3+2}\right| +$ constant.

10. The set of all values of x for which $x^2 - 3x + 2 > 0$ is

A. $(-\infty, 1)$

B. $(2, \infty)$

C. $(-\infty, 2) \cap (1, \infty)$

D. $(-\infty, 1) \cup (2, \infty)$

11. Consider the functions $f_1(x) = x^2$ and $f_2(x) = 4x^3 + 7$ defined on the real line. Then

A. f_1 is one-to-one and onto, but not f_2

B. f_2 is one-to-one and onto, but not f_1

C. both f_1 and f_2 are one-to-one and onto;

D. none of the above.

12. If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, $a > 0, b > 0$, then $f'(0)$ equals

A. $\left(\frac{b^2-a^2}{b^2}\right)\left(\frac{a}{b}\right)^{a+b-1}$

B. $\left(2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$

C. $2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}$

D. $\left(\frac{b^2-a^2}{ba}\right)$.

13. The linear programming problem

$$\begin{aligned}\max_{x,y} z &= 0.5x+1.5y\\ \text{subject to:}\quad x+y &\le 6\\ 3x+y &\le 15\\ x+3y &\le 15\\ x,y &\ge 0\end{aligned}$$

has

A. no solution;

B. a unique non-degenerate solution;

C. a corner solution;

D. infinitely many solutions.

14. Let $f(x;\theta)=\theta f(x;1)+(1-\theta)f(x;0)$, where θ is a constant satisfying $0<\theta<1$ Further, both $f(x;1)$ and $f(x;0)$ are probability density functions $(p\cdot d, f.)$. Then

A. $f(x;\theta)$ is a $p.d.f.$ for all values of θ

B. $f(x;\theta)$ is a $p.d.f.$ only for $0<\theta<\frac{1}{2}$

C. $f(x;\theta)$ is a $p.d.f.$ only for $\frac{1}{2}\le\theta<1$

D. $f(x;\theta)$ is not a $p.d.f.$ for any value of θ.

15. The correlation coefficient r for the following five pairs of observations satisfies

x	5	1	4	3	2
y	0	4	2	0	-1

A. $r>0$;

B. $r<-0.5$;

C. $-0.5<r<0$;

D. $r=0$.

16. An n -coordinated function f is called homothetic if it can be expressed as an increasing transformation of a homogeneous function of degree one. Let $f_1(x)=\sum_{i=1}^n x_i^r$, and $f_2(x)=\sum_{i=1}^n a_ix_i+b$, where $x_i>0$ for all $i, 0<r<1, a_i>0$ and b are constants. Then

A. f_1 is not homothetic but f_2 is;

B. f_2 is not homothetic but f_1 is;

C. both f_1 and f_2 are homothetic;

D. none of the above.

17. If $h(x) = \frac{1}{1-x}$, then $h(h(h(x))$ equals

    A. $\frac{1}{1-x}$

    B. x;

    C. $\frac{1}{x}$

    D. $1 - x$

18. The function $x|x| + \left(\frac{|x|}{x}\right)^3$ is

    A. continuous but not differentiable at $x = 0$

    B. differentiable at $x = 0$

    C. not continuous at $x = 0$;

    D. continuously differentiable at $x = 0$.

19. $\int \frac{2dx}{(x-2)(x-1)x}$ equals

    A. $\log \left|\frac{x(x-2)}{(x-1)^2}\right| +$ constant;

    B. $\log \left|\frac{(x-2)}{x(x-1)^2}\right| +$ constant;

    C. $\log \left|\frac{x^2}{(x-1)(x-2)}\right| +$ constant;

    D. $\log \left|\frac{(x-2)^2}{x(x-1)}\right| +$ constant.

20. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

    A. $1 - \frac{209}{552}$

    B. $1 - 14 \times \left(\frac{4}{5}\right)^{52}$

    C. $\left(\frac{4}{5}\right)^{50}$

    D. $\left(\frac{1}{5}\right)^{50}$

21. For any real number x, define $[x]$ as the highest integer value not greater than x. For example, $[0.5] = 0, [1] = 1$ and $[1.5] = 1$. Let $I = \int_0^{1/2} \{[x] + [x^2]\Big\} dx$. Then I equals

    A. 1

    B. $\frac{5-2\sqrt{2}}{2}$

    C. $2\sqrt{2}$

    D. none of these.

22. Every integer of the form $(n^3 - n)(n^2 - 4)$ (for $n = 3, 4, \ldots$) is

A. divisible by 6 but not always divisible by 12

B. divisible by 12 but not always divisible by 24

C. divisible by 24 but not always divisible by 120

D. divisible by 120 but not always divisible by 720 .

23. Two varieties of mango, A and B, are available at prices Rs. p_1 and Rs. p_2 per kg, respectively. One buyer buys 5 kg. of A and 10 kg. of B and another buyer spends Rs 100 on A and Rs. 150 on B. If the average expenditure per mango (irrespective of variety) is the same for the two buyers, then which of the following statements is the most appropriate?

A. $p_1 = p_2$

B. $p_2 = \frac{3}{4}p_1$

C. $p_1 = p_2$ or $p_2 = \frac{3}{4}p_1$

D. $\frac{3}{4} \leq \frac{p_2}{p_1} < 1$

24. For a given bivariate data set $(x_i, y_i; i = 1, 2, \ldots, n)$, the squared correlation coefficient (r^2) between x^2 and y is found to be 1. Which of the following statements is the most appropriate?

A. In the (x, y) scatter diagram, all points lie on a straight line.

B. In the (x, y) scatter diagram, all points lie on the curve $y = x^2$.

C. In the (x, y) scatter diagram, all points lie on the curve $y = a+bx^2, a > 0, b > 0$

D. In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2, a, b$ any real numbers.

25. The number of possible permutations of the integers 1 to 7 such that the numbers 1 and 2 always precede the number 3 and the numbers 6 and 7 always succeed the number 3 is

A. 720

B. 168

C. 84

D. none of these.

26. Suppose the real valued continuous function f defined on the set of non-negative real numbers satisfies the condition $f(x) = xf(x)$, then $f(2)$ equals

A. 1;

B. 2

C. 3

D. $f(1)$

27. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

    A. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$

    B. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$

    C. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$

    D. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$

28. Consider a square that has sides of length 2 units. Five points are placed anywhere inside this square. Which of the following statements is incorrect?

    A. There cannot be any two points whose distance is more than $2\sqrt{2}$.

    B. The square can be partitioned into four squares of side 1 unit each such that at least one unit square has two points that lies on or inside it.

    C. At least two points can be found whose distance is less than $\sqrt{2}$.

    D. Statements (a), (b) and (c) are all incorrect.

29. Given that f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

    A. $f(x)$ is an increasing function;

    B. $f(x)$ is a decreasing function;

    C. $|f(x)|$ is an increasing function;

    D. $|f(x)|$ is a decreasing function.

30. Let p, q, r, s be four arbitrary positive numbers. Then the value of $\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ is at least as large as

    A. 81

    B. 91

    C. 101.

    D. None of these.

3 ISI PEA 2008

1. $\int \frac{dx}{x+x\log x}$ equals

   A. $\log|x+x\log x|+$ constant

   B. $\log|1+x\log x|+$ constant

   C. $\log|\log x|+$ constant

   D. $\log|1+\log x|+$ constant.

2. The inverse of the function $\sqrt{-1+x}$ is

   A. $\frac{1}{\sqrt{x-1}}$,

   B. x^2+1,

   C. $\sqrt{x-1}$,

   D. none of these.

3. The domain of continuity of the function $f(x)=\sqrt{x}+\frac{x+1}{x-1}-\frac{x+1}{x^2+1}$ is

   A. $[0,1)$

   B. $(1,\infty)$

   C. $[0,1)\cup(1,\infty)$,

   D. none of these

4. Consider the following linear programme: minimise $x-2y$ subject to

$$x+3y\geq 3$$

$$3x+y\geq 3$$
$$x+y\leq 3$$

   An optimal solution of the above programme is given by

   A. $x=\frac{3}{4}, y=\frac{3}{4}$

   B. $x=0, y=3$

   C. $x=-1, y=3$

   D. none of the above.

5. Consider two functions $f_1:\{a_1,a_2,a_3\}\rightarrow\{b_1,b_2,b_3,b_4\}$ and $f_2:\{b_1,b_2,b_3,b_4\}\rightarrow\{c_1,c_2,c_3\}$. The function f_1 is defined by $f_1(a_1)=b_1, f_1(a_2)=b_2, f_1(a_3)=b_3$ and the function f_2 is defined by $f_2(b_1)=c_1, f_2(b_2)=c_2, f_2(b_3)=f_2(b_4)=c_3$. Then the mapping $f_2\circ f_1:\{a_1,a_2,a_3\}\rightarrow\{c_1,c_2,c_3\}$ is

   A. a composite and one $-$ to $-$ one function but not an onto function.

   B. a composite and onto function but not a one $-$ to $-$ one function.

   C. a composite, one $-$ to $-$ one and onto function.

   D. not a function.

6. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}, t > 0, t \neq 1$ then the relation between x and y is

   A. $y^x = x^{\frac{1}{y}}$,

   B. $x^{\frac{1}{y}} = y^{\frac{1}{x}}$,

   C. $x^y = y^x$,

   D. $x^y = y^{\frac{1}{x}}$.

7. The maximum value of $T = 2x_B + 3x_S$ subject to the constraint $20x_B + 15x_S \leq 900$ where $x_B \geq 0$ and $x_S \geq 0$, is

   A. 150,

   B. 180,

   C. 200,

   D. none of these.

8. The value of $\int_0^2 [x]^n f'(x)dx$, where $[x]$ stands for the integral part of x, n is a positive integer and f' is the derivative of the function f, is

   A. $(n + 2^n)(f(2) - f(0))$,

   B. $(1 + 2^n)(f(2) - f(1))$

   C. $2^n f(2) - (2^n - 1) f(1) - f(0)$,

   D. none of these.

9. A surveyor found that in a society of 10,000 adult literates 21% completed college education, 42% completed university education and remaining 37% completed only school education. Of those who went to college 61% reads newspapers regularly, 35% of those who went to the university and 70% of those who completed only school education are regular readers of newspapers. Then the percentage of those who read newspapers regularly completed only school education is

   A. 40%,

   B. 52%,

   C. 35%,

   D. none of these.

10. The function $f(x) = x|x|e^{-x}$ defined on the real line is

    A. continuous but not differentiable at zero,

    B. differentiable only at zero,

    C. differentiable everywhere,

    D. differentiable only at finitely many points.

11. Let X be the set of positive integers denoting the number of tries it takes the Indian cricket team to win the World Cup. The team has equal odds for winning or losing any match. What is the probability that they will win in odd number of matches?

A. 1/4,

B. 1/2,

C. 2/3

D. 3/4

12. Three persons X, Y, Z were asked to find the mean of 5000 numbers, of which 500 are unities. Each one did his own simplification. $X's$ method: Divide the set of number into 5 equal parts, calculate the mean for each part and then take the mean of these. $Y's$ method: Divide the set into 2000 and 3000 numbers and follow the procedure of A. Z' s method: Calculate the mean of 4500 numbers (which are $\neq 1$) and then add 1 . Then

    A. all methods are correct,

    B. $X's$ method is correct, but Y and $Z's$ methods are wrong,

    C. $X's$ and $Y's$ methods are correct but Z_s' methods is wrong,

    D. none is correct.

13. The number of ways in which six letters can be placed in six directed envelopes such that exactly four letters are placed in correct envelopes and exactly two letters are placed in wrong envelopes is

    A. 1,

    B. 15,

    C. 135 .

    D. None of these.

14. The set of all values of x for which the inequality $|x-3|+|x+2|<11$ holds is

    A. $(-3,2)$,

    B. $(-5,2)$,

    C. $(-5,6)$,

    D. none of these.

15. The function $f(x)=x^4-4x^3+16x$ has

    A. a unique maximum but no minimum,

    B. a unique minimum but no maximum,

    C. a unique maximum and a unique minimum,

    D. neither a maximum nor a minimum.

16. Consider the number $K(n)=(n+3)\left(n^2+6n+8\right)$ defined for integers n. Which of the following statements is correct?

    A. $K(n)$ is always divisible by 4

    B. $K(n)$ is always divisible by 5

C. $K(n)$ is always divisible by 6

D. All Statements are incorrect.

17. 25 books are placed at random on a shelf. The probability that a particular pair of books shall be always together is

A. $\frac{2}{25}$

B. $\frac{1}{25}$,

C. $\frac{1}{300}$

D. $\frac{1}{600}$

18. $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If -1 is a root of the equation, the other root is

A. $\frac{4}{5}$,

B. $\frac{8}{5}$,

C. $\frac{6}{5}$,

D. $\frac{3}{5}$.

19. The correlation coefficients between two variables X and Y obtained from the two equations $2x + 3y - 1 = 0$ and $5x - 2y + 3 = 0$ are

A. equal but have opposite signs,

B. $-\frac{2}{3}$ and $\frac{2}{5}$,

C. $\frac{1}{2}$ and $-\frac{3}{5}$,

D. Cannot say.

20. If a, b, c, d are positive real numbers then $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$ is always

A. less than $\sqrt{2}$.

B. less than 2 but greater than or equal to $\sqrt{2}$,

C. less than 4 but greater than or equal 2

D. greater than or equal to 4 .

21. The range of value of x for which the inequality $\log_{(2-x)}(x-3) \geq -1$ holds is

A. $2 < x < 3$,

B. $x > 3$,

C. $x < 2$,

D. no such x exists.

22. The equation $5x^3 - 5x^2 + 2x - 1$ has

A. all roots between 1 and 2 ,

B. all negative roots,

C. a root between 0 and 1 ,

D. all roots greater than 2 .

23. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \quad (k > 0, 0 \leq x \leq \infty)$$

Then, a equals

A. $\frac{k^3}{2}$,

B. $\frac{k}{2}$,

C. $\frac{k^2}{2}$,

D. k

24. Let $x = r$ be the mode of the distribution with probability mass function $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Then which of the following inequalities hold.

A. $(n+1)p - 1 < r < (n+1)p$,

B. $r < (n+1)p - 1$

C. $r > (n+1)p$

D. $r < np$.

25. Let $y = (y_1, \ldots, y_n)$ be a set of n observations with $y_1 \leq y_2 \leq \ldots \leq y_n$. Let $y' = (y_1, y_2, \ldots, y_j + \delta, \ldots, y_k - \delta, \ldots, y_n)$ where $y_k - \delta > y_{k-1} > \ldots > y_{j+1} > y_j + \delta$ $\delta > 0$. Let σ : standard deviation of y and σ' : standard deviation of y'. Then

A. $\sigma < \sigma'$,

B. $\sigma' < \sigma$,

C. $\sigma' = \sigma$,

D. nothing can be said.

26. Let x be a r.v. with pdf $f(x)$ and let $F(x)$ be the distribution function. Let $r(x) = \frac{xf(x)}{1-F(x)}$. Then for $x < e^{\mu}$ and $f(x) = \frac{e^{-\frac{(\log x - \mu)^2}{2}}}{x\sqrt{2\pi}}$, the function $r(x)$ is

A. increasing in x,

B. decreasing in x

C. constant,

D. none of the above.

27. A square matrix of order n is said to be a bistochastic matrix if all of its entries are non-negative and each of its rows and columns sum to 1. Let $y_{nx1} = P_{n \times n} x_{nx1}$ where elements of y are some rearrangements of the elements of x. Then

A. P is bistochastic with diagonal elements 1 ,

B. P cannot be bistochastic,

C. P is bistochastic with elements 0 and 1 ,

D. P is a unit matrix.

28. Let $f_1(x) = \frac{x}{x+1}$. Define $f_n(x) = f_1\left(f_{n-1}(x)\right)$, where $n \geq 2$. Then $f_n(x)$ is

A. decreasing in n,

B. increasing in n,

C. initially decreasing in n and then increasing in n,

D. initially increasing in n and then decreasing n.

29. $\lim_{n\to\infty} \frac{1-x^{-2n}}{1+x^{-2n}}, x > 0$ equals

A. 1,

B. -1,

C. 0 .

D. The limit does not exist.

30. Consider the function $f(x_1, x_2) = \max\{6 - x_1, 7 - x_2\}$. The solution (x_1^*, x_2^*) to the optimization problem minimize $f(x_1, x_2)$ subject to $x_1 + x_2 = 21$ is

A. $(x_1^* = 10.5, x_2^* = 10.5)$,

B. $(x_1^* = 11, x_2^* = 10)$

C. $(x_1^* = 10, x_2^* = 11)$,

D. None of these.

4 ISI PEA 2009

1. An infinite geometric series has first term 1 and sum 4. It common ratio is

    A. $\frac{1}{2}$

    B. $\frac{3}{4}$

    C. 1

    D. $\frac{1}{3}$

2. A continuous random variable X has a probability density function $f(x) = 3x^2$ with $0 \leq x \leq 1$. If $P(X \leq a) = P(x > a)$, then a is:

    A. $\frac{1}{\sqrt{6}}$

    B. $\left(\frac{1}{3}\right)^{\frac{1}{2}}$

    C. $\frac{1}{2}$

    D. $\left(\frac{1}{2}\right)^{\text{j}}$

3. If $f(x) = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x + \ldots}}}$ then $f'(x)$ equals to

    A. $\frac{f(x)-1}{2f(x)+1}$.

    B. $\frac{f^2(x)-f(x)}{2f(x)-1}$

    C. $\frac{2f(x)+1}{f^2(x)+f(x)}$

    D. $\frac{f(x)}{2f(x)+1}$

4. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$ is

    A. $\frac{1}{6}$

    B. 0

    C. $\frac{1}{4}$

    D. not well defined

5. If $X = 2^{65}$ and $Y = 2^{64} + 2^{63} + \ldots + 2^1 + 2^0$, then

    A. $Y = X + 2^{64}$.

    B. $X = Y$.

    C. $Y = X + 1$

    D. $Y = X - 1$

6. $\int_0^1 \frac{e^x}{e^x+1} dx =$

    A. $\log(1 + e)$

    B. log2.

C. $\log \frac{1+e}{2}$.

D. $2 \log(1 + e)$

7. There is a box with ten balls. Each ball has a number between 1 and 10 written on it. No two balls have the same number. Two balls are drawn (simultaneously) at random from the box. What is the probability of choosing two balls with odd numbers?

A. $\frac{1}{9}$.

B. $\frac{1}{2}$

C. $\frac{2}{9}$

D. $\frac{1}{3}$

8. A box contains 100 balls. Some of them are white and the remaining are red. Let X and Y denote the number of white and red balls respectively. The correlation between X and Y is

A. 0.

B. 1 .

C. -1.

D. some real number between $-\frac{1}{2}$ and $\frac{1}{2}$.

9. Let f, g and h be real valued functions defined as follows: $f(x) = x(1 - x)$ $g(x) = \frac{x}{2}$ and $h(x) = \min\{f(x), g(x)\}$ with $0 \leq x \leq 1$. Then h is

A. continuous and differentiable

B. differentiable but not continuous

C. continuous but not differentiable

D. neither continuous nor differentiable

10. In how many ways can three persons, each throwing a single die once, make a score of 8?

A. 5

B. 15

C. 21

D. 30

11. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 55 - 7x$$

for every $x \in \Re$, then $f(3)$ equals

A. 40

B. 32

C. 26

D. 10

12. Two persons, A and B, make an appointment to meet at the train station between 4 P.M. and 5 P.M.. They agree that each is to wait not more than 15 minutes for the other. Assuming that each is independently equally likely to arrive at any point during the hour, find the probability that they meet.

A. $\frac{15}{16}$

B. $\frac{7}{16}$

C. $\frac{5}{24}$

D. $\frac{22}{175}$

13. If x_1, x_2, x_3 are positive real numbers, then

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1}$$

is always

A. ≤ 3

B. $\leq 3^{\frac{1}{3}}$

C. ≥ 3

D. 3

14. $\lim_{n\to\infty} \frac{1^2+2^2+...+n^2}{n^3}$ equals

A. 0

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. 1 .

15. Suppose b is an odd integer and the following two polynomial equations have a common root.

$$x^2 - 7x + 12 = 0$$

$x^2 - 8x + b = 0$ The root of $x^2 - 8x + b = 0$ that is not a root of $x^2 - 7x + 12 = 0$ is

A. 2

B. 3

C. 4

D. 5

16. Suppose $n \geq 9$ is an integer. Let $\mu = n^{\frac{1}{2}} + n^{\frac{1}{3}} + n^{\frac{1}{4}}$. Then, which of the following relationships between n and μ is correct?

A. $n = \mu$

B. $n > \mu$

C. $n < \mu$

D. None of the above.

17. Which of the following functions $f : \Re \to \Re$ satisfies the relation $f(x+y) = f(x)+f(y)$?

A. $f(z) = z^2$

B. $f(z) = az$ for some real number a

C. $f(z) = \log z$

D. $f(z) = e^z$

18. For what value of a does the following equation have a unique solution?

$$\begin{vmatrix} x & a & 2 \\ 2 & x & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

A. 0

B. 1

C. 2

D. 4

19. Let

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

where l, m, n, a, b, c are non-zero numbers. Then $\frac{dy}{dx}$ equals

A.

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

B.

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix}$$

C.

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

D.

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l-a & m-b & n-c \\ 1 & 1 & 1 \end{vmatrix}$$

20. If $f(x) = |x-1| + |x-2| + |x-3|$, then $f(x)$ is differentiable at

    A. 0

    B. 1

    C. 2

    D. 3

21. If $(x-a)^2 + (y-b)^2 = c^2$, then $1 + \left[\frac{du}{dx}\right]^2$ is independent of

    A. a

    B. b

    C. c

    D. Both b and c

22. A student is browsing in a second-hand bookshop and finds n books of interest. The shop has m copies of each of these n bools. Assuming he never wants duplicate copies of any book, and that he selects at least one book, how many ways can he make a selection? For example, if there is one book of interest with two copies, then he can make a selection in 2 ways.

    A. $(m+1)^n - 1$

    B. nm

    C. $2^{nm} - 1$

    D. $\frac{nm!}{(m!)(nm-m)!} - 1$

23. Determine all values of the constants A and B such that the following function is continuous for all values of x.

$$f(x) = \begin{cases} Ax - B & \text{if } x \leq -1 \\ 2x^2 + 3Ax + B & \text{if } -1 < x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

    A. $A = B = 0$

    B. $A = \frac{3}{4}, B = -\frac{1}{4}$

    C. $A = \frac{1}{4}, B = \frac{3}{4}$

    D. $A = \frac{1}{2}, B = \frac{1}{2}$

24. The value of $\lim_{x \to \infty} \left(3^x + 3^{2x}\right)^{\frac{1}{2}}$ is

    A. 0

    B. 1

    C. e

    D. 9

25. A computer while calculating correlation coefficient between two random variables X and Y from 25 pairs of observations obtained the following results: $\sum X = 125, \sum X^2 = 650, \sum Y = 100, \sum Y^2 = 460, \sum XY = 508$. It was later discovered that at the time of inputing, the pair $(X = 8, Y = 12)$ had been wrongly input as $(X = 6, Y = 14)$ and the pair $(X = 6, Y = 8)$ had been wrongly input as $(X = 8, Y = 6)$. Calculate the value of the correlation coefficient with the correct data.

   A. $\frac{4}{5}$

   B. $\frac{2}{3}$

   C. 1

   D. $\frac{5}{6}$

26. The point on the curve $y = x^2 - 1$ which is nearest to the point (2,-0.5) is

   A. $(1, 0)$

   B. $(2, 3)$

   C. $(0, -1)$

   D. None of the above

27. If a probability density function of a random variable X is given by $f(x) = kx(2-x), 0 \leq x \leq 2$, then mean of X is

   A. $\frac{1}{2}$

   B. 1

   C. $\frac{1}{5}$

   D. $\frac{3}{4}$

28. Suppose X is the set of all integers greater than or equal to 8. Let $f : X \to \Re$. and $f(x + y) = f(xy)$ for all $x, y \geq 4$. If $f(8) = 9$, then $f(9) =$

   A. 8 .

   B. 9.

   C. 64 .

   D. 81 .

29. Let $f : \Re \to \Re$ be defined by $f(x) = (x-1)(x-2)(x-3)$. Which of the following is true about f?

   A. It decreases on the interval $\left[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}\right]$

   B. It increases on the interval $\left[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}\right]$

   C. It decreases on the interval $\left(-\infty, 2 - 3^{-\frac{1}{2}}\right]$

   D. It decreases on the interval [2,3]

30. A box with no top is to be made from a rectangular sheet of cardboard measuring 8 metres by 5 metres by cutting squares of side x metres out of each corner and folding up the sides. The largest possible volume in cubic metres of such a box is

    A. 15

    B. 12

    C. 20

    D. 18

5 ISI PEA 2010

1. The value of $100\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{99.100}\right]$

    A. is 99 ,

    B. is 100

    C. is 101

    D. is $\frac{(100)^2}{99}$.

2. The function $f(x)=x(\sqrt{x}+\sqrt{x+9})$ is

    A. continuously differentiable at $x=0$,

    B. continuous but not differentiable at $x=0$,

    C. differentiable but the derivative is not continuous at $x=0$,

    D. not differentiable at $x=0$.

3. Consider a GP series whose first term is 1 and the common ratio is a positive integer $r(>1)$. Consider an AP series whose first term is 1 and whose $(r+2)^{\text{th}}$ term coincides with the third term of the GP series. Then the common difference of the AP series is

    A. $r-1$

    B. r

    C. $r+1$

    D. $r+2$

4. The first three terms of the binomial expansion $(1+x)^n$ are $1, -9, \frac{297}{8}$ respectively. What is the value of n?

    A. 5

    B. 8

    C. 10

    D. 12

5. Given $\log_p x=\alpha$ and $\log_q x=\beta$, the value of $\log_{\frac{p}{q}} x$ equals

    A. $\frac{\alpha\beta}{\beta-\alpha}$

    B. $\frac{\beta-\alpha}{\alpha\beta}$

    C. $\frac{\alpha-\beta}{\alpha\beta}$,

    D. $\frac{\alpha\beta}{\alpha-\beta}$

6. Let $P=\{1,2,3,4,5\}$ and $Q=\{1,2\}$. The total number of subsets X of P such that $X\cap Q=\{2\}$ is

    A. 6

B. 7

C. 8

D. 9.

7. An unbiased coin is tossed until a head appears. The expected number of tosses required is

A. 1

B. 2

C. 4

D. ∞.

8. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x \geq c \\ 0 & x < c \end{cases}$$

Then the expectation of X is

A. 0

B. ∞

C. $\frac{1}{c}$

D. $\frac{1}{c^2}$

9. The number of real solutions of the equation $x^2 - 5|x| + 4 = 0$ is

A. two

B. three

C. four

D. None of these

10. Range of the function $f(x) = \frac{x^2}{1+x^2}$ is

A. [0,1)

B. (0,1)

C. [0,1]

D. (0,1]

11. If a, b, c are in AP, then the value of the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

A. $b^2 - 4ac$

B. $ab + bc + ca$

C. $2b - a - c$

D. $3b + a + c$

12. If $a < b < c < d$, then the equation $(x-a)(x-b) + 2(x-c)(x-d) = 0$ has
    A. both the roots in the interval $[a, b]$
    B. both the roots in the interval $[c, d]$,
    C. one root in the interval (a, b) and the other root in the interval (c, d)
    D. one root in the interval $[a, b]$ and the other root in the interval $[c, d]$.
    E. None of the above

13. Let f and g be two differentiable functions on (0,1) such that $f(0) = 2$, $f(1) = 6, g(0) = 0$ and $g(1) = 2$. Then there exists $\theta \in (0, 1)$ such that $f'(\theta)$ equals
    A. $\frac{1}{2}g'(\theta)$
    B. $2g'(\theta)$
    C. $6g'(\theta)$
    D. $\frac{1}{6}g'(\theta)$

14. The minimum value of $\log_x a + \log_a x$, for $1 < a < x$, is
    A. less than 1 ,
    B. greater than 2 ,
    C. greater than 1 but less than 2 .
    D. None of these.

15. The value of $\int_4^9 \frac{1}{2x(1+\sqrt{x})}dx$ equals
    A. $\log_e 3 - \log_e 2$
    B. $2\log_e 2 - \log_e 3$
    C. $2\log_e 3 - 3\log_e 2$
    D. $3\log_e 3 - 2\log_e 2$.

16. The inverse of the function $f(x) = \frac{1}{1+x}, x > 0$, is
    A. $(1 + x)$
    B. $\frac{1+x}{x}$
    C. $\frac{1-x}{x}$,
    D. $\frac{x}{1+x}$

17. Let $X_i, i = 1, 2, \ldots, n$ be identically distributed with variance σ^2. Let $\operatorname{cov}(X_i, X_j) = \rho$ for all $i \neq j$. Define $\bar{X}_n = \frac{1}{n}\sum X_i$ and let $a_n = \operatorname{Var}\left(\bar{X}_n\right)$ Then $\lim_{n\to\infty} a_n$ equals
    A. 0 ,
    B. ρ,
    C. $\sigma^2 + \rho$

D. $\sigma^2 + \rho^2$.

18. Let X be a Normally distributed random variable with mean 0 and variance 1. Let $\Phi(.)$ be the cumulative distribution function of the variable X. Then the expectation of $\Phi(X)$ is

    A. $-\frac{1}{2}$,

    B. 0 ,

    C. $\frac{1}{2}$,

    D. 1 .

19. Consider any finite integer $r \geq 2$. Then $\lim_{x \to 0} \left[\frac{\log_e \left(\sum_{k=0}^{r} x^k \right)}{\left(\sum_{k=1}^{\infty} \frac{x^k}{k!} \right)} \right]$ equals

    A. 0 ,

    B. 1

    C. e,

    D. $\log_e 2$.

20. Consider 5 boxes, each containing 6 balls labelled 1,2,3,4,5,6 . Suppose one ball is drawn from each of the boxes. Denote by b_i, the label of the ball drawn from the i -th box, $i = 1, 2, 3, 4, 5$. Then the number of ways in which the balls can be chosen such that $b_1 < b_2 < b_3 < b_4 < b_5$ is

    A. 1

    B. 2

    C. 5

    D. 6

21. The sum $\sum_{r=0}^{m} \binom{n+r}{r}$ equals

    A. $\begin{pmatrix} n+m+1 \\ n+m \end{pmatrix}$

    B. $(n+m+1) \begin{pmatrix} n+m \\ n+1 \end{pmatrix}$

    C. $\begin{pmatrix} n+m+1 \\ n \end{pmatrix}$

    D. $\begin{pmatrix} n+m+1 \\ n+1 \end{pmatrix}$

22. Consider the following 2 -variable linear regression where the error ϵ_i 's are independently and identically distributed with mean 0 and variance 1;

$$y_i = \alpha + \beta (x_i - \bar{x}) + \epsilon_i, \quad i = 1, 2, \ldots, n$$

Let $\hat{\alpha}$ and $\hat{\beta}$ be ordinary least squares estimates of α and β respectively. Then the correlation coefficient between $\hat{\alpha}$ and $\hat{\beta}$ is

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

23. Let f be a real valued continuous function on $[0,3]$. Suppose that $f(x)$ takes only rational values and $f(1)=1$. Then $f(2)$ equals

A. 2

B. 4

C. 8

D. None of these

24. Consider the function $f\left(x_1,x_2\right)=\int_0^{\sqrt{x_1^2+x_2^2}} e^{-\left(w^2/\left(x_1^2+x_2^2\right)\right)}dw$ with the prop- erty that $f(0,0)=0$. Then the function $f\left(x_1,x_2\right)$ is

A. homogeneous of degree -1

B. homogeneous of degree $\frac{1}{2}$,

C. homogeneous of degree 1

D. None of these.

25. If $f(1)=0, f'(x)>f(x)$ for all $x>1$, then $f(x)$ is

A. positive valued for all $x>1$,

B. negative valued for all $x>1$,

C. positive valued on (1,2) but negative valued on $[2,\infty)$

D. None of these.

26. Consider the constrained optimization problem

$$\max_{x\geq 0, y\geq 0}(ax+by) \text{ subject to } (cx+dy)\leq 100$$

where a,b,c,d are positive real numbers such that $\frac{d}{b}>\frac{(c+d)}{(a+b)}$. The unique solution (x^*,y^*) to this constrained optimization problem is

A. $\left(x^*=\frac{100}{a}, y^*=0\right)$

B. $\left(x^*=\frac{100}{c}, y^*=0\right)$

C. $\left(x^*=0, y^*=\frac{100}{b}\right)$,

D. $\left(x^*=0, y^*=\frac{100}{d}\right)$.

27. For any real number x, let $[x]$ be the largest integer not exceeding x. The domain of definition of the function $f(x)=(\sqrt{|[|x|-2]|-3})^{-1}$ is

A. [-6,6]

B. $(-\infty, -6) \cup (+6, \infty)$

C. $(-\infty, -6] \cup [+6, \infty)$

D. None of these.

28. Let $f : \Re \to \Re$ and $g : \Re \to \Re$ be defined as

$$f(x) = \begin{cases} -1 \text{ if } x < -\dfrac{1}{2} \\ -\dfrac{1}{2} \text{ if } -\dfrac{1}{2} \leq x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$$

and $g(x) = 1 + x - [x]$, where $[x]$ is the largest integer not exceeding x. Then $f(g(x))$ equals

A. -1

B. $-\frac{1}{2}$,

C. 0 ,

D. 1 .

29. If f is a real valued function and $a_1 f(x) + a_2 f(-x) = b_1 - b_2 x$ for all x with $a_1 \neq a_2$ and $b_2 \neq 0$. Then $f\left(\frac{b_1}{b_2}\right)$ equals

A. 0,

B. $-\left(\frac{2a_2 b_1}{a_1^2 - a_2^2}\right)$

C. $\frac{2a_2 b_1}{a_1^2 - a_2^2}$

D. More information is required to find the exact value of $f\left(\frac{b_1}{b_2}\right)$.

30. For all $x, y \in (0, \infty)$, a function $f : (0, \infty) \to \Re$ satisfies the inequality

$$|f(x) - f(y)| \leq |x - y|^3$$

Then f is

A. an increasing function,

B. a decreasing function,

C. a constant function.

D. None of these.

6 ISI PEA 2011

1. The expression $\sqrt{13+3\sqrt{23/3}}+\sqrt{13-3\sqrt{23/3}}$ is
   A. A natural number,
   B. A rational number but not a natural number,
   C. An irrational number not exceeding 6 ,
   D. An irrational number exceeding 6 .
2. The domain of definition of the function $f(x)=\frac{\sqrt{(x+3)}}{(x^2+5x+4)}$ is
   A. $(-\infty,\infty)\backslash\{-1,-4\}$
   B. $(-0,\infty)\backslash\{-1,-4\}$
   C. $(-1,\infty)\backslash\{-4\}$
   D. None of these.
3. The value of
$$\log_4 2-\log_8 2+\log_{16} 2-\ldots\ldots$$
   A. $\log_e 2$,
   B. $1-\log_e 2$,
   C. $\log_e 2-1$,
   D. None of these.
4. The function $\max\{1,x,x^2\}$, where x is any real number, has
   A. Discontinuity at one point only,
   B. Discontinuity at two points only,
   C. Discontinuity at three points only,
   D. No point of discontinuity.
5. If $x,y,z>0$ are in HP, then $\frac{x-y}{y-z}$ equals
   A. $\frac{x}{y}$,
   B. $\frac{y}{z}$
   C. $\frac{x}{z}$
   D. None of these.
6. The function $f(x)=\frac{x}{1+|x|}$, where x is any real number is,
   A. Everywhere differentiable but the derivative has a point of discontinuity.
   B. Everywhere differentiable except at 0.
   C. Everywhere continuously differentiable.

D. Everywhere differentiable but the derivative has 2 points of discontinuity.

7. Let the function $f : R_{++} \to R_{++}$ be such that $f(1) = 3$ and $f'(1) = 9$, where R_{++} is the positive part of the real line. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

   A. 3 ,

   B. e^2,

   C. 2,

   D. e^3.

8. Let $f, g : [0,\infty) \to [0,\infty)$ be decreasing and increasing respectively. Define $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

   A. Nonpositive for $x \geq 1$, positive otherwise,

   B. Always negative,

   C. Always positive,

   D. Positive for $x \geq 1$, nonpositive otherwise.

9. A committee consisting of 3 men and 2 women is to be formed out of 6 men and 4 women. In how many ways this can be done if Mr. X and Mrs. Y are not to be included together?

   A. 120 ,

   B. 140

   C. 90

   D. 60

10. The number of continuous functions f satisfying $xf(y)+yf(x) = (x+y)f(x)f(y)$, where x and y are any real numbers, is

    A. 1 ,

    B. 2,

    C. 3

    D. None of these.

11. If the positive numbers $x_1, \ldots, x_n$ are in AP, then

$$\frac{1}{\sqrt{x_1+\sqrt{x_2}}} + \frac{1}{\sqrt{x_2}+\sqrt{x_3}} + \ldots\ldots + \frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}} \text{ equals}$$

    A. $\frac{n}{\sqrt{x_1+\sqrt{x_n}}}$,

    B. $\frac{1}{\sqrt{x_1}+\sqrt{x_n}}$,

    C. $\frac{2n}{\sqrt{x_1}+\sqrt{x_n}}$,

D. None of these.

12. If x, y, z are any real numbers, then which of the following is always true?

    A. $\max\{x, y\} < \max\{x, y, z\}$

    B. $\max\{x, y\} > \max\{x, y, z\}$

    C. $\max\{x, y\} = \frac{x+y+|x-y|}{2}$

    D. None of these.

13. If $x_1, x_2, x_3, x_4 > 0$ and $\sum_{i=1}^{4} x_i = 2$, then $P = (x_1 + x_2)(x_3 + x_4)$ is

    A. Bounded between zero and one,

    B. Bounded between one and two,

    C. Bounded between two and three,

    D. Bounded between three and four.

14. Everybody in a room shakes hand with everybody else. Total number of handshakes is 91. Then the number of persons in the room is

    A. 11

    B. 12

    C. 13 ,

    D. 14

15. The number of ways in which 6 pencils can be distributed between two boys such that each boy gets at least one pencil is

    A. 30 ,

    B. 60

    C. 62

    D. 64

16. Number of continuous functions characterized by the equation $xf(x) + 2f(-x) = -1$, where x is any real number, is

    A. 1,

    B. 2

    C. 3,

    D. None of these

17. The value of the function $f(x) = x + \int_0 (xy^2 + x^2y) f(y)dy$ is $px + qx^2$, where

    A. $p = 80, q = 180$

    B. $p = 40, q = 140$

    C. $\mathrm{p} = 50, \mathrm{q} = 150$

D. None of these.

18. If x and y are real numbers such that $x^2 + y^2 = 1$, then the maximum value of $|x| + |y|$ is

A. $\frac{1}{2}$,

B. $\sqrt{2}$

C. $\frac{1}{\sqrt{2}}$

D. 2 .

19. The number of onto functions from $A = \{p, q, r, s\}$ to $B = \{p, r\}$ is

A. 16

B. 2,

C. 8,

D. 14

20. If the coefficients of $(2r + 5)$ th and $(r - 6)$ th terms in the expansion of $(1 + x)^{39}$ are equal, then ${}^{r}C_{12}$ equals

A. 45

B. 91,

C. 63,

D. None of these.

21. If $X = \begin{bmatrix} C & 2 \\ 1 & C \end{bmatrix}$ and $|X^7| = 128$, then the value of C is

A. ± 5,

B. ± 1

C. ± 2

D. None of these.

22. Let $f(x) = Ax^2 + Bx + C$, where A, B, C re real numbers. If $f(x)$ is an integer whenever x is an integer, then

A. $2A$ and $A + B$ are integers, but C is not an integer.

B. $A + B$ and C are integers, but $2A$ is not an integer.

C. $2A, A + B$ and C are all integers.

D. None of these.

23. Four persons board a lift on the ground floor of a seven-storey building. The number of ways in which they leave the lift, such that each of them gets down at different floors, is

A. 360 ,

B. 60

C. 120,

D. 240 .

24. The number of vectors (x, x_1, x_2) , where $x, x_1, x_2 > 0$, for which

$$|\log(xx_1)| + |\log(xx_2)| + \left|\log\left(\frac{x}{x_1}\right)\right| + \left|\log\left(\frac{x}{x_2}\right)\right|$$

$= |\log x_1 + \log x_2|$ holds, is

A. One,

B. Two,

C. Three,

D. None of these.

25. In a sample of households actually invaded by small pox, 70% of the inhabitants are attacked and 85% had been vaccinated. The minimum percentage of households (among those vaccinated) that must have been attacked [Numbers expressed as nearest integer value] is

A. 55 ,

B. 65

C. 30

D. 15

26. In an analysis of bivariate data (X and Y) the following results were obtained. Variance of $X\ (\sigma_x^2) = 9$, product of the regression coefficient of Y on X and X on Y is 0.36, and the regression coefficient from the regression of Y on $X\ (\beta_{yx})$ is 0.8 The variance of Y is

A. 16

B. 4

C. 1.69

D. 3

27. For comparing the wear and tear quality of two brands of automobile tyres, two samples of 50 customers using two types of tyres under similar conditions were selected. The number of kilometers x_1 and x_2 until the tyres became worn out, was obtained from each of them for the tyres used by them. The sample results were as follows: $\bar{x}_1 = 13,200$ km, $\bar{x}_2 = 13,650$ km $S_{x1} = 300$ km, $S_{x2} = 400$ km. What would you conclude about the two brands of tyres (at 5% level of significance) as far as the wear and tear quality is concerned?

A. The two brands are alike

B. The two brands are not the same,

C. Nothing can be concluded,

D. The given data are inadequate to perform a test.

28. A continuous random variable x has the following probability density function: $f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1}$ for $x > x_0, \alpha > 1$ The distribution function and the mean of x are given respectively by

A. $1-\left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha-1}{\alpha}x_0$

B. $1-\left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha-1}{\alpha}x_0$

C. $1-\left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha x_0}{\alpha-1}$

D. $1-\left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha x_0}{\alpha-1}$

29. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\begin{pmatrix} n \\ 0 \end{pmatrix}, \begin{pmatrix} n \\ 1 \end{pmatrix}, \ldots, \begin{pmatrix} n \\ n \end{pmatrix}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

A. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$

B. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$

C. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$

D. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

30. Let $\{X_i\}$ be a sequence of $i.i.d$ random variables such that $X_i = 1$ with probability p $= 0$ with probability $1-p$ Define $y = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} X_i = 100 \\ 0 \text{ otherwise} \end{cases}$ Then $E\left(y^2\right)$ is

A. ∞,

B. $\begin{pmatrix} n \\ 100 \end{pmatrix} p^{100}(1-p)^{n-100}$,

C. np,

D. $(np)^2$.

7 ISI PEA 2012

1. Kupamonduk, the frog, lives in a well 14 feet deep. One fine morning she has an urge to see the world, and starts to climb out of her well. Every day she climbs up by 5 feet when there is light, but slides back by 3 feet in the dark. How many days will she take to climb out of the well?

   A. 3 ,

   B. 8 ,

   C. 6

   D. None of the above.

2. The derivative of $f(x) = |x|^2$ at $x = 0$ is,

   A. -1

   B. Non-existent,

   C. 0 ,

   D. 1/2

3. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathcal{N}$ define $A_n = \{(n+1)k : k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals

   A. A_3

   B. A_4

   C. A_5

   D. A_6.

4. Let $S = \{a, b, c\}$ be a set such that a, b and c are distinct real numbers. Then $\min\{\max\{a, b\}, \max\{b, c\}, \max\{c, a\}\}$ is always

   A. the highest number in S,

   B. the second highest number in S,

   C. the lowest number in S,

   D. the arithmetic mean of the three numbers in S.

5. The sequence $< -4^{-n} >, n = 1, 2, \cdots$, is

   A. Unbounded and monotone increasing,

   B. Unbounded and monotone decreasing,

   C. Bounded and convergent,

   D. Bounded but not convergent.

6. $\int \frac{x}{7x^2+2} dx$ equals

   A. $\frac{1}{14} \ln(7x^2 + 2) +$ constant

B. $7x^2 + 2$

C. $\ln x+$ constant,

D. None of the above.

7. The number of real roots of the equation

$$2(x-1)^2 = (x-3)^2 + (x+1)^2 - 8$$

is

A. Zero,

B. One,

C. Two,

D. None of the above.

8. The three vectors [0,1],[1,0] and [1000,1000] are

A. Dependent,

B. Independent,

C. Pairwise orthogonal,

D. None of the above.

9. The function $f(.)$ is increasing over $[a,b]$. Then $[f(.)]^n$, where n is an odd integer greater than 1, is necessarily

A. Increasing over $[a,b]$

B. Decreasing over $[a,b]$

C. Increasing over $[a,b]$ if and only if $f(.)$ is positive over $[a,b]$

D. None of the above.

10. The determinant of the matrix $\left|\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right\|$ is

A. 21

B. -16

C. 0 ,

D. 14 .

11. In what ratio should a given line be divided into two parts, so that the area of the rectangle formed by the two parts as the sides is the maximum possible?

A. 1 is to 1 ,

B. 1 is to 4 ,

C. 3 is to 2 ,

D. None of the above.

12. Suppose (x^*, y^*) solves:

$$\text{Minimize } ax + by$$

subject to

$$x^\alpha + y^\alpha = M$$

and $x, y \geq 0$, where $a > b > 0, M > 0$ and $\alpha > 1$. Then, the solution is

A. $\frac{x*^{\alpha-1}}{y*^{\alpha-1}} = \frac{a}{b}$

B. $x^* = 0, y^* = M^{\frac{1}{\alpha}}$

C. $y^* = 0, x^* = M^{\frac{1}{\alpha}}$

D. None of the above.

13. Three boys and two girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is

A. $4_{P_2} \times 3!$

B. $3_{P_2} \times 2!$

C. $2! \times 3!$

D. None of the above.

14. The domain of x for which $\sqrt{x} + \sqrt{3-x} + \sqrt{x^2 - 4x}$ is real is,

A. [0,3]

B. (0,3)

C. $\{0\}$,

D. None of the above.

15. P(x) is a quadratic polynomial such that $P(1) = P(-1)$. Then

A. The two roots sum to zero,

B. The two roots sum to 1 ,

C. One root is twice the other,

D. None of the above.

16. The expression $\sqrt{11 + 6\sqrt{2}} + \sqrt{11 - 6\sqrt{2}}$ is

A. Positive and an even integer,

B. Positive and an odd integer,

C. Positive and irrational,

D. None of the above.

17. What is the maximum value of $a(1-a)b(1-b)c(1-c)$, where a, b, c vary over all positive fractional values?

A. 1

B. $\frac{1}{8}$

C. $\frac{1}{27}$

D. $\frac{1}{64}$.

18. There are four modes of transportation in Delhi: (A) Auto-rickshaw, (B) Bus, (C) Car, and (D) Delhi-Metro. The probability of using transports A, B, C, D by an individual is $\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{2}{9}$ respectively. The probability that he arrives late at work if he uses transportation A, B, C, D is $\frac{5}{7}, \frac{4}{7}, \frac{6}{7}$, and $\frac{6}{7}$ respectively. What is the probability that he used transport A if he reached office on time?

A. $\frac{1}{9}$,

B. $\frac{1}{7}$

C. $\frac{3}{7}$

D. $\frac{2}{9}$

19. What is the least (strictly) positive value of the expression $a^3 + b^3 + c^3 - 3abc$, where a, b, c vary over all strictly positive integers? (You may use the identity $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)\left((a - b)^2 + (b - c)^2 + (c - a)^2\right).$)

A. 2 ,

B. 3 ,

C. 4

D. 8 .

20. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is,

A. -0.75 ,

B. Belongs to the interval [-1,-0.5] ,

C. Belongs to the interval [0.5,1] ,

D. None of the above.

21. Consider the following linear programming problem: Maximize $a+b$ subject to $a+2b \leq 4$
$a + 6b \leq 6$

$a - 2b \leq 2$ $a, b \geq 0$ An optimal solution is:

A. $a = 4, b = 0$

B. $a = 0, b = 1$

C. $a = 3, b = 1/2$

D. None of the above.

22. The value of $\int_{-4}^{-1} \frac{1}{x} dx$ equals,

A. $\ln 4$

B. Undefined,

C. $\ln(-4) - \ln(-1)$

D. None of the above.

23. Given $x \geq y \geq z$, and $x + y + z = 9$, the maximum value of $x + 3y + 5z$ is

A. 27

B. 42 ,

C. 21 ,

D. 18 .

24. A car with six sparkplugs is known to have two malfunctioning ones. If two plugs are pulled out at random, what is the probability of getting at least one malfunctioning plug.

A. 1/15

B. 7/15

C. 8/15

D. 9/15.

25. Suppose there is a multiple choice test which has 20 questions. Each question has two possible responses - true or false. Moreover, only one of them is correct. Suppose a student answers each of them randomly.Which one of the following statements is correct?

A. The probability of getting 15 correct answers is less than the probability of getting 5 correct answers,

B. The probability of getting 15 correct answers is more than the probability of getting 5 correct answers,

C. The probability of getting 15 correct answers is equal to the probability of getting 5 correct answers,

D. The answer depends on such things as the order of the questions.

26. From a group of 6 men and 5 women, how many different committees consisting of three men and two women can be formed when it is known that 2 of the men do not want to be on the committee together?

A. 160

B. 80 ,

C. 120

D. 200 .

27. Consider any two consecutive integers a and b that are both greater than 1. The sum $(a^2 + b^2)$ is

A. Always even,

B. Always a prime number,

C. Never a prime number,

D. None of the above statements is correct.

28. The number of real non-negative roots of the equation

$$x^2 - 3|x| - 10 = 0$$

is,

A. 2 ,

B. 1

C. 0 ,

D. 3

29. Let $< a^n >$ and $< b^n >, n = 1, 2, \cdots$, be two different sequences, where $< a^n >$ is convergent and $< b^n >$ is divergent. Then the sequence $< a^n + b^n >$ is

A. Convergent,

B. Divergent,

C. Undefined,

D. None of the above.

30. Consider the function

$$f(x) = \frac{|x|}{1 + |x|}$$

This function is,

A. Increasing in x when $x \geq 0$,

B. Decreasing in x,

C. Increasing in x for all real x,

D. None of the above.

8 ISI PEA 2013

1. Let $f(x)=\frac{1-x}{1+x}, x \neq -1$. Then $f\left(f\left(\frac{1}{x}\right)\right), x \neq 0$ and $x \neq -1$, is

   A. 1 ,

   B. x,

   C. x^2,

   D. $\frac{1}{x}$.

2. The limiting value of $\frac{1.2+2.3+\ldots+n(n+1)}{n^3}$ as $n \rightarrow \infty$ is,

   A. 0 ,

   B. 1

   C. 1/3

   D. 1/2

3. Suppose $a_1, a_2, \ldots, a_n$ are n positive real numbers with $a_1 a_2 \ldots a_n=1$ Then the minimum value of $(1+a_1)(1+a_2)\ldots(1+a_n)$ is

   A. 2^n,

   B. 2^{2n},

   C. 1

   D. None of the above.

4. Let the random variable X follow a Binomial distribution with parame- ters n and p where $n(>1)$ is an integer and $0<p<1$. Suppose further that the probability of $X=0$ is the same as the probability of $X=1$ Then the value of p is

   A. $\frac{1}{n}$,

   B. $\frac{1}{n+1}$

   C. $\frac{n}{n+1}$

   D. $\frac{n-1}{n+1}$.

5. Let X be a random variable such that $E\left(X^2\right)=E(X)=1$. Then $E\left(X^{100}\right)$ is

   A. 1 ,

   B. 2^{100},

   C. 0,

   D. None of the above.

6. If α and β are the roots of the equation $x^2-ax+b=0$, then the quadratic equation whose roots are $\alpha+\beta+\alpha\beta$ and $\alpha\beta-\alpha-\beta$ is

   A. $x^2-2ax+a^2-b^2=0$

   B. $x^2-2ax-a^2+b^2=0$

C. $x^2 - 2bx - a^2 + b^2 = 0$

D. $x^2 - 2bx + a^2 - b^2 = 0$

7. Suppose $f(x) = 2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1$ where x is real and $x \neq 0$. Then the solutions of $f(x) = 0$ are such that their product is

   A. 1 ,

   B. 2

   C. -1

   D. -2 .

8. Toss a fair coin 43 times. What is the number of cases where number of 'Head'¿ number of 'Tail'?

   A. 2^{43},

   B. $2^{43} - 43$

   C. 2^{42}

   D. None of the above.

9. The minimum number of real roots of $f(x) = |x|^3 + a|x|^2 + b|x| + c$ where a, b and c are real, is

   A. 0 ,

   B. 2

   C. 3 ,

   D. 6.

10. Suppose $f(x, y)$ where x and y are real, is a differentiable function satisfying the following properties:

    (i) $f(x+k, y) = f(x, y) + ky$ (ii) $f(x, y+k) = f(x, y) + kx$; and (iii) $f(x, 0) = m$, where m is a constant. Then $f(x, y)$ is given by

    A. $m + xy$

    B. $m + x + y$

    C. mxy

    D. None of the above.

11. Let $I = \int_2^{343} \{x - [x]\}^2 dx$ where $[x]$ denotes the largest integer less than or equal to x. Then the value of I is

    A. $\frac{343}{3}$,

    B. $\frac{343}{2}$,

    C. $\frac{341}{3}$,

    D. None of the above.

12. The coefficients of three consecutive terms in the expression of $(1+x)^n$ are 165,330 and 462. Then the value of n is

    A. 10

    B. 11 ,

    C. 12 ,

    D. 13 .

13. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in

    A. $\left[\frac{1}{2},1\right]$

    B. [-1,1]

    C. $\left[-\frac{1}{2},\frac{1}{2}\right]$

    D. $\left[-\frac{1}{2},1\right]$

14. Let the function $f(x)$ be defined as $f(x)=|x-4|+|x-5|$. Then which of the following statements is true?

    A. $f(x)$ is differentiable at all points,

    B. $f(x)$ is differentiable at $x=4$, but not at $x=5$

    C. $f(x)$ is differentiable at $x=5$ but not at $x=4$,

    D. None of the above.

15. The value of the integral $\int_0^1\int_0^x x^2e^{xy}dxdy$ is

    A. e

    B. $\frac{e}{2}$,

    C. $\frac{1}{2}(e-1)$

    D. $\frac{1}{2}(e-2)$

16. Let $\mathcal{N}=\{1,2,\ldots\}$ be a set of natural numbers. For each $x\in\mathcal{N}$, define $A_n=\{(n+1)k, k\in\mathcal{N}\}$. Then $A_1\cap A_2$ equals

    A. A_2

    B. A_4

    C. A_5

    D. A_6

17. $\lim_{x\to 0}\left\{\frac{1}{x}\left(\sqrt{1+x+x^2}-1\right)\right\}$ is

    A. 0 ,

    B. 1 ,

    C. $\frac{1}{2}$,

    D. Non-existent.

18. The value of $\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \ldots + (n+1)\binom{n}{n}$ equals

   A. $2^n + n2^{n-1}$

   B. $2^n - n2^{n-1}$

   C. 2^n

   D. 2^{n+2}.

19. The rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is

   A. 1

   B. 2 ,

   C. 3 ,

   D. 4

20. Suppose an odd positive integer $2n+1$ is written as a sum of two integers so that their product is maximum. Then the integers are

   A. $2n$ and 1 ,

   B. $n+2$ and $n-1$

   C. $2n-1$ and 2

   D. None of the above.

21. If $|a| < 1, |b| < 1$, then the series $a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \ldots\ldots$ converges to

   A. $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$

   B. $\frac{a(a+b)}{1-a(a+b)}$

   C. $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$,

   D. $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

22. Suppose $f(x) = x^3 - 6x^2 + 24x$. Then which of the following statements is true?

   A. $f(x)$ has a maxima but no minima,

   B. $f(x)$ has a minima but no maxima,

   C. $f(x)$ has a maxima and a minima,

   D. $f(x)$ has neither a maxima nor a minima.

23. An urn contains 5 red balls, 4 black balls and 2 white balls. A player draws 2 balls one after another with replacement. Then the probability of getting at least one red ball or at least one white ball is

A. $\frac{105}{121}$

B. $\frac{67}{121}$

C. $\frac{20}{121}$

D. None of the above.

24. If $\log_t x = \frac{1}{t-1}$ and $\log_t y = \frac{t}{t-1}$, where $\log_t x$ stand for logarithm of x to the base t. Then the relation between x and y is

A. $y^x = x^{1/y}$

B. $x^{1/y} = y^{1/x}$

C. $x^y = y^x$,

D. $x^y = y^{1/x}$

25. Suppose $\frac{f''(x)}{f'(x)} = 1$ for all x. Also, $f(0) = e^2$ and $f(1) = e^3$. Then $\int_{-2}^{2} f(x)dx$ equals

A. $2e^2$

B. $e^2 - e^{-2}$

C. $e^4 - 1$

D. None of the above.

26. The minimum value of the objective function $z = 5x + 7y$, where $x \geq 0$ and $y \geq 0$, subject to the constraints

$2x + 3y \geq 6, \quad 3x - y \leq 15, \quad -x + y \leq 4$, and $2x + 5y \leq 27$ is

A. 14 ,

B. 15 ,

C. 25 ,

D. 28 .

27. Suppose A is a 2×2 matrix given as $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$ Then the matrix $A^2 - 3A - 13I$, where I is the 2×2 identity matrix, equals

A. I

B. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 5 \\ 3 & 0 \end{pmatrix}$

C. None of the above.

28. The number of permutations of the letters a, b, c, and d such that b does not follow a, c does not follow b, and d does not follow c is

A. 14 ,

B. 13 ,

C. 12 ,

D. 11

29. Given n observations $x_1, x_2, \ldots, x_n$, which of the following statements is true?

    A. The mean deviation about arithmetic mean can exceed the stan- dard deviation,

    B. The mean deviation about arithmetic mean cannot exceed the stan- dard deviation,

    C. The root mean square deviation about a point A is least when A is the median,

    D. The mean deviation about a point A is minimum when A is the arithmetic mean.

30. Consider the following classical linear regression of y on x,

$$y_i = \beta x_i + u_i, i = 1, 2, \ldots, \ldots, n$$

where $E(u_i) = 0, V(u_i) = \sigma^2$ for all i, and $u_i's$ are homoscedastic and non-autocorrelated. Now, let $\hat{u}_i$ be the ordinary least square estimate of u_i. Then which of the following statements is true?

    A. $\sum_{i=1}^{n} \hat{u}_i = 0$

    B. $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i = 0$

    C. $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i \neq 0$

    D. $\sum_{i=1}^{n} x_i \hat{u}_i = 0$

9 ISI PEA 2014

1. Let $f(x)=\frac{1-x}{1+x}, x \neq -1$. Then $f\left(f\left(\frac{1}{x}\right)\right), x \neq 0$ and $x \neq -1$, is

    A. 1

    B. x,

    C. x^2,

    D. $\frac{1}{x}$.

2. What is the value of the following definite integral?

$$2 \int_0^{\frac{\pi}{2}} e^x \cos(x) dx$$

    A. $e^{\frac{\pi}{2}}$

    B. $e^{\frac{x}{2}}-1$

    C. 0 .

    D. $e^{\frac{\pi}{2}}+1$

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as follows:

$$f(x)=|x-1|+(x-1)$$

Which of the following is not true for f?

    A. $f(x)=f\left(x^{\prime}\right)$ for all $x, x^{\prime}<1$

    B. $f(x)=2 f(1)$ for all $x>1$

    C. f is not differentiable at 1 .

    D. The derivative of f at $x=2$ is 2 .

4. Population of a city is 40% male and 60% female. Suppose also that 50% of males and 30% of females in the city smoke. The probability that a smoker in the city is male is closest to

    A. 0.5 .

    B. 0.46

    C. 0.53 .

    D. 0.7 .

5. A blue and a red die are thrown simultaneously. We define three events as follows: - Event E : the sum of the numbers on the two dice is 7 . - Event F : the number on the blue die equals 4 . - Event G: the number on the red die equals 3 . Which of the following statements is true?

    A. E and F are disjoint events.

B. E and F are independent events.

C. E and F are not independent events.

D. Probability of E is more than the probability of F.

6. Let $p > 2$ be a prime number. Consider the following set containing 2×2 matrices of integers:
$$T_p = \left\{ A = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \{0, 1, \ldots, p-1\} \right\}$$
A matrix $A \in T_p$ is p -special if determinant of A is not divisible by p. How many matrices in T_p are p -special?

A. $(p-1)^2$

B. $2p-1$

C. p^2.

D. $p^2 - p + 1$

7. A "good" word is any seven letter word consisting of letters from $\{A, B, C\}$ (some letters may be absent and some letter can be present more than once), with the restriction that A cannot be followed by B, B cannot be followed by C, and C cannot be followed by A. How many good words are there?

A. 192 .

B. 128 .

C. 96 .

D. 64 .

8. Let n be a positive integer and $0 < a < b < \infty$. The total number of real roots of the equation $(x-a)^{2n+1} + (x-b)^{2n+1} = 0$ is

A. 1 .

B. 3

C. $2n-1$

D. $2n+1$

9. Consider the optimization problem below:
$$\begin{aligned} \max\nolimits_{x,y} \; & x + y \\ \text{subject to } & 2x + y \leq 14 \\ & -x + 2y \leq 8 \\ & 2x - y \leq 10 \\ & x, y \geq 0 \end{aligned}$$
The value of the objective function at optimal solution of this optimization problem:

A. does not exist.

B. is 8 .

C. is 10 .

D. is unbounded.

10. A random variable X is distributed in $[0,1]$. Mr. Fox believes that X follows a distribution with cumulative density function (cdf) $F : [0,1] \rightarrow [0,1]$ and Mr. Goat believes that X follows a distribution with cdf $G : [0,1] \rightarrow [0,1]$. Assume F and G are differentiable, $F \neq G$ and $F(x) \leq G(x)$ for all $x \in [0,1]$. Let $\mathbb{E}_F[X]$ and $\mathbb{E}_G[X]$ be the expected values of X for Mr. Fox and Mr. Goat respectively. Which of the following is true?

    A. $\mathbb{E}_F[X] \leq \mathbb{E}_G[X]$

    B. $\mathbb{E}_F[X] \geq \mathbb{E}_G[X]$

    C. $\mathbb{E}_F[X] = \mathbb{E}_G[X]$

    D. None of the above.

11. Let $f : [0,2] \rightarrow [0,1]$ be a function defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \leq \alpha \\ \frac{1}{2} & \text{if } x \in (\alpha, 2] \end{cases}$$

    where $\alpha \in (0,2)$. Suppose X is a random variable distributed in [0,2] with probability density function f. What is the probability that the realized value of X is greater than 1?

    A. 1 .

    B. 0.

    C. $\frac{1}{2}$.

    D. $\frac{3}{4}$.

12. The value of the expression

$$\sum_{k=1}^{100} \int_0^1 \frac{x^k}{k} dx$$

    is

    A. $\frac{100}{101}$

    B. $\frac{1}{99}$.

    C. 1

    D. $\frac{99}{100}$

13. Consider the following system of inequalities.

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_2 - x_3 &\leq -2 \\ x_3 - x_4 &\leq 10 \\ x_4 - x_2 &\leq \alpha \\ x_4 - x_3 &\leq -4 \end{aligned}$$

where α is a real number. A value of α for which this system has a solution is

A. -16

B. -12

C. -10

D. None of the above.

14. A fair coin is tossed infinite number of times. The probability that a head turns up for the first time after even number of tosses is

A. $\frac{1}{3}$.

B. $\frac{1}{2}$.

C. $\frac{2}{3}$.

D. $\frac{3}{4}$.

15. An entrance examination has 10 "true-false" questions. A student answers all the questions randomly and his probability of choosing the correct answer is 0.5. Each correct answer fetches a score of 1 to the student, while each incorrect answer fetches a score of zero. What is the probability that the student gets the mean score?

A. $\frac{1}{4}$.

B. $\frac{63}{256}$.

C. $\frac{1}{2}$.

D. $\frac{1}{8}$.

16. For any positive integer k, let S_k denote the sum of the infinite geometric progression whose first term is $\frac{(k-1)}{k!}$ and common ratio is $\frac{1}{k}$. The value of the expression $\sum_{k=1}^{\infty} S_k$ is

A. e.

B. $1+e$

C. $2+e$

D. e^2.

17. Let $G(x)=\int_0^x te^t dt$ for all non-negative real number x. What is the value of $\lim_{x\to 0}\frac{1}{x}G'(x)$, where $G'(x)$ is the derivative of G at x

A. 0.

B. 1

C. e.

D. None of the above.

18. Let $\alpha \in (0,1)$ and $f(x)=x^\alpha+(1-x)^\alpha$ for all $x\in[0,1]$. Then the maximum value of f is

A. 1

B. greater than 2 .

C. in (1,2)

D. 2 .

19. Let n be a positive integer. What is the value of the expression

$$\sum_{k=1}^{n} kC(n,k)$$

where $C(n,k)$ denotes the number of ways to choose k out of n objects?

A. $n2^{n-1}$.

B. $n2^{n-2}$.

C. 2^{n}.

D. $n2^{n}$.

20. The first term of an arithmetic progression is a and common difference is $d \in (0,1)$. Suppose the k -th term of this arithmetic progression equals the sum of the infinite geometric progression whose first term is a and common ratio is d. If $a > 2$ is a prime number, then which of the following is a possible value of d?

A. $\frac{1}{2}$.

B. $\frac{1}{3}$.

C. $\frac{1}{5}$.

D. $\frac{1}{9}$.

21. In period 1, a chicken gives birth to 2 chickens (so, there are three chickens after period 1). In period 2, each chicken born in period 1 either gives birth to 2 chickens or does not give birth to any chicken. If a chicken does not give birth to any chicken in a period, it does not give birth in any other subsequent periods. Continuing in this manner, in period $(k+1)$, a chicken born in period k either gives birth to 2 chickens or does not give birth to any chicken. This process is repeated for T periods - assume no chicken dies. After T periods, there are in total 31 chickens. The maximum and the minimum possible values of T are respectively

A. 12 and 4 .

B. 15 and 4

C. 15 and 5 .

D. 12 and 5 .

22. Let a and p be positive integers. Consider the following matrix

$$A = \begin{bmatrix} p & 1 & 1 \\ 0 & p & a \\ 0 & a & 2 \end{bmatrix}$$

If determinant of A is 0, then a possible value of p is

A. 1

B. 2 .

C. 4

D. None of the above.

23. For what value of α does the equation $(x-1)(x^2-7x+\alpha)=0$ have exactly two unique roots?

A. 6 .

B. 10 .

C. 12 .

D. None of the above.

24. What is the value of the following infinite series?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \log_e 3^k$$

A. $\log_e 2$

B. $\log_e 2 \log_e 3$

C. $\log_e 6$

D. $\log_e 5$.

25. There are 20 persons at a party. Each person shakes hands with some of the persons at the party. Let K be the number of persons who shook hands with odd number of persons. What is a possible value of K?

A. 19.

B. 1.

C. 20 .

D. All of the above.

26. Two independent random variables X and Y are uniformly distributed in the interval $[0,1]$. For a $z \in [0,1]$, we are told that probability that $\max(X,Y) \leq z$ is equal to the probability that $\min(X,Y) \leq (1-z)$ What is the value of z?

A. $\frac{1}{2}$.

B. $\frac{1}{\sqrt{2}}$

C. any value in $\left[\frac{1}{2}, 1\right]$

D. None of the above.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies for all $x, y \in \mathbb{R}$

$$f(x+y)f(x-y) = (f(x)+f(y))^2 - 4x^2 f(y)$$

Which of the following is not possible for f?

A. $f(0) = 0$

B. $f(3) = 9$

C. $f(5) = 0$

D. $f(2) = 2$

28. Consider the following function $f : \mathbb{R} \to \mathbb{Z}$, where $\mathbb{R}$ is the set of all real numbers and $\mathbb{Z}$ is the set of all integers.

$$f(x) = \lceil x \rceil$$

where $\lceil x \rceil$ is the smallest integer that is larger than x. Now, define a new function g as follows. For any $x \in \mathbb{R}, g(x) = |f(x)| - f(|x|)$, where $|x|$ gives the absolute value of x. What is the range of g?

A. {0,1}

B. [-1,1]

C. {-1,0,1}

D. {-1,0}

29. The value of $\lim_{x \to -1} \frac{x+1}{|x+1|}$ is.

A. 1.

B. -1

C. 0.

D. None of the above.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = 2$ if $x \leq 2$ and $f(x) = a^2 - 3a$ if $x > 2$, where a is a positive integer. Which of the following is true?

A. f is continuous everywhere for some value of a.

B. f is not continuous.

C. f is differentiable at $x = 2$.

D. None of the above.

10 ISI PEA 2015

1. $\lim_{x\to 0+}\frac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimalpart of x, is

    A. 0

    B. 1

    C. non-existent

    D. none of these

2. $f : [0,1] \to [0,1]$ is continuous.Then it is true that

    A. $f(0) = 0, f(1) = 1$

    B. f is differentiable only at $x = \frac{1}{2}$

    C. $f'(x)$ is constant for all $x \in (0,1)$

    D. $f(x) = x$ for at least one $x \in [0,1]$

3. $f(x) = |x-2| + |x-4|$. Then f is

    A. continuously differentiable at x = 2

    B. differentiable but not continuously differentiable at $x = 2$

    C. f has both left and right derivatives at $x = 2$

    D. none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry.Then the number of students who passed in exactly one subject is

    A. 30

    B. 25

    C. 10

    D. none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if

    A. $a = b = c = 0$

    B. $a = c = 0, b$ is any non-zero real number

    C. $a = b = 0, c$ is any non-zero real number

    D. $a = 0$ and either $b = 0$ or $c = 0$

6. If the positive numbers x, y, z are in harmonic progression, then $\log(x+z)+\log(x-2y+\text{z})$ equals

    A. $4\log(x-z)$

B. $3\log(x-z)$

C. $2\log(x-z)$

D. $\log(x-z)$

7. If $f(x+2y, x-2y) = xy$, then $f(x,y)$ equals

A. $\frac{x^2-y^2}{8}$

B. $\frac{x^2-y^2}{4}$

C. $\frac{x^2+y^2}{4}$

D. none of these

8. The domain of the function $f(x) = \sqrt{x^2-1} - \log(\sqrt{1-x}), x \geq 0$,is

A. $(-\infty, -1)$

B. (-1,0)

C. null set

D. none of these

9. The graph of the function $y = \log\left(1-2x+x^2\right)$ intersects the x axis at

A. 0,2

B. 0,-2

C. 2

D. 0

10. The sum of two positive integers is 100 .The minimum value of the sum of their reciprocals is

A. $\frac{3}{25}$

B. $\frac{6}{25}$

C. $\frac{1}{25}$

D. none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

A. $\left(\frac{3}{4}, \infty\right)$

B. $\left[\frac{3}{4}, \infty\right)$

C. $(7, \infty)$

D. $[7, \infty)$

12. The function $f: R \to R$ satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$, where R is the real line, and $f(1) = 7$. Then $\sum_{r=1}^{n} f(r)$ equals

A. $\frac{7n}{2}$

B. $\frac{7(n+1)}{2}$

C. $\frac{7n(n+1)}{2}$

D. $7n(n+1)$

13. Let f and g be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$ Suppose that for all $x \in (0,1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

    A. 1

    B. 3

    C. -2

    D. -1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on [1,2] .Then it is always possible to get a $k \in (1,2)$ such that

    A. $k = 2a + b$

    B. $k = a + 2b$

    C. $k = 3a + b$

    D. none of these

15. In a sequence the first term is $\frac{1}{3}$. The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500$^{\text{th}}$ term is

    A. $\frac{1}{503}$

    B. $\frac{1}{501}$

    C. $\frac{1}{502}$

    D. none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

    A. 6

    B. 18

    C. 27

    D. 36

17. Let a be a positive integer greater than 2 .The number of values of x for which

$$\int_a^x (x+y)dy = 0 \text{ holds is}$$

    A. 1

    B. 2

C. a

D. $a+1$

18. Let (x^*, y^*) be a solution to any optimization problem $\max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \leq c_1$. Let (x', y) be a solution to the same optimization problem $\max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \leq c_1$ with an added constraint that $g_2(x,y) \leq c_2$. Then which one of the following statements is always true?

   A. $f(x^*, y^*) \geq f(x', y)$

   B. $f(x^*, y^*) \leq f(x', y)$

   C. $|f(x^*, y^*)| \geq |f(x', y)|$

   D. $|f(x^*, y^*)| \leq |f(x', y)|$

19. Let (x^*, y^*) be a real solution to: $\max_{(x,y)\in\Re^2} \sqrt[x]{x} + y$ subject to $px + y \leq m$, where $m > 0, p > 0$ and $y^* \in (0, m)$. Then which one of the following statements is true?

   A. x^* depends only on p

   B. x^* depends only on m

   C. x^* depends on both p and m

   D. x^* is independent of both p and m.

20. Let $0 < a_1 < a_2 < 1$ and let $f(x; a_1, a_2) = -|x - a_1| - |x - a_2|$.Let X be the set of all values of x for which $f(x; a_1, a_2)$ achieves its maximum. Then

   A. $X = \left\{x \mid x \in \left\{\frac{a_1}{2}, \frac{1+a_2}{2}\right\}\right\}$

   B. $X = \{x \mid x \in \{a_1, a_2\}\}$

   C. $X = \left\{x \mid x \in \left\{0, \frac{a_1+a_2}{2}, 1\right\}\right\}$

   D. $X = \{x \mid x \in [a_1, a_2]\}$

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:

   A. P(A/B) > P(A) always

   B. P(A/B) < P(A) always

   C. P(A/B) > P(B) always

   D. None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:

   A. A and B are necessarily independent

   B. A and B are necessarily dependent

   C. A and B are necessarily equally likely

   D. None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).

$$100, 100, 111, 114, 165, 210, 225, 225, 230,$$
$$575, 1200, 1900, 2100, 2100, 2650, 3300$$

Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:

A. Mean will increase by Rs. 200,000 but the median will not change

B. Both mean and median will be increased by Rs. 200,000

C. Mean and standard deviation will both be changed

D. Standard deviation will be increased but the median will be unchanged

24. Let Pr(X = 2) = 1. Define $\mu_{2n} = E(X - \mu)^{2n}, \mu = E(X)$. Then:

A. $\mu_{2n} = 2$

B. $\mu_{2n} = 0$

C. $\mu_{2n} > 0$

D. None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:

A. Mean is greater than median

B. Mean is smaller than median

C. Mean and median are same

D. None of the above

26. Puja and Priya play a fair game (i.e. winning probability is 1/2 for both players) repeatedly for one rupee per game. If originally Puja has a rupees and Priya has b rupees (where $a > b$), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

A. 1

B. 0

C. b/(a + b)

D. $a/(a + b)$

27. An urn contains w white balls and b black balls ($w > 0$) and ($b > 0$).The balls are thoroughly mixed and two are drawn, one after the other, without replacement. Let Wi denote the outcome 'white on the i -th draw' for $i = 1, 2$. Which one of the following is true?

A. P(W2) = P(W1) = w/(w + b)

B. P(W2) = P(W1) = (w − 1)/(w + b − 1)

C. $\mathrm{P(W1) = w/(w+b), P(W2) = (w-1)/(w+b-1)}$

D. $\mathrm{P(W1) = w/(w+b), P(W2) = \{w(w-1)\}/\{(w-b)(w+b-1)\}}$

28. A bag contains four pieces of paper, each labeled with one of the digits $1, 2, 3, 4$, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

A. $\frac{3/4}{6}$

B. 1/2

C. 1/4

D. 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y = [XI. Which of the following is true?

A. The variables X and Yare independent and Pearson's correlation coefficient between X and Y is 0.

B. The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.

C. The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1

D. The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1

30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7: 20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

A. 5/20

B. 25/400

C. 10/20

D. 7/16

11 ISI PEA 2016

1. Consider the polynomial $P(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \{1, 2, \ldots, 9\}$. If $P(10) = 5861$, then the value of c is

   A. 1.

   B. 2

   C. 6

   D. 5 .

2. Let $A \subset \mathbb{R}, f : A \to \mathbb{R}$ be a twice continuously differentiable function, and $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$

   A. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a sufficient condition for x^* to be a point of local maximum of f on A

   B. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a necessary condition for x^* to be a point of local maximum of f on A

   C. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is necessary and sufficient for x^* to be a point of local maximum of f on A

   D. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is neither necessary nor sufficient for x^* to be a point of local maximum of f on A.

3. You are given five observations x_1, x_2, x_3, x_4, x_5 on a variable x, ordered from lowest to highest. Suppose x_5 is increased. Then,

   A. The mean, median, and variance, all increase.

   B. The median and the variance increase but the mean is unchanged.

   C. The variance increases but the mean and the median are unchanged.

   D. None of the above.

4. Suppose the sum of coefficients in the expansion $(x + y)^n$ is 4096. The largest coefficient in the expansion is:

   A. 924

   B. 1024

   C. 824

   D. 724

5. There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. I choose a card with equal probability, then a side of that card with equal probability. If the side I choose of the card is green, what is the probability that the other side is green?

   A. $\frac{1}{3}$.

   B. $\frac{1}{2}$.

C. $\frac{2}{3}$.

D. $\frac{3}{4}$.

6. The value of

$$\int_0^{\frac{\pi}{2}} x \sin x dx$$

is:

A. 0.

B. -1

C. $\frac{1}{2}$.

D. 1

7. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} ax + b & \text{if } x \geq 0 \\ \sin 2x & \text{if } x < 0 \end{cases}$$

For what values of a and b is f continuous but not differentiable?

A. $a = 2, b = 0$.

B. $a = 2, b = 1$

C. $a = 1, b = 1$

D. $a = 1, b = 0$

8. A student wished to regress household food consumption on household income. By mistake the student regressed household income on household food consumption and found R^2 to be 0.35. The R^2 in the correct regression of household food consumption on household income is

A. 0.65

B. 0.35

C. $1 - (.35)^2$.

D. None of the above.

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

Which of the following statements is true?

A. $(x = 1, y = 0)$ is a local maximum of f

B. $(x = 1, y = 0)$ is a local minimum of f.

C. $(x = 1, y = 0)$ is neither a local maximum nor a local minimum of f.

D. $(x = 0, y = 0)$ is a global maximum of f.

10. Let

$$f(x) = \frac{x+\sqrt{3}}{1-\sqrt{3}x}$$

for all $x \neq \frac{1}{\sqrt{3}}$. What is the value of $f(f(x))$?

A. $\frac{x-\sqrt{3}}{1+\sqrt{3}x}$.

B. $\frac{x^2+2\sqrt{3}x+3}{1-2\sqrt{3}x+3x}$

C. $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

D. $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

11. The continuous random variable X has probability density $f(x)$ where

$$f(x) = \begin{cases} a & \text{if } 0 \leq x < k \\ b & \text{if } k \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $a > b > 0$ and $0 < k < 1$. Then $E(X)$ is given by:

A. $\frac{b(1-a)^2}{2a(a-b)}$.

B. $\frac{1}{2}$.

C. $\frac{a-b}{(a+b)}$

D. $\frac{1-2b+ab}{2(a-b)}$.

12. The set of values of x for which $x^2 - 3|x| + 2 < 0$ is given by:

A. $\{x : x < -2\} \cup \{x : x > 1\}$

B. $\{x : -2 < x < -1\} \cup \{x : 1 < x < 2\}$

C. $\{x : x < -1\} \cup \{x : x > 2\}$

D. None of the above.

13. The system of linear equations

$$\begin{aligned} (4d-1)x + y + z &= 0 \\ -y + z &= 0 \\ (4d-1)z &= 0 \end{aligned}$$

has a non-zero solution if:

A. $d = \frac{1}{4}$.

B. $d = 0$.

C. $d \neq \frac{1}{4}$.

D. $d = 1$.

14. Suppose F is a cumulative distribution function of a random variable x distributed in [0,1] defined as follows:

$$F(x) = \begin{cases} ax + b & \text{if } x \geq a \\ x^2 - x + 1 & \text{otherwise} \end{cases}$$

where $a \in (0,1)$ and b is a real number. Which of the following is true?

A. F is continuous in (0,1)

B. F is differentiable in (0,1)

C. F is not continuous at $x = a$.

D. None of the above.

E. Incorrect option provided for this question.

15. The solution of the optimization problem

$$\max_{x,y} 3xy - y^3$$

subject to

$$\begin{aligned} &2x + 5y \geq 20 \\ &x - 2y = 5 \\ &x, y \geq 0 \end{aligned}$$

is given by:

A. $x = 19, y = 7$.

B. $x = 45, y = 20$

C. $x = 15, y = 5$

D. None of the above.

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Let g be the inverse of the function f. If $f'(1) = g(1) = 1$, then $g'(1)$ equals to

A. 0.

B. $\frac{1}{2}$.

C. -1 .

D. 1 .

17. Consider a quadratic polynomial $P(x)$. Suppose $P(1) = -3, P(-1) = -9, P(-2) = 0$. Then, which of the following is true.

A. $P\left(\frac{1}{2}\right) = 0$

B. $P\left(\frac{5}{2}\right) = 0$

C. $P\left(\frac{5}{4}\right) = 0$

D. $P\left(\frac{3}{4}\right) = 0$

18. For any positive integers k, ℓ with $k \geq \ell$, let $C(k, \ell)$ denote the number of ways in which ℓ distinct objects can be chosen from k objects. Consider $n \geq 3$ distinct points on a circle and join every pair of points by a line segment. If we pick three of these line segments uniformly at random, what is the probability that we choose a triangle?

    A. $\frac{C(n,2)}{C(C(n,2),3)}$

    B. $\frac{C(n,3)}{C(C(n,2),3)}$

    C. $\frac{2}{n-1}$

    D. $\frac{C(n,3)}{C(C(n,2),2)}$.

19. Let $X = \left\{(x, y) \in \mathbb{R}^2 : x + y \leq 1, 2x + \frac{y}{2} \leq 1, x \geq 0, y \geq 0\right\}$. Consider the optimization problem of maximizing a function $f(x) = ax + by$, where a, b are real numbers, subject to the constraint that $(x, y) \in X$. Which of the following is not an optimal value of f for any value of a and b?

    A. $x = 0, y = 1$

    B. $x = \frac{1}{3}, y = \frac{2}{3}$

    C. $x = \frac{1}{4}, y = \frac{1}{4}$

    D. $x = \frac{1}{2}, y = 0$

20. Let $F : [0, 1] \to \mathbb{R}$ be a differentiable function such that its derivative $F'(x)$ is increasing in x. Which of the following is true for every $x, y \in [0, 1]$ with $x > y$?

    A. $F(x) - F(y) = (x - y)F'(x)$

    B. $F(x) - F(y) \geq (x - y)F'(x)$

    C. $F(x) - F(y) \leq (x - y)F'(x)$

    D. $F(x) - F(y) = F'(x) - F'(y)$

21. A bag contains N balls of which $a(a < N)$ are red. Two balls are drawn from the bag without replacement. Let p_1 denote the probability that the first ball is red and p_2 the probability that the second ball is red. Which of the following statements is true?

    A. $p_1 > p_2$

    B. $p_1 < p_2$

    C. $p_2 = \frac{a-1}{N-1}$

    D. $p_2 = \frac{a}{N}$.

22. Let $t = x + \sqrt{x^2 + 2bx + c}$ where $b^2 > c$. Which of the following statements is true?

    A. $\frac{dx}{dt} = \frac{t-x}{t+b}$

    B. $\frac{dx}{dt} = \frac{t+2x}{2t+b}$.

    C. $\frac{dx}{dt} = \frac{1}{2x+b}$.

    D. None of the above.

23. Let A be an $n \times n$ matrix whose entry on the i -th row and j -th column is $\min(i, j)$. The determinant of A is:

    A. n.

    B. 1

    C. $n!$

    D. 0.

24. What is the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 10$?

    A. 66 .

    B. 55 .

    C. 100 .

    D. None of the above.

25. For $b > 0$, the value of

$$\int_b^{2b} \frac{xdx}{x^2+b^2}$$

    A. $\frac{1}{b}$.

    B. $\ln 4b^2$.

    C. $\frac{1}{2} \ln \left(\frac{5}{2}\right)$

    D. None of the above.

26. Let f and g be functions on $\mathbb{R}^2$ defined respectively by

$$f(x, y) = \frac{1}{3}x^3 - \frac{3}{2}y^2 + 2x$$

and

$$g(x, y) = x - y$$

Consider the problems of maximizing and minimizing f on the constraint set $C = \{(x, y) \in \mathbb{R}^2 : g(x, y) = 0\}$

    A. f has a maximum at $(x = 1, y = 1)$, and a minimum at $(x = 2, y = 2)$.

    B. f has a maximum at $(x = 1, y = 1)$, but does not have a minimum.

    C. f has a minimum at $(x = 2, y = 2)$, but does not have a maximum.

    D. f has neither a maximum nor a minimum.

27. A particular men's competition has an unlimited number of rounds. In each round, every participant has to complete a task. The probability of a participant completing the task in a round is p. If a participant fails to complete the task in a round, he is eliminated from the competition. He participates in every round before being elimi- nated. The competition begins with three participants. The probability that all three participants are eliminated in the same round is:

    A. $\frac{(1-p)^3}{1-p^3}$.

B. $\frac{1}{3}(1-p)$

C. $\frac{1}{p^3}$.

D. None of the above.

28. Three married couples sit down at a round table at which there are six chairs. All of the possible seating arrangements of the six people are equally likely. The probability that each husband sits next to his wife is:

    A. $\frac{2}{15}$.

    B. $\frac{1}{3}$.

    C. $\frac{4}{15}$.

    D. None of the above.

29. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function. For every $x, y, z \in \mathbf{R}$, we know that $f(x,y) + f(y,z) + f(z,x) = 0$. Then, for every $x, y \in \mathbf{R}^2$, $f(x,y) - f(x,0) + f(y,0) =$

    A. 0.

    B. 1

    C. -1 .

    D. None of the above.

30. The minimum value of the expression below for $x > 0$ is:

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$$

    A. 1 .

    B. 3 .

    C. 6

    D. 12 .

12 ISI PEA 2017

1. The dimension of the space spanned by the vectors $(-1, 0, 1, 2)$, $(-2, -1, 0, 1)$, $(-3, 2, 0, 1)$ and $(0, 0, -1, 1)$ is
   - A. 1
   - B. 2
   - C. 3
   - D. 4

2. How many onto functions are there from a set A with $m > 2$ elements to a set B with 2 elements?
   - A. 2^m
   - B. $2^m - 1$
   - C. $2^{m-1} - 2$
   - D. $2^m - 2$

3. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is
   - A. quasiconcave and concave
   - B. concave but not quasiconcave
   - C. quasiconcave but not concave
   - D. none of the above

4. A function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is
   - A. homogeneous of degree 0
   - B. homogeneous of degree 1
   - C. homogeneous of degree 2
   - D. not homothetic

5. You have n observations on rainfall in centimeters (cm) at a certain location, denoted by x, and you calculate the standard deviation, variance and coefficient of variation (CV). Now, if instead, you were given the same observations measured in millimeters (mm), then
   - A. the standard deviation and CV would increase by a factor of 10, and the variance by a factor of 100
   - B. the standard deviation would increase by a factor of 10, the variance by a factor of 100 and the CV would be unchanged
   - C. the standard deviation would increase by a factor of 10, the variance and CV by a factor of 100
   - D. none of the above

6. You have n observations on rainfall in centimeters (cm) at two locations, denoted by x and y respectively, and you calculate the covariance, correlation coefficient r, and the slope coefficient b of the regression of y on x. Now, if instead, you were given the same observations measured in millimeters (mm), then

   A. the covariance would increase by a factor of 10, b by a factor of 100, and r would be unchanged

   B. the covariance and b would increase by a factor of 100, and r would be unchanged

   C. the covariance would increase by a factor of 100 and b and r would be unchanged

   D. none of the above

7. Let $0 < p < 1$. Any solution (x^*, y^*) of the constrained maximization problem

$$\max_{x,y} \left(\frac{-1}{x} + y \right)$$
$$\text{subject to}$$
$$px + y \leq 10$$
$$x, y \geq 0$$

   must satisfy

   A. $y^* = 10 - p$

   B. $x^* = 10/p$

   C. $x^* = 1/\sqrt{p}$

   D. none of the above

8. Suppose the matrix equation $Ax = b$ has no solution, where A is 3×3 non-zero matrix of real numbers and b is a 3×1 vector of real numbers. Then

   A. The set of vectors x for which $Ax = 0$ is a plane

   B. The set of vectors x for which $Ax = 0$ is a line

   C. The rank of A is 3

   D. $Ax = 0$ has a non-zero solution

9. k people get off a plane and walk into a hall where they are assigned to at most n queues. The number of ways in which this can be done is

   A. ${}^nC_k$

   B. ${}^nP_k$

   C. $n^k k!$

   D. $n(n+1)\ldots(n+k-1)$

10. If $\Pr(A) = \Pr(B) = p$, then $\Pr(A \cap B)$ must be

A. greater than p^2

B. equal to p^2

C. less than or equal to p^2

D. none of the above

11. If $\Pr(A^c) = \alpha$ and $\Pr(B^c) = \beta$ (where A^c denotes the event "not A"), then $\Pr(A \cap B)$ must be

    A. $1 - \alpha\beta$

    B. $(1 - \alpha)(1 - \beta)$

    C. greater than or equal to $1 - \alpha - \beta$

    D. none of the above

12. The density function of a normal distribution with mean μ and standard deviation σ has inflection points at

    A. μ

    B. $\mu - \sigma, \mu + \sigma$

    C. $\mu - 2\sigma, \mu + 2\sigma$

    D. nowhere

13. In how many ways can five objects be placed in a row if two of them cannot be placed next to each other?

    A. 36

    B. 60

    C. 72

    D. 24

14. Suppose $x = 0$ is the only solution to the matrix equation $Ax = 0$ where A is $m \times n$, x is $n \times 1$, and 0 is $m \times 1$. Then, of the two statements (i) The rank of A is n, and (ii) $m \geq n$

    A. Only (i) must be true

    B. Only (ii) must be true

    C. Both (i) and (ii) must be true

    D. Neither (i) nor (ii) has to be true

15. Mr. A is selling raffle tickets which cost 1 rupee per ticket. In the queue for tickets there are n people. One of them has only a 2-rupee coin while all the rest have 1-rupee coins. Each person in the queue wants to buy exactly one ticket and each arrangement in the queue is equally likely to occur. Initially, Mr. A has no coins and enough tickets for everyone in the queue. He stops selling tickets as soon as he is unable to give the required change. The probability that he can sell tickets to all people in the queue is

A. $\frac{n-2}{n}$

B. $\frac{1}{n}$

C. $\frac{n-1}{n}$

D. $\frac{n-1}{n+1}$

16. Out of 800 families with five children each, how many families would you expect to have either 2 or 3 boys. Assume equal probabilities for boys and girls.

A. 400

B. 450

C. 500

D. 550

17. The function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

is

A. concave

B. convex

C. neither concave nor convex

D. both concave and convex

18. As $n \to \infty$, the sequence $\left\{\frac{n^2+1}{2n^2+3}\right\}$

A. diverges

B. converges to 1/3

C. converges to 1/2

D. neither converges nor diverges

19. The function $x^{1/3}$ is

A. differentiable at $x = 0$

B. continuous at $x = 0$

C. concave

D. none of the above

20. The function $\sin(\log x)$, where $x > 0$

A. is increasing

B. is bounded and converges to a real number as $x \to \infty$

C. is bounded but does not converge as $x \to \infty$

D. none of the above

21. For any two functions $f_1 : [0,1] \to \mathbb{R}$ and $f_2 : [0,1] \to \mathbb{R}$, define the function $g : [0,1] \to \mathbb{R}$ as $g(x) = \max(f_1(x), f_2(x))$ for all $x \in [0,1]$.

    A. If f_1 and f_2 are linear, then g is linear

    B. If f_1 and f_2 are differentiable, then g is differentiable

    C. If f_1 and f_2 are convex, then g is convex

    D. none of the above

22. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as

$$f(x) = x^3 - 3x$$

Find the maximum value of the $f(x)$ on the set of real numbers satisfying $x^4 + 36 \leq 13x^2$.

    A. 18

    B. -2

    C. 2

    D. 52

23. A monkey is sitting on 0 on the real line in period 0. In every period $t \in \{0, 1, 2, \ldots\}$ it moves 1 to the right with probability p and 1 to the left with probability $1 - p$, where $p \in \left[\frac{1}{2}, 1\right]$. Let π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_k for any positive integer k is

    A. p^k

    B. 1

    C. $\frac{p^k}{(1-p)^k}$

    D. $\frac{p}{k}$

24. Refer to the previous question. Suppose $p = \frac{1}{2}$ and π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_0 is

    A. 0

    B. $\frac{1}{2^k}$

    C. $\frac{1}{2}$

    D. 1

25. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function with $f'(x) > 0$ for all $x \in \mathbb{R}$ and satisfying the property

$$\lim_{x \to -\infty} f(x) \geq 0$$

Which of the following must be true?

A. $f(1) < 0$

B. $f(1) > 0$

C. $f(1) = 0$

D. none of the above

26. For what values of x is

$$x^2 - 3x - 2 < 10 - 2x$$

A. $4 < x < 9$

B. $x < 0$

C. $-3 < x < 4$

D. none of the above

27. $\displaystyle\int_e^{e^2} \frac{1}{x(\log x)^3} dx =$

A. 3/8

B. 5/8

C. 6/5

D. $-4/5$

28. The solution of the system of equations

$$\begin{aligned} x - 2y + z &= 7 \\ 2x - y + 4z &= 17 \\ 3x - 2y + 2z &= 14 \end{aligned}$$

is

A. $x = 4, y = -1, z = 3$

B. $x = 2, y = 4, z = 3$

C. $x = 2, y = -1, z = 5$

D. none of the above

29. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function with non-zero second partial derivatives. Suppose that for every $x \in \mathbb{R}$, there is a unique value of y, say $y^*(x)$, that solves the problem $\max_{y \in \mathbb{R}} \ f(x, y)$ Then y^* is increasing in x if

A. f is strictly concave

B. f is strictly convex

C. $\frac{\partial^2 f}{\partial x \partial y} > 0$

D. $\frac{\partial^2 f}{\partial x \partial y} < 0$

30. $\displaystyle\int 3^{\sqrt{2x+1}} dx =$

A. $\dfrac{3^{\sqrt{2x+1}}}{\ln 3} + \dfrac{\sqrt{2x+1}}{\ln 3} + c$

B. $\dfrac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{\ln 3} - \dfrac{3^{\sqrt{2x+1}}}{(\ln 3)^2} + c$

C. $\dfrac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{(\ln 3)^2} - \dfrac{3^{\sqrt{2x+1}}}{\ln 3} + c$

D. none of the above

13 ISI PEA 2018

1. Suppose that the level of savings varies positively with the level of income and that savings is identically equal to investment. Then the IS curve:

   A. slopes positively.

   B. slopes negatively.

   C. is vertical.

   D. does not exist.

2. Consider the Solow growth model without technological progress. Suppose that the rate of growth of the labor force is 2%. Then, in the steady-state equilibrium:

   A. per capita income grows at the rate of 2%.

   B. per capita consumption grows at the rate of 2%.

   C. wage per unit of labor grows at the rate of 2%.

   D. total income grows at the rate of 2%.

3. Consider a Simple Keynesian Model for a closed economy with government. Suppose there does not exist any public sector enterprise in the economy. Income earners are divided into two groups, Group 1 and Group 2, such that the saving propensity of the former is less than that of the latter. Aggregate planned investment is an increasing function of GDP (Y). Start with an initial equilibrium situation. Now, suppose the government imposes and collects additional taxes from Group 1 and uses the tax revenue so generated to make transfer payments to Group 2. Following this:

   A. aggregate saving in the economy remains unchanged.

   B. aggregate saving in the economy declines.

   C. aggregate saving in the economy rises.

   D. aggregate saving in the economy may change either way.

4. Suppose, in an economy, the level of consumption is fixed, while the level of investment varies inversely with the rate of interest. Then the IS curve is:

   A. positively sloped.

   B. negatively sloped.

   C. vertical.

   D. horizontal.

5. Suppose, in an economy, the demand function for labor is given by:

$$L^d = 100 - 5w$$

   whereas the supply function for labor is given by:

$$L^s = 5w$$

where w denotes the real wage rate. Total labor endowment in this economy is 80 units. Suppose further that the real wage rate is flexible. Then involuntary unemployment in this economy is:

A. 30 .

B. 50 .

C. 70 .

D. 0.

6. Consider again the economy specified in Question 5. Suppose now that the real wage rate is mandated by the government to be at least 11. Then total unemployment will be:

A. 35 .

B. 0.

C. 30 .

D. 10 .

7. Consider a macro-economy defined by the following equations:

$$\begin{aligned} &M = kPy + L(r) \\ &S(r) = I(r) \\ &y = \bar{y} \end{aligned}$$

where M, P, y and r represent, respectively, money supply, the price level, output and the interest rate, while k and $\bar{y}$ are positive constants. Furthermore, $S(r)$ is the savings function, $I(r)$ is the investment demand function and $L(r)$ is the speculative demand for money function, with , $S'(r) > 0, I'(r) < 0$ and $L'(r) < 0$. Then, an increase in M must:

A. increase P proportionately.

B. reduce P.

C. increase P more than proportionately.

D. increase P less than proportionately.

8. Two individuals, X and Y, have to share Rs. 100. The shares of X and Y are denoted by x and y respectively, $x, y \geq 0, x+y = 100$. Their utility functions are $U_X(x,y) = x + \left(\frac{1}{4}\right) y$ and $U_Y(x,y) = y + \left(\frac{1}{2}\right) x$. The social welfare function is $W(U_X, U_Y) = \min\{U_X, U_Y\}$. Then the social welfare maximizing allocation is:

A. (44,56)

B. (48,52) .

C. (50,50) .

D. (60,40) .

9. Consider two consumers. They consume one private good (X) and a public good (G). Consumption of the public good depends on the sum of their simultaneously and non-cooperatively chosen contributions towards the public good out of their incomes. Thus, if g_1 and g_2 are their contributions, then the consumption of the public good is $g = g_1 + g_2$. Let the utility function of consumer $i(i = 1, 2)$ be $U_i(x_i, g) = x_i g$. The price of the private good is $p > 0$ and the income of each consumer is $M > 0$. Then the consumers' equilibrium contributions towards the public good will be:

   A. $\left(\frac{M}{2}, \frac{M}{2}\right)$.

   B. $\left(\frac{M}{3}, \frac{M}{3}\right)$

   C. $\left(\frac{M}{4}, \frac{M}{4}\right)$.

   D. $\left(\frac{M}{p}, \frac{M}{p}\right)$

10. Consider two firms, 1 and 2 , producing a homogeneous product and competing in Cournot fashion. Both firms produce at constant marginal cost, but firm 1 has a lower marginal cost than firm 2. Specifically, firm 1 requires one unit of labour and one unit of raw material to produce one unit of output, while firm 2 requires two units of labour and one unit of raw material to produce one unit of output. There is no fixed cost. The prices of labour and material are given and the market demand for the product is determined according to the function $q = A - bp$, where q is the quantity demanded at price p and $A, b > 0$. Now, suppose the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 1 will:

    A. increase.

    B. decrease.

    C. remain unchanged.

    D. go up or down depending on the parameters.

11. Considered again the problem in Question 10. As before, suppose that the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 2 will:

    A. increase.

    B. decrease.

    C. remain unchanged.

    D. go up or down depending on the parameters.

12. Consider a firm which initially operates only in market A as a monopolist and faces market demand $Q = 20 - p$. Given its cost function $C(Q) = \frac{1}{4}Q^2$, it charges a monopoly price P_m in this market. Now suppose that, in addition to selling as a monopolist in market A, the firm starts selling its products in a competitive market, B, at price $\bar{p} = 6$ Under this situation the firm charges P_m^* in market A. Then:

    A. $P_m^* > P_m$

    B. $P_m^* < P_m$

C. $P_m^* = P_m$

D. given the available information we cannot say whether $P_m^* > P_m$ or $P_m^* < P_m$

13. Two consumers, A and B, have utility functions $U_A = \min\{x_A, y_A\}$ and $U_B = x_B + y_B$, respectively. Their endowments vectors are $e_A = (100, 100)$ and $e_B = (50, 0)$ Consider a competitive equilibrium price vector (P_X, P_Y). Then,

A. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is the unique equilibrium price vector.

B. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is one of the many possible equilibrium price vectors.

C. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is never an equilibrium price vector.

D. an equilibrium price vector does not exist.

14. Suppose a firm is a monopsonist in the labor market and faces separate labor supply functions for male and female workers. The labor supply function for male workers is given by $l_M = (w_M)^k$, where l_M is the amount of male labor available when the wage offered to male workers is w_M, and k is a positive constant. Analogously, the labor supply function for female workers is given by $l_F = w_F$. Male and female workers are perfect substitutes for one another. The firm produces one unit of output from each unit of labor it employs, and sells its output in a competitive market at a price of p per unit. The firm can pay male and female workers differently if it chooses to. Suppose the firm decides to pay male workers more than female workers. Then it must be the case that:

A. $k < \frac{1}{2}$

B. $\frac{1}{2} \le k < 1$

C. $k = 1$

D. $k > 1$

15. Consider the problem in Question 14, and assume that the firm pays male workers more than female workers. Suppose further that $p > 2$. Then the firm must:

A. hire more male workers than female workers.

B. hire more female workers than male workers.

C. hire identical numbers of male and female workers.

D. hire more females than males if $2 < p \le 4$, but more males than females if $p > 4$.

16. Consider the system of linear equations:

$$\begin{aligned}(4a-1)x + y + z &= 0\\ -y + z &= 0\\ (4a-1)z &= 0\end{aligned}$$

The value of a for which this system has a non-trivial solution (i.e., a solution other than (0,0,0)) is:

A. $\frac{1}{2}$.

B. $\frac{1}{4}$.

C. $\frac{3}{4}$.

D. 1 .

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex and differentiable function with $f(0) = 1$, where $\mathbb{R}$ denotes the set of real numbers. If the derivative of f at 2 is 2, then the maximum value of $f(2)$ is:

A. 3 .

B. 5 .

C. 10.

D. ∞.

18. Consider the equation $2x + 5y = 103$. Then how many pairs of positive integer values can(x, y) take such that $x > y$?

A. 7.

B. 8 .

C. 13 .

D. 14 .

19. Let X be a discrete random variable with probability mass function (PMF) $f(x)$ such that

$$\begin{array}{l} f(x) > 0 \quad \text{if } x = 0, 1, \ldots, n, \text{ and} \\ f(x) = 0 \text{ otherwise} \end{array}$$

where n is a finite integer. If $\text{Prob}(X \geq m \mid X \leq m) = f(m)$, then the value of m is:

A. 0.

B. 1.

C. $n - 1$

D. none of the above.

20. Consider the function $f(x) = 2ax \log_c x - ax^2$ where $a \neq 0$. Then

A. the function has a maximum at $x = 1$.

B. the function has a minimum at $x = 1$.

C. the point $x = 1$ is a point of inflexion.

D. none of the above.

21. Let $f : [0, 10] \to [10, 20]$ be a continuous and twice differentiable function such that $f(0) = 10$ and $f(10) = 20$. Suppose $|f'(x)| \leq 1$ for all $x \in [0, 10]$. Then, the value of $f''(5)$ is

A. 0.

B. $\frac{1}{2}$.

C. 1 .

D. cannot be determined from the given information.

22. Consider the system of linear equations:

$$
\begin{aligned}
x + 2ay + az &= 0 \\
x + 3by + bz &= 0 \\
x + 4cy + cz &= 0
\end{aligned}
$$

Suppose that this system has a non-zero solution. Then a, b, c

A. are in arithmetic progression.

B. are in geometric progression.

C. are in harmonic progression.

D. satisfy $2a + 3b + 4c = 0$

23. Let a, b, c be real numbers. Consider the function $f(x_1, x_2) = \min\{a - x_1, b - x_2\}$.Let (x_1^*, x_2^*) be the solution to the maximization problem

$$\max f(x_1, x_2) \text{ subject to } x_1 + x_2 = c$$

Then $x_1^* - x_2^*$ equals

A. $\frac{c+a-b}{2}$.

B. $\frac{c+b-a}{2}$.

C. $a - b$

D. $b - a$.

24. Suppose that you have 10 different books, two identical bags and a box. The bags can each contain three books and the box can contain four books. The number of ways in which you can pack all the books is

A. $\frac{104}{2!3!3!4!}$

B. $\frac{10t}{3!3!4!}$

C. $\frac{10!}{2!3!4!}$

D. none of the above.

25. Real numbers $a_1, a_2, \ldots, a_{99}$ form an arithmetic progression. Suppose that

$$a_2 + a_5 + a_8 + \ldots + a_{98} = 205$$

Then the value of $\sum_{k=1}^{99} a_k$ is

A. 612 .

B. 615 .

C. 618 .

D. none of the above.

26. A stone is thrown into a circular pond of radius 1 meter. Suppose the stone falls uniformly at random on the area of the pond. The expected distance of the stone from the center of the pond is

A. $\frac{1}{3}$.
B. $\frac{1}{2}$.
C. $\frac{2}{3}$.
D. $\frac{1}{\sqrt{2}}$.

27. Suppose that there are n stairs, where n is some positive integer. A person standing at the bottom wants to reach the top. The person can climb either 1 stair or 2 stairs at a time. Let T_n be the total number of ways in which the person can reach the top. For instance, $T_1 = 1$ and $T_2 = 2$. Then, which one of the following statements is true for every $n > 2$?

A. $T_n = n$.
B. $T_n = 2T_{n-1}$
C. $T_n = T_{\mathrm{n}-1} + T_{n-2}$
D. $T_n = \sum_{k=1}^{n-1} T_k$

28. Let $Y_1, Y_2, \ldots, Y_n$ be the income of n individuals with $E(Y_i) = \mu$ and $\operatorname{Var}(Y_i) = \sigma^2$ for all $i = 1, 2, \ldots n$. These n individuals form m groups, each of size k. It is known that individuals within the same group are correlated but two individuals in different groups are always independent. Assume that when individuals are correlated, the correlation coefficient is the same for all pairs. Consider the random variable $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$. The limiting variance of $\bar{Y}$ when m is large but k is finite is

A. 0.
B. $\frac{1}{k}$
C. 1 .
D. $\frac{\sigma^2}{k}$.

29. A person makes repeated attempts to destroy a target. Attempts are made independently of each other. The probability of destroying the target in any attempt is 0.8 . Given that he fails to destroy the target in the first five attempts, the probability that the target is destroyed in the 8 -th attempt is

A. 0.032 .
B. 0.064
C. 0.128 .
D. 0.160 .

30. Let E and F be two events such that $0 < \operatorname{Prob}(E) < 1$ and $\operatorname{Prob}(E \mid F) + \operatorname{Prob}(E \mid F^c) = 1$. Then

A. E and F are mutually exclusive.

B. $\text{Prob}\left(E^{c} \mid F\right)+\text{Prob}\left(E^{c} \mid F^{c}\right)=1$

C. E and F are independent.

D. $\text{Prob}(E \mid F)+\text{Prob}\left(E^{c} \mid F^{c}\right)=1$

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1. Robinson Crusoe will live this period (period 1) and the next period (period 2) as the only inhabitant of his island completely isolated from the rest of the world. His only income is a crop of 100 coconuts that he harvests at the beginning of each period. Coconuts not consumed in the current period spoil at the rate of 20% per period. Crusoe's preference over consumption in period 1 (c_1) and consumption in period 2 (c_2) is given by the utility function $u(c_1, c_2) = \min\{5c_1, 6c_2\}$. Crusoe's utility maximizing consumption choice is given by

   A. $c_1 = \frac{200\times 6}{11}, c_2 = \frac{200\times 5}{11}$.

   B. $c_1 = 90, c_2 = 108$

   C. $c_1 = 100, c_2 = 100$

   D. none of the above.

2. The domestic supply and demand equations for a commodity in a country are as follows: Supply: $P = 50+Q$, Demand: $P = 200-2Q$, where P is the price in rupees per kilogram and Q is the quantity in thousands of kilograms. The country is a small producer in the world market where the price (which will not be affected by anything done by this country) is Rs. 60 per kilogram. The government of this country introduces a "Permit Policy" which works as follows. The government issues a fixed number of Permits - each Permit allows its owner to sell exactly 100 kilograms of the commodity in this country's market. An exporter from a foreign country cannot sell this commodity in this country unless she purchases such a Permit. Suppose the government issues 300 Permits. What is the maximum price an exporter is willing to pay for a Permit?

   A. Rs. 3000 .

   B. Rs. 2000 .

   C. Rs. 1500 .

   D. Rs. 1000 .

3. SeaTel provides cellular phone service in Delhi and has some monopoly power in the sense that it has its captive customer base with each customer's weekly demand being given by: $Q = 60 - P$, where Q denotes hours of cell phone calls per week and P is the price per hour. SeaTel's total cost of providing cell phone service is given by $C = 20Q$, so that the marginal cost is $MC = 20$. Suppose SeaTel offers a "Call-As-Much-As-You-Wish" deal: it charges only a flat weekly access fee, and once a customer pays the flat access fee, he/she can call as much as he/she wishes with- out paying any extra usage fee per hour. The weekly access fee that SeaTel should charge to maximize its profit is given by

   A. 1800

   B. 1200 .

   C. 800

   D. 40 .

4. A bus stop has to be located on the interval $[0,1]$. There are three individuals located at points 0.2,0.3 and 0.9 on the interval. If the bus stop is located at point x, then the utility of an individual located at y is $-|y-x|$, that is, the negative of the distance between the bus stop and the individual's location. A relocation of the bus stop is said to be Pareto improving if at least one individual is better off and no individual is worse off from the relocation. A location of the bus stop is said to be Pareto efficient if there does not exist any Pareto improving relocation. Then

   A. 0.5 is the only Pareto efficient location.

   B. $\frac{0.2+0.3+0.9}{3}$ is the only Pareto efficient location.

   C. Median of 0.2,0.3 and 0.9 is the only Pareto efficient location.

   D. none of the above.

5. Consider three goods: a cable television, b a fish in international waters, and c a burger. Also consider four descriptions of the goods: (A) non-rival and non-excludable, (B) rival and excludable, (C) non-rival and excludable, and (D) rival and nonexcludable. In what follows we match goods to possible descrip- tions. Choose the correct match.

   A. $\text{(a)} - \text{(A)}, \text{(b)} - \text{(C)}, \text{(c)} - \text{(B)}$

   B. $\text{(a)} - \text{(C)}, \text{(b)} - \text{(D)}, \text{(c)} - \text{(A)}$

   C. $\text{(a)} - (C), \text{(b)} - (B), \text{(c)} - (A)$

   D. $\text{(a)} - \text{(C)}, \text{(b)} - \text{(D)}, \text{(c)} - \text{(B)}$

6. Consider an economy consisting of three individuals -1,2 and 3 , two goods $-$A and B, and a single monopoly firm that can produce both goods at zero cost. Each individual would like to buy exactly 1 unit of the goods A and B, if at all. An individual's valuation of a good is defined as the maximum amount she is willing to pay for one unit of the good. Individual 1 's valuation of good A is Rs. 10 and that of good B is Rs. 1. Individual 2's valuation of good A is Rs. 1, and that of good B is Rs. 10 . Individual 3 's valuation is Rs. 7 for good A, and Rs. 7 good B. The firm can charge a single price p_A for good A, a single price p_B for good B, and a bundled price p_{AB} such that if an individual pays p_{AB} then she gets the bundle consisting of one unit each of goods A and B. If the monopolist sets p_A, p_B and p_{AB} to maximize its profit then

   A. $p_A = 11, p_B = 11, p_{AB} = 11$

   B. $p_A = 11, p_B = 11, p_{AB} = 14$

   C. $p_A = 10, p_B = 10, p_{AB} = 11$

   D. none of the above.

7. Consider a Bertrand duopoly with two firms, 1 and 2. Both firms produce the same good that has a market demand func- tion $p = 10 - q$. The market is equally shared in case the firms charge the same price, otherwise the lower priced firm gets the entire demand. A firm must satisfy all the demand coming to it. The cost function of firm 1 is $3q_1$, that of firm 2 is $2q_2$. Suppose prices vary along the following grid, $\{0, 0.1, 0.2, \ldots\}$. The Bertrand equilibrium is given by

A. $p_1 = 2, p_2 = 2$

B. $p_1 = 3, p_2 = 2$

C. $p_1 = 3, p_2 = 2.9$

D. $p_1 = 3, p_2 = 3$

8. Consider a monopolist with a market demand function $p = 20 - q$. It is a multi-plant monopolist with two plants, plant 1 and plant 2, where the plant specific cost function of plant $i, i = 1, 2$, is

$$c_i(q_i) = \begin{cases} 2 + 4q_i, & \text{if } q_i > 0 \\ 0, & \text{otherwise} \end{cases}$$

The optimal monopoly profit is given by

A. 60 .

B. 64

C. 68 .

D. 62 .

9. Consider a closed economy in which an individual's labour supply (L) to firms is determined by the amount which maximizes her utility function $u(C, L) = C^{\alpha}(1 - L)^{\beta}$, where $\alpha, \beta > 0, \alpha + \beta < 1$ and C is consumption expenditure which is taken to be equal to wage income (wL). Then

A. labour supply does not depend on the wage rate w.

B. labour supply is directly proportional to the wage rate w.

C. labour supply is inversely proportional to the wage rate w.

D. more information is needed to derive the labour supply.

10. In the scenario described in Question 9, assume that the economy is Keynesian, that is, investment expenditure (I) is autonomous and output (Y) is determined by aggregate demand, $Y = C + I$. The aggregate production function is given by $Y = AL^{\theta}$, where $A > 0$ is a productivity parameter and $0 < \theta < 1$. [Note that the firm's employment of labour is obtained by equating the marginal product of labour to w.] Then the marginal propensity to consume is

A. $\frac{a+\beta}{\theta}$.

B. $\frac{g}{\theta}$.

C. α.

D. θ

11. Consider a Solow growth model (in continuous time) with a production function with labour augmenting technological change, $Y_t = F(K_t, A_t L_t)$, where Y_t denotes output, K_t denotes the capital stock, A_t denotes the level of total factor productivity (TFP), and L_t denotes the stock of the labour force. Assume that L_t grows at the rate $n > 0$ and A_t grows at the rate $g > 0$, that is, $\frac{\dot{L}}{L} = n$ and $\dot{A} = g$, and the capital accumulation

equation is given by $\dot{K} = sY_t - \delta K_t$, where $s \in [0,1]$ is the exogenous savings rate, and $\delta \in [0,1]$ is the depreciation rate of capital. [Note that for any variable $x, \dot{x}$ denotes $\frac{dx}{dt}$,] Define capital in efficiency units to be $Z \equiv \frac{K}{AL}$. Then the expression for $\frac{2}{2}$ is given by

A. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (n+g)$

B. $\frac{z}{Z} = \frac{\Delta f(Z)}{Z} - (\delta + n + g)$

C. $\frac{\dot{2}}{2} = \frac{\Delta f(Z)}{Z}$

D. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - n$

12. In the Solow growth model described in Question 11 , the growth rate of Y at the steady state is given by

A. $n + g$

B. $\delta + n + g$

C. zero.

D. n.

13. Consider an IS-LM model where the IS curve is represented by $0.25Y = 500 + G - i$, and money demand function is given by $\frac{M}{P} = \frac{2Y}{e^1}$. The notations are standard: Y denotes output, G denotes government expenditure, i denotes the interest rate, P is the price level and e is the exponential. Suppose the government wants to increase spending and therefore the central bank decides to change the money supply accordingly such that the interest rate remains the same in the short run. Then the change in money supply satisfies the following condition:

A. $\frac{dM}{dG} = e$

B. $\frac{dM}{dG} = \frac{Y_c}{M}$

C. $\frac{dM}{dG} = \frac{Y}{M}$

D. $\frac{dM}{dG} = \frac{4M}{Y}$.

14. An agent lives for two periods. Her utility from consumption in period $1\,(c_1)$ and consumption in period $2\,(c_2)$ is given by $u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. The agent earns incomes w_1 in period 1 and w_2 in period 2. The rate of interest is $r > 0$ The agent chooses c_1 and c_2 so as to maximize $u(c_1, c_2)$ subject to her budget constraint. Consider a temporary increase in income where w_1 increases but the agent does not change her expectations about w_2. Then the marginal propensity to consume of present consumption with respect to w_1, $\frac{dc_1}{du_1}$, is given by

A. $\frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\right)$

B. $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$

C. $\frac{1}{1+\beta}$

D. 1

15. In the scenario described in Question 14, consider a permanent increase in income where w_1 increases and the agent expects that w_2 will also increase by the same amount. Then $\frac{dc_1}{dw_1}$ is given by

    A. $\frac{1}{1+\beta}\left(1+\frac{1}{1+r}\right)$

    B. $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$

    C. $\frac{1}{1+\beta}$

    D. 1

16. For what values of a are the vectors $(0,1,a),(a,1,0),(1,a,1)$ in $\mathbb{R}^3$ linearly dependent?

    A. 0 .

    B. 1

    C. 2

    D. $\sqrt{2}$.

17. Which of the following set of vectors form a basis of $\mathbb{R}^2$?

    A. {(2,1)}

    B. {(1,1),(2,2)}

    C. {(1,1),(1,2),(2,1)}

    D. {(1,1),(2,3)}

18. If a candidate is good he is selected in MSQE examination with probability 0.9. If a candidate is bad he is selected in MSQE examination with probability 0.2. Suppose every candidate is equally likely to be good or bad. If you meet a candidate who is selected in the MSQE examination, what is the probability that he will be good?

    A. $\frac{11}{20}$.

    B. $\frac{9}{10}$

    C. $\frac{9}{11}$.

    D. $\frac{11}{12}$.

19. Let $S_1 = \{2,3,4,\ldots,9\}$. First, an integer s_1 is drawn uniformly at random from S_1. Then s_1 and all its factors are removed from S_1. Let the new set be S_2. Next an integer s_2 is drawn uniformly at random from S_2. Then s_2 and all its factors are removed from S_2. Let the new set be S_3. Finally, an integer s_3 is drawn uniformly at random from S_3. What is the probability that $s_1 = 2, s_2 = 3, s_3 = 5$?

    A. $\frac{1}{8}$

    B. $\frac{1}{64}$

    C. $\frac{1}{16}$.

    D. $\frac{1}{72}$.

20. Mr. A and B are independently tossing a coin. Their coins have a probability 0.25 of coming HEAD. After each of them tossed the coin twice, we see a total of 2HEADS. What is the probability that Mr. A had exactly one HEAD?

    A. $\frac{2}{3}$.

    B. $\frac{1}{2}$.

    C. $\frac{1}{4}$.

    D. $\frac{1}{3}$.

21. Consider the following function $f : \mathbb{R} \to \mathbb{R}$.

$$f(x) = \begin{cases} x & \text{if } x \le e \\ x(\log_c x) & \text{if } x > e \end{cases}$$

    Which of the following is true for f?

    A. f is not continuous at e

    B. f is not differentiable at e.

    C. f is neither continuous nor differentiable at e.

    D. f is continuous and differentiable at e.

22. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous and weakly increasing function such that $\int_{-1}^{1} f(x)dx = 2\int_{-1}^{1} f(-x)dx$. Suppose $f(-1) = 0$ then $f(1)$ is

    A. 0.

    B. 1

    C. $\frac{1}{2}$.

    D. none of the above.

23. Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ be a twice continuously differentiable function. Let $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$. Consider the following two statements: (i) if $\frac{\partial^2 f}{\partial^2 x}(x^*) \le 0$, then x^* is a point of local maximum of f; (ii) if x^* is a point of local maximum of f, then $\frac{\partial^2 f}{\partial^2 x}(x^*) < 0$. Which of the following is true?

    A. both (i) and (ii) are correct.

    B. both (i) and (ii) are incorrect.

    C. (i) is correct but (ii) is incorrect.

    D. (ii) is correct but (i) is incorrect.

24. Consider the function $f(x) = e^x$ for all $x \in \mathbb{R}$. Which of the following is true?

    A. f is quasi-convex.

    B. f is quasi-concave.

    C. f is neither quasi-convex nor quasi-concave.

    D. f is both quasi-convex and quasi-concave.

25. Consider the following matrix A.

$$A=\left[\begin{array}{ccc} x & 0 & k \\ 1 & x & k-3 \\ 0 & 1 & 1 \end{array}\right]$$

Suppose determinant of A is zero for two distinct real values of x. What is the least positive integer value of k?

A. 1 .

B. 9.

C. 10.

D. 8 .

26. Define the following function on the set of all positive integers.

$$f(n)=\left\{\begin{array}{ll} 2 \times 4 \times \ldots \times(n-3) \times(n-1) & \text { if } n \text { is odd } \\ 1 \times 3 \times \ldots \times(n-3) \times(n-1) & \text { if } n \text { is even. } \end{array}\right.$$

What is the value of $f(n+2) f(n+1)$?

A. $n!$.

B. $(n+1)!$

C. $(n+2)!$

D. $(n+2)(n!)$

27. The sequence $\left\{x_n\right\}_{n \geq 0}$ is defined as follows. We set $x_0=1$ and $x_n=\sum_{j=0}^{n-1} x_j$ for each integer $n \geq 1$. Then the value of the expression $\sum_{j=0}^{\infty} \frac{1}{x_j}$ is equal to

A. ∞.

B. 2 .

C. 3 .

D. $\frac{7}{4}$.

28. For what values of p does the following quadratic equation have more than two solutions (variable in this equation is x)?

$$\left(p^2-16\right) x^2-\left(p^2-4 p\right) x+\left(p^2-5 p+4\right)=0$$

A. No such value of p exists.

B. -4 and 4

C. 1 and 4

D. 4

29. Consider the square with vertices A, B, C, D. Call a pair of vertices in the square adjacent if they are connected by an edge. You have four colours: RED, BLUE, GREEN, YELLOW. How many ways can you colour the vertices A, B, C, D such that no adjacent vertices share the same colour?

    A. 84

    B. 24.

    C. 72

    D. 108 .

30. Two players P_1 and P_2 are playing a game which involves filling the entries of an $n \times n$ matrix, where $n \geq 2$ is an even integer. Starting with P_1, each player takes turn to fill an unfilled entry of the matrix with a real number. The game ends when all entries are filled. Player P_1 wins if the determinant of the final matrix is non-zero. Else, player P_2 wins. A player $i \in \{1, 2\}$ has a winning strategy if irrespective of what the other player does, i wins by following this strategy. Which of the following is true?

    A. Player 1 has a winning strategy.

    B. Player 2 has a winning strategy.

    C. No player has a winning strategy.

    D. None of the above.

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1. Consider the functions

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

A. f is differentiable at zero but g is not differentiable at zero

B. g is differentiable at zero but f is not differentiable at zero

C. f and g are both differentiable at zero

D. Neither f nor g is differentiable at zero

2. How many ordered pairs of numbers (x, y) are there where x,$y \in \{1.2, \ldots .100\}$. such that $|x - y| \leq 50$?

A. 2550

B. 5050

C. 7550

D. None of the other options are correct

3. Let ABC be a right angled isosceles triangle with angle $\angle ABC$ being right-angled. Let D be the mid-point of $AB.E$ be the foot of the perpendicular drawn from D to the side AC, and F be the foot of the perpendicular drawn from E to the side BC. What is the value of $\frac{FC}{BC}$?

A. $\frac{1}{\sqrt{2}}$

B. $\frac{3}{4}$

C. $2 - \sqrt{2}$

D. None of the other options are correct

4. Suppose that there are 30 MCQ type questions where each question has four options: $A, B.C.D$. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, and 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5 , option B is correct is 0.3 , option C is currect is 0.2 , and option D is correct is 0. Two students Gupi and Baglan how no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5, but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

A. Both Gupi and Bagha have expected scores more than 30

B. Gupi's expected score is ≥ 30 and Bagha's expected scure is strictly less than 30

C. Gupi's experted score is less than or equal to 30 and Bagha's expected score is strictly more than 30

D. None of the above options are correct

5. Evaluate: $\lim_{x\to\infty}\left[e^{3x}-5x\right]^{\frac{1}{x}}$

A. e^3

B. 3

C. 1

D. None of the other options are correct

6. Suppose $f(x)=\left\{\begin{array}{ll}\frac{|x-3|}{x-3}, & \text{for } x\neq 3\\ 0. & \text{for } x=3\end{array}\right\}$. Then. $\lim_{x\to 3} f(x)$:

A. is -1

B. is 0

C. does not exist

D. is 1

7. Consider the following system of equations in x, y, z :

$$\begin{aligned} x+2y-3z&=a\\ 2x+6y-11z&=b\\ x-2y+7z&=c \end{aligned}$$

For what values of a, b, c, does the above system have no solution?

A. $c+2b-5a\neq 0$

B. $c+2b-5a=0$

C. $c+2b-4a=0$

D. None of the other options are correct

8. The sequence x_n is given by the formula: for every positive integer n

$$x_n=n^3-9n^2+631$$

The largest value of n such that $x_n>x_{n+1}$ is

A. 1

B. 5

C. 6

D. None of the other options are correct

9. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D?

   A. 10

   B. 1

   C. 0

   D. None of the other eptions are correct

10. Suppose we are given a 4×4 apure matrix A, which satisfies $A_{ij} = 0$ if $i < j$. Suppose the each diagonal entry A_{ii} is drawn uniformly at random fron $\{0, 1, \ldots, 9\}$. What is the probability that A has full rank?

   A. $\frac{1}{10^4}$

   B. $\frac{3}{5}$

   C. $1 - \frac{1}{10^4}$

   D. $\left(\frac{9}{10}\right)^4$

11. Let a and b be two real numbers where $b \neq 0$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function which satisfies

$$g(g(x)) = \text{a}g(x) + bx \quad \forall x \in \mathbb{R}$$

   Which of the following must be true?

   A. g is strictly increasing

   B. g is strictly decreasing

   C. $\lim_{x\to\infty} g(x)$ is finite

   D. Either (A) or (B)

12. For every positive integer n, let $S(n)$ denote the sum of digits in n. For instance, $s(387) = 3 + 8 + 7 = 18$. The value of the sum

$$S(1) + S(2) + \ldots + S(99)$$

   is

   A. 450

   B. 495

   C. 900

   D. 990

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 plaving cards, with four suits of 13 cards each, which las been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

   A. $\frac{{}^4C_1\,{}^{13}C_5}{{}^{52}C_5}$

B. $\frac{^4C_2\,^{13}C_4}{^{52}C_5}$

C. $\frac{^4P_1\,^{13}C_5}{^{52}P_5}$

D. $\frac{^4C_1\,^{12}C_5}{^{52}C_5}$

14. Evaluate: $\int x^n \ln x dx$, where $n > 1$

A. $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)^2} x^{n+1} + c$

B. $\ln x \frac{x^{n+1}}{2(n+1)} - \frac{1}{(n+1)^2} x^{n+1} + c$

C. $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)} x^{n+1} + c$

D. None of the other options are correct

15. The area of the region bounded by the curve $y = \ln(x)$, the $Y-$ axis. and the lince $y = 1$ and $y = -1$ is

A. $\frac{e}{2}$

B. 2

C. $e - \frac{1}{e}$

D. None of the other options are correct

16. A price discriminating monopolist finds that a person's demand for its product depends on the person's age. The inverse demand function of someone of age y, can be written as $p = A(y) - q$ where $A(y)$ is an increasing function of y. The product cannot be resold from one buyer to another and the monopolist knows the ages of its consumers. (This is often the case with online subscriptions) . If the monopolist maximizes its profits, then

A. older people will pay higher prices and purchase more of this product compared to younger people

B. everyone pays the same price but older people consume more

C. older people will pay higher prices compared to the younger people but everyone will consume the same quantity of the product

D. None of the other options are correct

17. Pam's family consists of herself and her 3 sisters. They own a small fartn in the agricultural sector in Agri-land. The value of their total output is \$4000 which is divided equally amongst the four. The urban sector has two kinds of jobs: informal sector (which anyone can get) pays \$500 and formal sector jobs give \$1200. Pam would like to maximize her own total income and calculates her own expected returns to migration. The proportion of formal sector jobs to urban labor force that would deter her from migrating is:

A. Less than 2/3

B. More than 5/6

C. More than $\frac{1}{2}$

D. Less than 5/7

18. Hu and Li are two dealers of used tractors in a rural area of China. Hu sells high quality second hand tractors while Li sells low quality ones. Hu would be willing to sell his high quality tractor at \$8000 while Lu would sell his low quality one for \$5000, Consumers are willing to pay up to \$10,000 for a high quality tractor and \$7000 for a low quality one. They expect a 50% chance of buying a high quality second-hand tractor. In order to signal the quality of their tractors Hu and Li can offer warranties. The cost of warranty for a high quality tractor is 500Y and 1000Y for a low quality one (Y is the number of years of warranty). What is the optimal number of years of warranty that Hu should offer so that consumers know his tractors are of good quality?

    A. Less than 2 years

    B. 0.5 years

    C. More than 1.5 years

    D. 3 years

19. Suppose the capacity curve for each laborer is described as follows: for all payments up to \$100, capacity is zero and then begins to rise by 2 units for each additional \$ paid. This happens until the payment rises to \$500. Thereafter, an additional \$ payment increases work capacity by only 1.1 units, until total income paid is \$1000. At this point, additional payments have no effect on work capacity. Assume all income is spent on nutrition. Suppose you are an employer faced by the above capacity curve of your workers. You need 8000 units of work or capacity units. How many workers would you hire and how much would you pay each worker so that you get 8000 units of work at minimum cost?

    A. 5 workers: \$1000 per worket

    B. 10 workers: \$700 yer worker

    C. 10 workers; \$500 per worker

    D. 15 workers; \$400 per worleer

20. Suppose you were to believe that money illusion exists, that is as prices and income rise proportionally, then people buy more. Which of the following statements about demand should not be true?

    A. Demand functions are downward sloping

    B. Demand functions are homogeneous of degree zero

    C. Demand has a positive vertical intercept

    D. Demand functions are homogeneous of degree one

21. Consider a Bertrand price competition model between two profit maximizing widget producers, savy A and B. The marginal cost of producing a widget is 4 for each producer. Each widget producer has a capacity constraint to produce only 5 widgets. There are 8 identical individuals who demand 1 widget only, and value each widget at 6. If the firms are maximizing profits, then the following statement is true:

A. Firm A and B will charge 1

B. Firm A and B will charge 6

C. Firm A and B will change greater than or equal to 5

D. None of the other options are correct

22. The government estimates the market demand (Q_D) and market supply (Q_S) for turnips to be the following: $Q_d = 30 - 2P, Q_S = 4$ where P is the per unit price and Q is the quantity measured in kilograms. The government aims to increase the market price of turnips to \$8 per unit to improve the welfare of domestic producers of turnships. It is considering three possible choices: i) per unit subsidy, ii) a price floor and purchase of any surplus production, iii) a production quota. Which of these policies should the government adopt if it aims to mazimize the producer's welfare but minimize the loss of efficiency?

    A. A production quota

    B. A price subsidy

    C. Either a price subsidy or price floor

    D. Either a production quota or price floor

23. A monopolist faces a demand curve $q = \frac{5}{p}$. Her cost function is $C(q) = 3q$. Suppose, in the same market, there are some competitive suppliers ready to sell the good at price $p = 5$. The monopolist's profit maximizing price and output could be given by

    A. $p = 3, q = \frac{5}{3}$

    B. $p = 3.01, q = \frac{5}{3.01}$

    C. $p = 2.99, q = \frac{5}{2.99}$

    D. $p = 4.99, q = \frac{5}{4.99}$

24. The consumption function is given by $C = AY^{\beta}$ with $\beta = 0.5$ and $A = 0.3$. The marginal propensity to save is

    A. equal to 0.5

    B. increasing in income. Y

    C. equal to 0.3

    D. equal to 0.7

25. The production function is given by $Y = AL$. The wage rigidity constraint is given by $W \geq B$. The labour endowment is given by C. Here, A, B, and C are finite and positive constants. Assume that the entire labour endowment is supplied. If $A > B$, then in a labour market equilibrium

    A. $L = C$

    B. $L = 0$

    C. $0 < L < C$

D. None of the other options are correct

26. Consider the Mundell-Flemming model with perfect capital mobility and a flexible exchange rate in the short run. A monetary expansion leads to ____________in output , a fiscal expansion leads to ____________in output

    A. decrease; no change

    B. increase, decrease

    C. increase; no change

    D. increase; increase

27. Mr. X has an exogenous income W and his utility from consumption is $u(.)$. Mr. X know the an accident can occur with probability p and if it occurs, the monetary equivalent to the damage is T. Mr. X can however affect the accident probability p through the prevention effort e. In particular, e can take two values - zero and a and an assumption is that $p(0) > p(a)$, that is by putting prevention effort. probability of occurring an accident can be reduced. Let us also assume that if Mr. X puts an effort e, the disutility from the effort is Ae^2 where A is the per unit effort cost. What is the critical value of A, A^*. below which the effort will be undertaken, and above which the effort will not be undertaken, by Mr. X?

    A. $A^* = \frac{|p(a)-p(0)||u(W-T)-u(W)]}{a^2}$

    B. $A^* = \frac{|p(a)-p(0)]a^2}{u(W-T)-u(W)}$

    C. $A^* = \frac{\frac{p(a)}{p(0)}}{\frac{u(w-T)}{u(w)}a^2}$

    D. $A^* = \frac{p(a)p(0)a^2}{u(W-T)u(W)}$

28. Labor supply in macro models results from individual decision making. Let c denote an individual's consumption and L denote labor supply. Assume that individuals solve the following optimization problem

$$\underset{\{c,L\}}{\text{Max}} U(c, L) = \log c - \frac{1}{2}\frac{1}{b}L^2$$

subject to $c + \bar{S} = wL$ where $U(\cdot)$ is the utility function, $b > 0$ is a constant, $\bar{S}$ is a constant exogenous level of savings, and w is the real wage the person can earn in the labor market. Derive the optimal labor supply. It is

    A. increasing in w; increasing in c

    B. decreasing in w; decreasing in c

    C. increasing in w; decreasing in c

    D. decreasing in w; increasing in c

29. Consider a Solow economy that begins in steady state. Then a strong eathquake destroys half the capital stock. The steady state level of capital ____________, the level of output ____________on impact, and the growth rate of the economy ____________as the economy approaches its steady state.

A. decreases; decreases; decreases

B. remains the same; decreases; decreases

C. remains the same; decreases; increases

D. decreases; remains the same, decreases

30. Suppose the economy is characterized by the following equatious

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y$$

where C = Consumption, c_0 = Autonomous Consumption, $c_1 \in [0.1]$ Y_D = Disposable Income, Y = Aggregate GDP, T = Taxes, I = Investinent , b_0 = Autonomous Investment, and $b_1 \in [0, 1]$. For the multiplier to be positive, what condition needs to be satisfied?

A. $b_1 + c_1 = 0$

B. $b_1 + c_1 = 1$

C. $b_1 + c_1 < 1$

D. $b_1 + c_1 > 1$

16 ISI PEB 2013

1. An agent earns w units of wage while young, and earns nothing while old. The agent lives for two periods and consumes in both the periods. The utility function for the agent is given by $u = \log c_1 + \log c_2$, where c_i is the consumption in period $i = 1, 2$. The agent faces a constant rate of interest r (net interest rate) at which it can freely lend or borrow,
   (a) Find out the level of saving of the agent while young.
   (b) What would be the consequence of a rise in the interest rate, r on the savings of the agent?

2. Consider a city that has a number of fast food stalls selling Masala Dosa (MD). All vendors have a marginal cost of Rs. $15/-$ per MD, and can sell at most 100MD a day.
   (a) If the price of an MD is Rs. $20/-$, how much does each vendor want to sell?
   (b) If demand for MD be $d(p) = 4400 - 120p$, where p denotes price per MD, and each vendor sells exactly 100 units of MD, then how many vendors selling MD are there in the market?
   (c) Suppose that the city authorities decide to restrict the number of vendors to 20. What would be the market price of MD in that case?
   (d) If the city authorities decide to issue permits to the vendors keeping the number unchanged at 20, what is the maximum that a vendor will be willing to pay for obtaining such a permit?

3. A firm is deciding whether to hire a worker for a day at a daily wage of Rs. $20/-$. If hired, the worker can work for a maximum of 10 hours during the day. The worker can be used to produce two intermediate inputs, 1 and 2, which can then be combined to produce a final good. If the worker produces only 1, then he can produce 10 units of input 1 in an hour. However, if the worker produces only 2, then he can produce 20 units of input 2 in an hour. Denoting the levels of production of the amount produced of the intermediate goods by k_1 and k_2, the production function of the final good is given by $\sqrt{k_1 k_2}$. Let the final product be sold at the end of the day at a per unit price of Rs. $1/-$ Solve for the firms optimal hiring, production and sale decision.

4. A monopolist has contracted with the government to sell as much of its output as it likes to the government at Rs. 100/ - per unit. Its sales to the government are positive, and it also sells its output to buyers at Rs. 150/ - per unit. What is the price elasticity of demand for the monopolists services in the private market?

5. Consider the following production function with usual notations.

$$Y = K^\alpha L^{1-\alpha} - \beta K + \theta L \text{ with } 0 < \alpha < 1, \beta > 0, \theta > 0$$

   Examine the validity of the following statements.
   (a) Production function satisfies constant returns to scale.

(b) The demand function for labour is defined for all non-negative wage rates.

(c) The demand function for capital is undefined when price of capital service is zero.

6. Suppose that due to technological progress labour requirement per unit of output is halved in a Simple Keynesian model where output is proportional to the level of employment. What happens to the equilibrium level of output and the equilibrium level of employment in this case? Consider a modified Keynesian model where consumption expenditure is proportional to labour income and wage-rate is given. Does technological progress produce a different effect on the equilibrium level of output in this case?

7. A positive investment multiplier does not exist in an open economy simple Keynesian model when the entire amount of investment goods is supplied from import. Examine the validity of this statement.

8. A consumer consumes two goods, x_1 and x_2, with the following utility function

$$U(x_1, x_2) = U_1(x_1) + U_2(x_2)$$

Suppose that the income elasticity is positive. It is claimed that in the above set-up all goods are normal. Prove or disprove this claim.

9. A consumer derives his market demand, say x, for the product X as $x = 10 + \frac{m}{10p_x}$, where $m > 0$ is his money income and p_x is the price per unit of X. Suppose that initially he has money income $m = 120$, and the price of the product is $p_x = 3$. Further, the price of the product is now changed to $p'_x = 2$. Find the price effect. Then decompose price effect into substitution effect and income effect.

10. Consider an otherwise identical Solow model of economic growth where the entire income is consumed.

(a) Analyse how wage and rental rate on capital would change over time.

(b) Can the economy attain steady state equilibrium?

17 ISI PEB 2014

1. Consider a firm that can sell in the domestic market where it is a monopolist, and/or in the export market. The domestic demand is given by $p = 10 - q$, and export price is 5. Suppose the firm has a constant marginal cost of 4 and a capacity constraint on output of 100.
   (a) Solve for the optimal production plan of the firm. [**15** marks]
   (b) Solve for the optimal production plan of the firm if its constant marginal cost is 6.[5 marks]
2. (a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by
   $$-\left[(10-a)^2 + (10-b)^2\right]$$
   Suppose prices of both the goods are equal to 1.
      i. Solve for the optimal consumption of the consumer when his income is 40.[10 marks]
      ii. What happens to his optimal consumption when his income goes down to 10. [5 marks]
   (b) A monopolist faces the demand curve $q = 60 - p$ where p is measured in rupees per unit and q in thousands of units. The monopolist's total cost of production is given by $C = \frac{1}{2}q^2$
      i. What is the deadweight loss due to monopoly? [3 marks]
      ii. Suppose a government could set a price ceiling (maximum price) that the monopolist can charge. Find the price ceiling that the government should set to minimize the deadweight loss. [2 marks]
3. (a) A cinema hall has a capacity of 150 seats. The owner can offer students a discount on the price when they show their student IDs. The demand for tickets from students is
   $$D_s = 220 - 40P_s$$
   where P_s is the price of tickets for students after the discount. The demand for tickets for non-students is
   $$D_n = 140 - 20P_n$$
   where P_n is the price of tickets for non-students.
      i. What is the maximum profit the owner can make? [**8** marks]
      ii. What is the maximum profit he could make if the demand functions of students and non-students were interchanged? [4 marks]
   (b) There are 11 traders and 6 identical (indivisible) chickens. Each trader wants to consume at most one chicken. There is also a (divisible) good called "money". Let D_i equal to 1 indicate that trader i consumes a chicken; 0 if he does not. Trader $i's$ utility function is given by $u_iD_i + m_i$, where u_i is the value he attaches to consuming

a chicken, m_i is the units of money that the trader has. The valuations for the 11 traders are:

$$u_1 = 10; u_2 = 8; u_3 = 7; u_4 = 4; u_5 = 3; u_6 = 1; u_7 = u_8 = 3$$

$u_9 = 5; u_{10} = 6; u_{11} = 8$ Initially each trader is endowed with 25 units of money. Traders 6,7,8,9,10,11 are endowed with one chicken each.

i. What is a possible equilibirum market price (units of money per chicken in a competitive market? [4 marks]

ii. Is the equilibrium unique? [4 marks]

4. (a) Consider a monopolist who faces a market demand for his product:

$$p(q) = 20 - q$$

where p is the price and q is the quantity. He has a production function given by

$$q = \min\left\{\frac{L}{2}, \frac{K}{3}\right\}$$

where L denotes labour and K denotes capital. There is a physical restriction on the availability of capital, that is, $\bar{K}$. Let both wage rates (w) and rental rates (r) be equal to 1. Find the monopoloy equilibrium quantity and price when (i) when $\bar{K} = 24; (ii)\bar{K} = 18$. [12 marks]

(b) Define Samuelson's Weak Axiom of Revealed Preference (WARP). [2 marks]

(c) Prove that WARP implies non-positivity of the own-price substitution effect and the demand theorem. [**6** marks]

5. Consider two firms: 1 and 2, with their output levels denoted by q_1 and q_2. Suppose both have identical and linear cost functions, $C(q_i) = q_i$. Let the market demand function be $q = 10 - p$, where q denotes aggregate output and p the market price.

(a) Suppose the firms simultaneously decide on their output levels. Define the equilibrium in this market. Solve for the reaction functions of the two fims. Using these, find the equilibrium. marks]

(b) Suppose the firms still compete over quantities, but both have a capacity constraint at an output level of 2. Find these reaction functions and the equilibrium in this case. [10 marks]

6. (a) Suppose the government subsidizes housing expenditures of low income families by providing them a rupee-for-rupee subsidy for their expenditure. The Lal family qualifies for this subsidy. They spend Rs. 250 on housing, and receive Rs. 250 as subsidy from the government. Recently, a new policy has been proposed to replace the earlier policy. The new policy proposes to provide each low income family with a lump-sum transfer of Rs. 250, which can be used for housing or other goods.

i. Explain graphically if the Lal family would prefer the current program over the proposed program. [6 marks]

ii. Can they be indifferent between the two programs? [3 marks]

iii. Does the optimal consumption of housing and other goods change compared to the subsidy scheme? If yes, how? [3 marks]

(b) A drug company company is a monopoly supplier of Drug X which is protected by a patent. The demand for the drug is

$$p = 100 - X$$

and the monopolist's cost function is

$$C = 25 + X^2$$

i. Determine the profit maximizing price and quantity of the monopolist. [2 marks]

ii. Suppose the patent expires at a certain point in time, and after that any new drug company can enter the market and produce Drug X, facing the same cost function. What will be the competitive equilibrium industry output and price? How many firms will be there in the market? [6 marks]

7. Assume that an economy's production function is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t is output at time t, K_t is the capital stock at time t and N is the fixed level of employment (number of workers), $\alpha \in (0,1)$ is the share of output paid to capital. The evolution of the capital stock is given by

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is investment at time t and $\delta \in [0,1]$ is the depreciation rate.

(a) Derive an expression for $\frac{Y}{N}$.[5 marks]

(b) How large is the effect of an increase in the savings rate on the steady state level of output per worker. [10 marks]

(c) What is the savings rate that would maximize steady state consumption per worker? [5 marks]

8. In an IS-LM model, graphically compare the effect of an expansionary monetary policy with an expansionary fiscal policy on investment (I) in (1) the short-run and (2) the medium run (where the aggregate supply and aggregate demand curves adjust). Assume that

$$I = I(i, Y)$$

where i is the interest rate and Y is the output. Also, $\frac{\partial I}{\partial i} < 0$ and $\frac{\partial I}{\partial Y} > 0$.[15 marks] Under which policy (expansionary monetary or fiscal), is the investment higher in the medium run? [5 marks]

9. Suppose the economy is characterized by the following equations:

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y$$

where C is consumption, Y is the income, Y_D is the disposable income, T is tax, I is investment, and c_0, c_1, b_0, b_1 are positive constants with $c_1 < 1, b_1 < 1$. Government spending is constant.

(a) Solve for equilibrium output. [5 marks]

(b) What is the value of the multiplier? For the multiplier to be positive, what condition must $c_1 + b_1$ satisfy? [5 marks]

(c) How will equilibrium output be affected when b_0 is changed? What will happen to saving? [5 marks]

(d) Instead of fixed T, suppose $T = t_0 + t_1 Y$, where $t_0 > 0$ and $t_1 \in (0, 1)$. What is the effect of increase in b_0 on equilibrium Y? Is it larger or smaller than the case where taxes are autonomous? [5 marks]

10. Consider an economy where a representitive agent lives for three periods. In the first period, she is young - this is the time when she gets education. In the second period, she is middle-aged and with the level of education acquired in the first period, she generates income. More specifically, if she has h units of education in the first period, she can earn $\bar{w}h$, in the second period, where $\bar{w}$ is the exogenously given wage rate.

The agent borrows funds for her education when she is young and repays with interest when she is middle aged. If in the first period, the agent borrows e, then the human capital h at the beginning of the second period becomes $h(e)$, where $\frac{dh}{de} > 0$ along with $\frac{d^2h}{de^2} < 0$

In the third period of her life, she consumes out of her savings made in the second period, that is, when she was middle aged. Assume that the exogenous rate of interest (gross) on saving or borrowing is $\bar{R}$. For simplicity, assume that an agent does not consume when she is young and, thus, the life time utility is $u\left(c^M\right) + \beta u\left(c^O\right)$, where c^M and c^O are the level of consumption when they are middle-aged and old respectively and $\beta \in (0, 1)$ is the discount factor.

(a) Write down the utility maximization problem of the agent and the first order conditions. [10 marks]

(b) How does the optimal level of education vary with the wage rate and the rate of interest? [10 marks]

18 ISI PEB 2015

1. Consider an agent in an economy with two goods X_1 and X_2.Suppose she has income 20.Suppose also that when she consumes amounts x_1 and x_2 of the two goods respectively, she gets utility
$$u\left(x_1, x_2\right) = 2x_1 + 32x_2 - 3x_2^2$$
   (a) Suppose the prices of X_1 and X_2 are each 1. What is the agent's optimal consumption bundle? [5 marks]
   (b) Suppose the price of X_2 increases to 4, all else remaining the same. Which consumption bundle does the agent choose now? [5 marks]
   (c) How much extra income must the agent be given to compensate her for the increase in price of X_2?[**10** marks]

2. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $1/2$, whereas the inverse demand function is given by: p = 1 − q. The official charge per connection is set at 0; thus, the state provides a subsidy of $1/2$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.
   (a) Find the equilibrium bribe rate per connection and the social surplus. [5 marks]
   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them. [10 marks]
   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to c, 0 < c < $1/2$. Find the range of values of c for which privatization increases consumers' surplus. [5 marks]

3. Suppose the borders of a state, B, coincide with the circumference of a circle of radius r > 0, and its population is distributed uniformly within its borders (so that the proportion of the population living within some region of B is simply the proportion of the state's total land mass contained in that region), with total population normalized to 1.For any resident of B, the cost of travelling a distance d is kd, with k > Every resident of B is endowed with an income of 10, and is willing to spend up to this amount to consume one unit of a good, G, which is imported from outside the state at zero transport cost. The Finance Minister of B has imposed an entry tax at the rate $100t\%$ on shipments of G brought into B. Thus, a unit of G costs $p(1+t)$ inside the borders of B, but can be purchased for just p outside; $p(1+t) < 10$. Individual residents of B have to decide whether to travel beyond its borders to consume the good or to purchase it inside the

state. Individuals can travel anywhere to shop and consume, but have to return to their place of origin afterwards.

(a) Find the proportion of the population of B which will evade the entry tax by shopping outside the state. [5 marks]

(b) Find the social welfare-maximizing tax rate. Also find the necessary and sufficient conditions for it to be identical to the revenue-maximizing tax rate. [5 marks]

(c) Assume that the revenue-maximizing tax rate is initially positive. Find the elasticity of tax revenue with respect to the external price of G, supposing that the Finance Minister always chooses the revenue-maximizing tax rate. [10 marks]

4. Suppose there are two firms, 1 and 2 , each producing chocolate, at 0 marginal cost. However, one firm's product is not identical to the product of the other. The inverse demand functions are as follows:

$$p_1 = A_1 - b_{11}q_1 - b_{12}q_2, p_2 = A_2 - b_{21}q_1 - b_{22}q_2$$

where p_1 and q_1 are respectively price obtained and quantity produced by firm 1 and 2 and p_2 and q_2 are respectively price obtained and quantity produced by firm 2 . $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are all positive.Assume the firms choose independently how much to produce.

(a) How much do the two firms produce, assuming both produce positive quantities? [10 marks]

(b) What conditions on the parameters $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are together both necessary and sufficient to ensure that both firms produce positive quantities? [5 marks]

(c) Under what set of conditions on these parameters does this model reduce to the standard Cournot model? [5 marks]

5. Suppose a firm manufactures a good with labor as the only input. Its production function is $Q = L$, where Q is output and L is total labor input employed. Suppose further that the firm is a monopolist in the product market and a monopsonist in the labor market. Workers may be male (M) or female (F); thus, $L = L_M + L_F$.Let the inverse demand function for output and the supply functions for gender-specific labor be respectively $p = A - \frac{Q}{2}; L_i = w_i^{\varepsilon_i}, \varepsilon_i > 0$; where p is the price received per unit of the good and w_i is the wage the firm pays to each unit of labor of gender $i, i \in \{M, F\}$.Let $\varepsilon_M \varepsilon_F = 1$. Suppose, in equilibrium, the firm is observed to hire both M and F workers, but pay M workers double the wage rate that it pays F workers.

(a) Derive the exact numerical value of the elasticity of supply of male labor. [10 marks]

(b) What happens to total male labor income as a proportion of total labor income when the output demand parameter A increases? Prove your claim. [10 marks]

6. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good(Y) by means of a production function $Y = F(L); F' > 0, F'' < 0$, and maximizes

profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate.The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by:

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\frac{M}{P} - d(L)$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour; with $d' > 0, d'' > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \bar{M} - PT$$

where $\bar{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G.The government's budget constraint is given by:

$$M - \bar{M} = PG - PT$$

Goods market clearing implies: $Y = C + G$

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e.,, $\frac{dC}{dG} < 0$). [15 marks]

(b) Show that everything else remaining the same, a rise in $\bar{M}$ leads to an equiproportionate rise in P. [5 marks]

7. Consider the Solow growth model in continuous time, where the exogenous rate of technological progress, g, is zero. Consider an intensive form production function given by:

$$f(k) = k^4 - 6k^3 + 11k^2 - 6k$$

where $k = \frac{K}{L}$ (the capital labour ratio).

(a) Specify the assumptions made with regard to the underlying extensive form production function $F(K, L)$ in the Solow growth model, and explain which ones among these assumptions are violated by (1). [10 marks]

(b) Graphically show that, with a suitable value of $(n + \delta)$, where n is the population growth rate, and $\delta \in [0, 1]$ is the depreciation rate on capital, there exist three steady state equilibria. [5 marks]

(c) Explain which of these steady state equilibria are locally unstable, and which are locally stable. Also explain whether any of these equilibria can be globally stable. [5 marks]

8. Consider a standard Solow model in discrete time, with the law of motion of capital is given by

$$K(t + 1) = (1 - \delta)K(t) + I(t)$$

where $I(t)$ is investment at time t and $K(t)$ is the capital stock at time t; the capital stock depreciates at the rate $\delta \in [0,1]$.Suppose output, $Y(t)$, is augmented by government spending, $G(t)$, in every period, and that the economy is closed; thus:

$$Y(t) = C(t) + I(t) + G(t)$$

where $C(t)$ is consumption at time t. Imagine that government spending is given by:

$$G(t) = \sigma Y(t)$$

where $\sigma \in [0,1]$

(a) Suppose that: $C(t) = (\emptyset - \lambda\sigma)Y(t)$; where $\lambda \in [0,1]$.Derive the effect of higher government spending (in the form of higher σ) on the steady state equilibrium. [10 marks]

(b) Does a higher σ lead to a lower value of the capital stock in every period (i.e, along the entire transition path)? Prove your claim. [10 marks]

19 ISI PEB 2016

1. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.
   (a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.
   (b) What is the competitive equilibrium in this economy? Justify your answer.
   (c) Are the perfectly competitive equilibria Pareto optimal?
   (d) Now consider another economy where everything is as before, apart from individual 2 's preferences, which are as follows: (a) among any two any bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy. [6] + [6] + [2] + [6]

2. Consider a monopolist who can sell in the domestic market, as well as in the export market. In the domestic market she faces a demand $p_d = 10 - q_d$, where p_d and q_d are domestic price and demand respectively. In the export market she can sell unlimited quantities at a price of 4. Suppose the monopolist has a single plant with cost function $\frac{q^2}{4}$.
   (a) Solve for total output, domestic sale and exports of the monopolist.
   (b) Solve for the domestic and world welfare at this equilibrium. [10] + [10]

3. A consumer consumes electricity, denoted by E, and butter, denoted by B. The per unit price of B is 1. To consume electricity the consumer has to pay a fixed charge R, and a per unit price of p. If consumption of $E \leq \frac{1}{2}$ then $p = 1$; otherwise $p = 2$. The utility function of the consumer is $3E + B$, and her income is $I > R$
   (a) Draw the consumer's budget line.
   (b) If $R = 0$ and $I = 1$, find the consumer's optimal consumption of E and B.
   (c) Consider a different pricing scheme where there is a rental charge of R and the price of E is 1 for any $X \leq 1/2$, and every additional unit beyond $\frac{1}{2}$ is priced at $p = 2$. Find the optimal consumption of B and E when $R = 1$ and $I = 3$.[7] + [7] + [6]

4. A monopoly publishing house publishes a magazine, earning revenue from selling the magazine, as well as by publishing advertisements. Thus $R = q \cdot p(q) + A(q)$, where R is total revenue, q denotes quantity, $p(q)$ is the inverse demand function, and $A(q)$ is the advertising revenue. Assume that $p(q)$ is decreasing and $A(q)$ is increasing in q. The cost of production $c(q)$ is also increasing in the quantity sold. Assume all functions are twice differentiable in q.
   (a) Derive the profit-maximising outcome.

(b) Is the marginal revenue curve necessarily negatively sloped?

(c) Can the monopolist fix the price of the magazine below the marginal cost of production? [7] + [7] + [6]

5. Consider a Solow style growth model where the production function is given by

$$Y_t = A_t F(K_t, H_t)$$

where Y_t = output of the final good, K_t is the capital stock, A_t = the level of technology, and H_t = the quantity of labor used in production (the labor force). Assume technology is equal to $A_t = A_0(1+\alpha)^t$ where $\alpha > 0$ is the growth rate of technology, A_0 is the time 0 level of technology, and $H_{t+1} = (1+n)H_t$, where $n > 0$ is the labour force growth rate. The production function is homogenous of degree 1 and satisfies the usual properties. (Assume that inputs are essential and Inada conditions hold). Assume that capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is the level of investment.

(a) Define $y_t = \frac{Y_i}{H_t}$. Show that

$$y_t = A_t f(k_t)$$

where $f(k) = F(k, 1)$

(b) Define $k_t = \frac{K_t}{H_t}$ and $i_t = \frac{I_t}{H_t}$. Show that

$$k_{t+1} = \frac{(1-\delta)k_t + i_t}{1+n}$$

(c) Suppose the savings rate is given by $s_t = \sigma y_t$ where $\sigma \in [0, 1]$. Derive the condition that determines the steady state capital stock when $\alpha = 0$. How many non-zero steady states are there?

(d) Let $\gamma_t = \frac{k_{t+1}}{k_t}$ be the gross growth rate. Suppose $\alpha = 0$. Derive an expression for γ_t and evaluate and discuss the sign for $\frac{d\gamma_t}{dk_t}$

(e) Let $f(k_t) = k_t^\theta$, $A_0 = 1$, and $\alpha > 0$. Along a balanced growth path show that $\frac{k_{t+1}}{k_t}$ and $\frac{y_{c+1}}{y_t}$ grow at the same rate. [2] + [3] + [5] + [5] + [5]

6. Consider the aggregate supply curve for an economy given by

$$P_t = P_t^e(1+\mu)F(u_t, z)$$

where P_t = actual price level at time period t, P_t^e = expected prices at time t, and the function, F, given by,

$$F(u_t, z) = 1 - \alpha u_t + z$$

captures the effects of the unemployment rate (u_t) at time t and the level of unemploment benefits (z) on the price level (through their effects on wages). Assume $\mu > 0$ denotes the monopoly markup. Assume μ and z are constant.

(a) Show that the aggregate supply curve can be transformed to be written in terms of π_t (the inflation rate) and the expected inflation rate, π_t^e, i.e. $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$

(b) Now assume that $\pi_t^e = \theta \pi_{t-1}$ where $\theta > 0$. What is this equation called ? Re-write the equation in the above bullet and interpret when $\theta = 1$ and $\theta \neq 1$.

(c) Let $\pi_t^e = \pi_{t-1}$ Derive the natura rate of unemployment, and express the change in the inflation rate in terms of the natural rate. Briefly interpret this equation. (iv) How would you think about wage indexation in this model ? Does wage indexation increase the effect of unemployment on inflation? Assume $\pi_t^e = \pi_{t-1}$.[8]+[3]+[6]+[6]

7. Consider an inter-temporal choice problem in which a consumer maximises utility,

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\delta}$$

where c_i is the consumption in period $i, i = 1, 2$, and δ is the discount factor (measure of the consumer's impatience), subject to

$$c_1 + \frac{c_2}{1+\delta} = Y_1 + \frac{Y_2}{1+r} \equiv W$$

where Y_i is the consumer's income in period $i = 1, 2$, and r is the rate of interest. Assume $c_i > 1 \forall i$

(a) Let $u(c_i) = \log(c_i)$. Find a condition such that there is consumption smoothing.

(b) Plot the two cases where (a) the consumer biases its consumption towards the future, and (b) where the consumer biases it consumption towards the present. Put c_2 on the vertical axis and c_1 on the horizontal axis.

(c) Suppose there is consumption smoothing. Solve for $c_1^* = c_1(r, Y_1, Y_2)$. Interpret this equation.

(d) Define Y_P, the permanent income, as that constant stream of income (Y_P, Y_P) which gives the same lifetime income as does the fluctuating income stream (Y_1, Y_2). What does this imply about the optimal choice of c_1, c_2, and Y_P? Interpret your result graphically. [5] + [6] + [4] + [5]

8. Consider a cake of size 1 which can be divided between two individuals, A and B. Let α(resp. β) be the amount allocated to A(resp. B), where $\alpha + \beta = 1$ and $0 \leq \alpha, \beta \leq 1$ Agents A's utility function is $u_A(\alpha) = \alpha$ and that of agent B is $u_B(\beta) = \beta$

(a) What is the set of Pareto optimal allocations in this economy?

(b) Suppose A is asked to cut the cake in two parts, after which B can choose which of the two segments to pick for herself, leaving the other segment for agent A. How should A cut the cake?

(c) Suppose A is altruistic, and his utility function puts weight on what B obtains, i.e. $u_A(\alpha, \beta) = \alpha + \mu\beta$, where μ is the weight on agent B 's utility. (a) If $0 < \mu < 1$, does the answer to either 8(i) or 8 (ii) change? (b) What if $\mu > 1$?[5] + [5] + [10]

9. A firm uses four inputs to produce an output with a production function

$$f(x_1, x_2, x_3, x_4) = \min\{x_1, x_2\} + \min\{x_3, x_4\}$$

   (a) Suppose that 1 unit of output is to be produced and factor prices are 1,2,3 and 4 for x_1, x_2, x_3 and x_4 respectively. Solve for the optimal factor demands.

   (b) Derive the cost function.

   (c) What kind of returns to scale does this technology exhibit? [6] + [8] + [6]

10. Consider an IS-LM model where the sectoral demand functions are given by

$$C = 90 + 0.75Y$$
$$G = 30, I = 300 - 50r$$
$$\left(\frac{M}{P}\right)_d = 0.25Y - 62.5r, \left(\frac{M}{P}\right)_s = 500$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

   (a) Now consider an initial situation where $Y = 2500, r = 1/5$. What is the change in the level of I that must occur before there is any change in the level of Y?

   (b) Draw a graph to explain your answer.

   (c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 2, Y = 2500)$? [10] + [5] + [5]

20 ISI PEB 2017

1. A researcher has 100 hours of work which have to be allocated between two research assistants, Aditya and Gaurav. If Aditya is allocated x hours of work, his utility is $-(x-20)^2$. If Gaurav is allocated x hours of work, his utility is $-(x-30)^2$. The researcher is considering two proposals: [I] Aditya works for 60 hours and Gaurav works for 40 hours. [II] Aditya works for 90 hours and Gaurav works for 10 hours.

   Which of the following statements is correct.

   A. Proposal I is Pareto efficient but Proposal II is not.

   B. Proposal II is Pareto efficient but Proposal I is not.

   C. Both proposals are Pareto efficient.

   D. Neither proposal is Pareto efficient.

2. The industry demand curve for tea is: $Q = 1800 - 200P$. The industry exhibits constant long run average cost (ATC) at all levels of output of Rs. 1.50 per unit of output. Which market form(s) - perfect competition, pure monopoly, and first-degree price discrimination - has the highest total market (that is, producer + consumer) surplus?

   A. perfect competition

   B. pure monopoly

   C. first degree price discrimination

   D. perfect competition and first degree price discrimination

3. The following information will be used in the next question also. OIL Inc. is a monopoly in the local oil refinement market. The demand for refined oil is

   $$Q = 75 - P$$

   where P is the price in Rupees and Q is the quantity, while the marginal cost of production is

   $$\text{MC} = 0.5Q$$

   The fixed cost is zero. Pollution is emitted in the refinement of oil which generates a marginal external cost (MEC) equal to Rs. 31 per unit. What is the level of Q that maximizes social surplus?

   A. 50

   B. $29\frac{1}{3}$

   C. 17.6

   D. 44

4. Refer to the previous question. Suppose the government decides to impose a per unit pollution fee on OIL Inc. At what level should the fee (in Rs./unit) be set to produce the level of output that maximizes social surplus? You may use the fact that the marginal revenue is given by: $\text{MR} = 75 - 2Q$.

A. 1/3
B. 2
C. 3/4
D. 5/3

5. Mr. X has an exogenous income W, and his utility function from consumption is given by $U(c)$. With probability p, an accident can occur. If it occurs, the monetary equivalent of the damage is T. Mr. X can however affect the accident probability p by taking prevention effort e. In particular e can take two values: 0 and $a > 0$. Assume that $p(0) > p(a)$. Let us also assume that the utility cost of effort is Ae^2. Calculate the value of A below which effort will be undertaken.
   A. $\dfrac{[p(a) - p(0)][U(W-T) - U(W)]}{a^2}$
   B. $\dfrac{p(a) - p(0)}{U(W-T) - U(W)}$
   C. $\dfrac{p(a)p(0)a^2}{U(W-T) - U(W)}$
   D. $\dfrac{p(a)/p(0)}{U(W-T)/U(W)}a^2$

6. Suppose Mr. X maximizes inter-temporal utility for 2 periods. His total utility is given by

$$\log(c_1) + \beta \log(c_2)$$

where $\beta \in (0, 1)$ and c_1 and c_2 are his consumption in period 1 and period 2, respectively. Suppose he earns a wage only in period 1 and it is given by W. He saves for the second period on which he enjoys a gross return of $(1 + r)$ where $r > 0$ is the net interest rate. Suppose the government implements a scheme where $T \geq 0$ is collected from agents (this also from Mr. X) in the first period, and gives the same amount T back in the second period. What is the optimum T for which his total utility is maximized?
   A. $T = 0$
   B. $T = \dfrac{W}{2\beta}$
   C. $T = \dfrac{\beta W}{2(1-\beta)}$
   D. $T = \dfrac{W}{2(1-\beta)}$

7. Suppose there is one company in an economy which has a fixed supply of shares in the short run. Suppose there is new information that causes expectations of lower profits. How does this new stock market equilibrium affect final output and the final price level of the economy if you assume that autonomous consumption spending and household wealth are positively related?

A. real GDP increases; price decreases

B. real GDP decreases, price increases

C. real GDP decreases, price decreases

D. real GDP increases, price stays constant

8. A monopolist faces a demand function, $p = 10 - q$. It has two plants at its disposal. The cost of producing q_1 in the first plant is $300 + q_1^2$ if $q_1 > 0$, and 0 otherwise. The cost of producing q_2 in the second plant is $200 + q_2^2$ if $q_2 > 0$, and 0 otherwise. What are the optimal production levels in two plants?

   A. 10 units in both plants

   B. 20 units in the first plant and 10 in the second.

   C. 0 units in the first plant and 15 in the second

   D. None of the above

9. Consider a firm facing three consumers, 1, 2, and 3, with the following valuations for two goods X and Y (All consumers consume at most 1 unit of X and 1 unit of Y)

Consumer	X	Y
1	7	1
2	4	5
3	1	6

   The firm can produce both the goods at a cost of zero. Suppose the firm can supply both goods at a constant per unit price of p_X for X, and p_Y for Y. It can also supply the two goods as a bundle, for a price of p_{XY}. The optimal vector of prices (p_X, p_Y, p_{XY}) is given by

   A. (7,6,9)

   B. (4,1,4)

   C. (7,7,7)

   D. None of the above

10. Two individuals, Bishal (B) and Julie (J), discover a stream of mountain spring water. They each separately decide to bottle some of this water and sell it. For simplicity, presume that the cost of production is zero. The market demand for bottled water is given by $P = 90 - 0.25Q$, where P is price per bottle and Q is the number of bottles. What would Bishal's output Q_B, Julie's output Q_J, and the market price be if the two individuals behaved as Cournot duopolists?

   A. $Q_B = 120; Q_J = 120; P = 42$

   B. $Q_B = 90; Q_J = 90; P = 30$

   C. $Q_B = 120; Q_J = 120; P = 30$

   D. $Q_B = 100; Q_J = 120; P = 30$

11. The next three questions (11,12,13) are to be answered based on the following information: Consider the following model of a closed economy:

$$\begin{aligned}\Delta Y &= \Delta C + \Delta I + \Delta G \\ \Delta C &= c\Delta Y_d \\ \Delta Y_d &= \Delta Y - \Delta T \\ \Delta T &= t\Delta Y + \Delta T_0\end{aligned}$$

where ΔY = change in GDP, ΔC = change in consumption, ΔI = change in private investment, ΔG = change in government spending, ΔY_d = change in disposable income (i.e. after tax income), ΔT = the change in total tax collections, $t \in (0,1)$ is the tax rate, and ΔT_0 = the change in that portion of tax collections that can be altered by government fiscal policy measures. The value of the balanced budget multiplier (in terms of G and T_0) is given by:

   A. $\frac{1}{1-c(1-t)}$
   B. $\frac{-c}{1-c(1-t)}$
   C. $\frac{1-c}{1-c(1-t)}$
   D. none of the above

12. Refer to the previous question. Suppose the marginal to consumer, $c = 0.8$ and $t = 0.375$. The value of the government expenditure multiplier is

   A. 2
   B. -1.6
   C. 0.4
   D. 0.5

13. Refer to the previous question. Suppose the marginal to consumer, $c = 0.8$ and $t = 0.375$. The value of the tax multiplier (with respect to T_0) is

   A. -1.6
   B. 2
   C. 0.4
   D. 0.3

14. In the IS-LM model, a policy plan to increase national savings (public and private) without changing the level of GDP, using any combination of fiscal and monetary policy involves

   A. contractionary fiscal policy, contractionary monetary policy
   B. expansionary fiscal policy, contractionary monetary policy
   C. contractionary fiscal policy, expansionary monetary policy
   D. expansionary fiscal policy, expansionary monetary policy

15. Consider the IS-LM-BP model with flexible exchange rates but with no capital mobility. Consider an increase in the money supply. At the new equilibrium, the interest rate is ..., the exchange rate is ..., and the level of GDP is ..., respectively.

    A. higher, lower, higher

    B. lower, higher, higher

    C. lower, higher, lower

    D. higher, lower, lower

16. Consider a Solow model of an economy that is characterized by the following parameters: population growth, n; the depreciation rate, δ; the level of technology, A; and the share of capital in output, α. Per-capita consumption is given by $c = (1-s)y$ where s is the exogenous savings rate, and $y = Ak^\alpha$, where y denotes output per-capita, and k denotes the per-capita capital stock. The economy's golden rule capital stock is determined by which of the following conditions?

    A. $\frac{\partial c}{\partial k} = Ak^\alpha - (n+\delta)k = 0$

    B. $\frac{\partial c}{\partial k} = \alpha Ak^{\alpha-1} - (n+\delta) = 0$

    C. $\frac{\partial c}{\partial k} = (n+\delta)k - sAk^\alpha = 0$

    D. none of the above

17. In the Ramsey model, also known as the optimal growth model, with population growth n and an exogenous rate of growth of technological progress g, the steady state growth rates of aggregate output Y, aggregate capital K and aggregate consumption C are

    A. $0, 0, 0$

    B. $n+g, n+g, n+g$

    C. $g, n+g, n$

    D. $n+g, n+g, g$

18. Consider the standard formulation of the Philips Curve,

$$\pi_t - \pi_t^e = -\alpha(u_t - u_n)$$

where π_t is the current inflation rate, π_t^e is the expected inflation rate, α is a parameter, u_n is the natural rate of unemployment. Suppose the economy has two types of labour contracts: a proportion, λ, that are indexed to actual inflation, π_t, and a proportion, $1-\lambda$, that are not indexed and simply respond to last year's inflation, π_{t-1}. Wage indexation (relative to no indexation) will ... the effect of unemployment on inflation.

    A. strongly decrease

    B. increase

    C. not change

    D. mildly decrease

19. Consider a Harrod-Domar style growth model with a (i) Leontief aggregate production function, (ii) no technological progress, and (iii) constant savings rate. Let K and L denote the level of capital and labor employed in the economy. Output Y is produced according to $Y = \min(AK, BL)$ where A and B are positive constants. Let $\overline{L}$ be the full employment level. Under what condition will there be positive unemployment.

    A. $AK > B\overline{L}$

    B. $AK < B\overline{L}$

    C. $AK = B\overline{L}$

    D. none of the above

20. The next two questions (20 and 21) are to be answered together. People in a certain city get utility from driving their cars but each car releases k units of pollution per km driven. The net utility of each person is his or her utility from driving , v, minus the total pollution generated by everyone else. Person i's net utility is given by:

    $U_i(x_1, \ldots, x_n) = v(x_i) - \sum_{j \neq i \ \& \ j=1}^{n} kx_j$

    where x_j is km driven by person j, n is the city population, and the utility of driving v has an inverted U-shape with $v(0) = 0, \lim_{x \to 0^+} v'(x) = \infty, v''(x) < 0$, and $v(\overline{x}) = 0$ for some $\overline{x} > 0$. In an unregulated city, an increase in population will

    A. increase the km driven per person

    B. decrease the km driven per person

    C. leave the km driven per person unchanged

    D. may or may not increase the km driven per person

21. Refer to the information given in the previous question. A city planner decides to impose a tax per km driven and sets the tax rate in order to maximize the total net utility of the residents. Then, if the population increases, the optimal tax will

    A. increase

    B. decrease

    C. stay unchanged

    D. may or may not increase

22. The production function: $F(L, K) = (L + 10)^{\frac{1}{2}} K^{\frac{1}{2}}$ has

    A. increasing returns to scale

    B. constant returns to scale

    C. decreasing returns to scale

    D. none of the above

23. Consider the production functions: $F(L, K) = L^{\frac{1}{2}} K^{\frac{2}{3}}$ and $G(L, K) = LK$ where L denotes labour and K denotes capital.

A. F is consistent with the law of diminishing returns to capital but G is not

B. G is consistent with the law of diminishing returns to capital but F is not

C. Both F and G are consistent with the law of diminishing returns to capital

D. Neither F nor G is consistent with the law of diminishing returns to capital

24. A public good is one that is non-rivalrous and non-excludable. Consider a cable TV channel and a congested city street.

    A. A cable TV channel is a public good but a congested city street is not

    B. A congested city street is a public good but a cable TV channel is not

    C. Neither is a public good

    D. Both are public goods

25. Firm A's cost of producing output level $y > 0$ is $c_A(y) = 1 + y$ while firm B's cost of producing of producing output level y is $c_B(y) = y(1 - y)^2$.

    A. A can operate in a perfectly competitive industry but B cannot

    B. B can operate in a perfectly competitive industry but A cannot

    C. Neither could operate in a perfectly competitive industry

    D. Either could operate in a perfectly competitive industry

26. Suppose we generally refer to a New Keynesian model as a model with a non vertical aggregate supply (AS) curve. Under sticky prices, the AS curve will be ..., and under sticky wages, the AS curve will be ..., respectively.

    A. horizontal, upward sloping

    B. upward sloping, upward sloping

    C. downward sloping, horizontal

    D. upward sloping, horizontal

27. With perfect capital mobility, and ..., monetary policy is ... at influencing output.

    A. fixed exchange rates, effective

    B. fixed exchange rates, ineffective

    C. flexible exchange rates, ineffective

    D. none of the above are correct

28. The next three questions (28,29 and 30) use the following information. Consider an economy with two goods x and y, and two consumers, A and B, with endowments (x, y) given by $(1, 0)$ and $(0, 1)$ respectively. A's utility is $U_A(x, y) = x + 2y$ while B's utility is $U_B(x, y) = 2x + y$.

    Using an Edgeworth box with x measured on the horizontal axis and y measured on the vertical axis, with A's origin in the bottom-left corner and B's origin in the top-right corner, the set of Pareto-optimal allocations is

A. a straight line segment

B. the bottom and right edges of the box

C. the left and top edges of the box

D. none of the above

29. Referring to the information given in the previous question, the following allocations are the ones that may be achieved in some competitive equilibrium.

A. $(0, 1)$

B. The line segment joining $(0, 1/2)$ to $(0, 1)$ and the line segment joining $(0, 1)$ to $(1/2, 1)$

C. The line segment joining $(1/2, 0)$ to $(1, 0)$ and the line segment joining $(1, 0)$ to $(1, 1/2)$

D. $(1, 0)$

30. Referring to the information given in the previous two questions, if the price of y is 1, then the price of x in a competitive equilibrium

A. must be $1/2$

B. must be 1

C. must be 2

D. could be any of the above

21 ISI PEB 2018

1. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $\frac{1}{2}$, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of $\frac{1}{2}$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.
   (a) Find the equilibrium bribe rate per connection and the social surplus.
   (b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them.
   (c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to $c, 0 < c < \frac{1}{2}$. Find the range of values of c for which privatization increases consumers' surplus.

2. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.
   (a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.
   (b) What is the competitive equilibrium in this economy? Justify your answer.
   (c) Are the perfectly competitive equilibria Pareto optimal?
   (d) Now consider another economy where everything is as before, apart from individual 2 's preferences, which are as follows: (a) among any two bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy.

3. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good (Y) by means of a production function $Y = F(L), F'(L) > 0, F''(L) < 0$, and maximizes profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate. The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by

$$U = \frac{1}{2} \ln C + \frac{1}{2} \ln\left(\frac{M}{P}\right) - d(L)$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour with $d'(L) > 0, d''(L) > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \bar{M} - PT$$

where $\bar{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G. The government's budget constraint is given by:

$$M - \bar{M} = PG - PT$$

Goods market clearing implies $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$).

(b) Show that everything else remaining the same, a rise in $\bar{M}$ leads to an equiproportionate rise in P.

4. Consider an IS-LM model where the sectoral demand functions are given by

$$\begin{aligned} C &= 90 + 0.75Y \\ G &= 30 \\ I &= 300 - 50r \\ \left(\frac{M}{P}\right)_d &= 0.25Y - 62.5r \\ \left(\frac{M}{P}\right)_s &= 500 \end{aligned}$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Consider an initial situation where $Y = 2500, r = \frac{1}{5}$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 0.2, Y = 2500)$?

5. Answer the following questions.

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \frac{|x|}{2x} \forall x \in \mathbb{R} \backslash \{0\}$$

Can $f(0)$ be defined in a way such that f is continuous at 0? Justify your answer.

(b) Consider the following optimization problem:

$$\max_{x\in[0,\beta]} x(1-x)$$

where $\beta \in [0,1]$. Let x^* be an optimal solution of the above optimization problem. For what values of β will we have $x^* = \beta$?

(c) A firm is producing two products a and b. The market price (per unit) of a and b are respectively 3 and 2 . The firm has resources to produce only 10 units of a and b together. Also, the quantity of a produced cannot exceed double the quantity of b produced. What is the revenue-maximizing production plan (i.e., how many units of a and b) of the firm?

6. Answer the following questions.

(a) A slip of paper is given to person A, who marks it with either $(+)$ or $(-)$. The probability of her writing $(+)$ is $\frac{1}{3}$. Then, the slip is passed sequentially to B, C, and D. Each of them either changes the sign on the slip with probability $\frac{2}{3}$ or leaves it as it is with probability $\frac{1}{3}$.

i. Compute the probability that the final sign is $(+)$ if A wrote $(+)$.

ii. Compute the probability that the final sign is $(+)$ if A wrote $(-)$.

iii. Compute the probability that A wrote $(+)$ if the final sign is $(+)$.

(b) There are n houses on a street numbered $h_1, \ldots, h_n$. Each house can either be painted BLUE or RED.

i. How many ways can the houses $h_1, \ldots, h_n$ be painted?

ii. Suppose $n \geq 4$ and the houses are situated on n points on a circle. There is an additional constraint on painting the houses: exactly two houses need to be painted BLUE and they cannot be next to each other. How many ways can the houses $h_1, \ldots, h_n$ be painted under this new constraint?

iii. How will your answer to the previous question change if the house are located on n points on a line.

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1. Ms. A earns Rs. 25,000 in period 1 and Rs. 15,000 in period 2. Mr. B earns Rs. 15,000 in period 1 and Rs. 30,000 in period 2 . They can borrow money at an interest rate of 200%, and can lend money at a rate of 0%. They like both consumption in period 1 (C_1) and consumption in period 2 (C_2), and their preferences are such that their chosen consumption bundles will always lie on their budget lines.

   (a) Write down the equations of their budget constraints and draw their budget lines in the same figure by plotting consumption in period 1 (C_1) (in thousand rupees) on x -axis and consumption in period 2 (C_2) (in thousand rupees) on y -axis.

   (b) Given the income profile and the market interest rates, Mr. B chooses to borrow Rs. 5,000 in period 1. Give an example of a consumption profile (that is, (C_1, C_2)) such that, if Ms. A chooses this profile, we would know for sure that Ms. A and Mr. B have different preferences for consumption in period 1 (C_1) and consumption in period 2 (C_2). Give a clear explanation for your answer.

   (c) Suppose now that Ms. A and Mr. B have the same preferences for C_1 and C_2, and, as in part (b), Mr.B borrows Rs. 5,000 in period 1

      i. Suppose that Ms. A chooses to be a lender in period one. Find out, with a clear explanation, the maximum amount that she will lend in period 1 consistent with the fact that they have the same preferences for C_1 and C_2

      ii. Explain clearly whether Mr.B is better off than Ms. A.

2. Consider an exchange economy with two agents 1 and 2 and two goods X and Y. There is one unit of both goods in the economy. An allocation is a pair $\{(X_1, Y_1), (X_2, Y_2)\}$ where $X_1+X_2 = 1, Y_1+Y_2 = 1$, and (X_1, Y_1) and (X_2, Y_2) are the consumption bundles of agents 1 and 2 respectively. The utility function for agent 1 is given by $u_1(X_1, Y_1) = X_1 \cdot Y_1$ and that of agent 2 by $u_2(X_2, Y_2) = 2X_2 + Y_2$

   (a) Describe the set of Pareto-efficient allocations in the economy.

   (b) An allocation $\{(X_1, Y_1), (X_2, Y_2)\}$ is envy-free if no agent strictly prefers the consumption bundle of the other agent to her own, that is, $u_1(X_1, Y_1) \geq u_1(X_2, Y_2)$ and $u_2(X_2, Y_2) \geq u_2(X_1, Y_1)$

      i. Consider each of the two statements below. Decide whether they are true or false. Justify your answer with a proof or a counter-example as appropriate.

         α) All Pareto-efficient allocations are envy-free.

         β) All envy-free allocations are Pareto-efficient.

      ii. Describe the set of envy-free allocations in the economy.

   (c) Suppose each agent has an endowment of half-unit of each good. Prove without direct computation that the competitive equilibrium allocation is both Pareto-efficient and envy-free.

3. Two flat-mates, 1 and 2, rent a flat and play their own music on the only CD player owned by the flat-owner. They both like their own music, but dislike the music played by the other person. Given the timing constraints, each one must play her own music

when the other person is also present. Let m_i denote the amount of music played by i, and Y_i denote her amount of money holding. Individual i 's utility function is

$$u_i\left(m_1, m_2, Y_i\right) = 8m_i - 2m_i^2 - \frac{3}{2}m_j^2 + Y_i, \quad i, j = 1, 2, i \neq j$$

(a) How much music would each individual play? What is the efficient amount of music for each individual? Is the amount of music actually played more or less than the efficient level? Explain the economic intuition for your answer.

(b) Suppose that individual 2 is considering to gift a headphone to her flat-mate on her birthday. Assume that she does not get any utility from just gift-giving. What is the maximum price she is willing to pay for the headphone?

(c) Suppose that the price of the headphone is Rs. 11. Does it make sense for the two flat-mates to jointly buy a headphone, sharing the price equally, and making a binding commitment that they would each listen to their own music only via the headphone?

(d) Now suppose that the CD player is owned by individual 1 so that she can prevent individual 2 from playing any music at all. Suppose individual 1 can offer a take-it-or-leave-it contract that looks like the following: "I shall play music at a level $\bar{m}_1$, and you can play music at the level $\bar{m}_2$ in return for a sum of Rs. T." In case the offered contract is rejected, individual 1 selects m_1 unilaterally, and individual 2 cannot play any music of her choice. Solve for the optimal levels of $\bar{m}_1, \bar{m}_2$ and T Discuss the economic intuition for your answer.

4. Consider a country where there are only two provinces $-A$ and B. The production function to produce a single output Y is given by $Y = F\left(N^A + N^B\right)$ where F is a concave function and N^i represents employees from province $i, i = A, B$. Wages paid to the employees are given by $W^i, i = A, B$. Price of the final good Y is denoted by P. The employers are price takers and take P, W^A and W^B as given.

   (a) Write down the expression for an employer's profit as a function of N^A and $N^B, \pi\left(N^A, N^B\right)$

   (b) An employer chooses N^A and N^B to maximize

   $$u\left(N^A, N^B\right) = u\left(\pi\left(N^A, N^B\right), N^A, N^B\right)$$

   where $\frac{\partial u}{\partial \pi} > 0, \frac{\partial u}{\partial NA} > 0$ and $\frac{\partial u}{\partial N^d} < 0$. The last two conditions on $u\left(N^A, N^B\right)$ imply that the employer prefers employees from province A but dislikes employees from province B Write down the first order conditions for the employer's maximization problem assuming an interior solution.

   (c) In equilibrium do the employees from different provinces get the same wage? If yes, explain your answer. If not, then determine, with a clear explanation, which employees are paid more and by how much.

5. Consider a concave utility function $u(c, l)$ where c represents consumption good and l represents labour supply (working hours, to be precise). While utility increases with the level of consumption good, increasing working hours reduces utility. Wage per hour

of labour is given by w, thus working for l hours will ensure wl amount of total wage which is denoted by y, that is, $y = wl$. Given this, the utility function can be written as $u\left(c, \frac{y}{u}\right)$. The price of the consumption good c is given by p. Also $\bar{L}$ is a fixed number of hours representing total time available to an agent and $\bar{L} - l$ represents leisure. [In all the figures you are asked to draw below, plot y on x -axis and c on y -axis.]

(a) Derive the slope of an indifference curve for the utility function $u\left(c, \frac{y}{w}\right)$ on the $y-c$ plane.

(b) Demonstrate the agent's utility maximizing choice of y and c in a figure by plotting her budget line and indifference curves for the utility function $u\left(c, \frac{y}{w}\right)$

(c) Experiment 1: Suppose there is an increase in w. Demon strate the agent's new utility maximizing choice of y and c in the same figure as in part b. [Show clearly how the agent's budget line and/or indifference curves change as a result of the increase in w.] Compare the old and new choices with a brief economic explanation.

(d) Experiment 2 : Suppose, instead of an increase in w, there is a tax imposed on income. That is, the after-tax income of the agent is $(1-\tau)y$ where τ is the proportional tax rate. In a new figure demonstrate the agent's new as well as old (as in part b) utility maximizing choices of y and c. [Show clearly how the agent's budget line and/or indifference curves change as a result of this proportional tax.] Compare the old and new choices with a brief economic explanation.

6. Consider an agent who lives for three periods but consumes only in periods two and three where the consumptions are denoted by c_2 and c_3 respectively. Her utility is given by $u(c_2, c_3) = \log(c_2) + \beta \log(c_3)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. She invests an amount e in education in the first period which she borrows from the market at a given interest rate $r > 0$. Her income in the second period is $w \cdot h(e)$ where w is a fixed wage rate per unit of human capital and $h(e)$ is the amount of human capital that results from investment in education (e) in the first period. Assume that $h(e)$ is an increasing and concave function of e. The agent repays her education loan in the second period. She has no income in the third period. But she can save (s) in the second period from her income on which she receives the return $s(1+r)$ in the third period to meet her consumption expenditure.

(a) Write down the agent's period 2 and period 3 budget con straints separately.

(b) Set up the agent's utility maximization problem by showing her choice variables clearly.

(c) Write down the first order conditions for the agent's utility maximization problem.

(d) Derive the ratio of consumptions in period 2 and period 3 ,

(e) Explain how investment in education, e, depends on the preference parameter β

7. Consider a street represented by the interval $[0,1]$. Three agents, $\{1,2,3\}$, live on this street. Agent $i \in \{1,2,3\}$ lives at $x_i \in [0,1]$ and assume that $x_1 \leq x_2 \leq x_3$. Suppose we locate a hospital at a point $p \in [0,1]$

(a) We say p is square-optimal if it minimizes $\sum_{i=1}^{3}(x_i - p)^2$ Derive the square optimal value of p

(b) We say p is absolute-optimal if it minimizes $\sum_{i=1}^{3}|x_i - p|$

i. Argue that if p is absolute-optimal, then $p \in [x_1, x_3]$

ii. Use this to derive an absolute-optimal p.

(c) Now suppose that n agents, $\{1, 2, 3, \ldots, n\}$, live on this street where $x_1 \leq x_2 \leq x_3 \ldots \leq x_n$ and n is an odd number. Derive an absolute-optimal p

8. Two random variables x_1 and x_2 are uniformly drawn from [0,1] Define the following function:

$$G(p) = p \times \text{ Probability } [p \geq \max(x_1, x_2)] \quad \forall p \in [0,1]$$

(a) Derive, with a clear explanation, the expression for $G(p)$.

(b) Plot $G(p)$

(c) Is G convex or concave in p? Give clear explanations for your answer.

(d) Find $\max_{p \in [0,1]} G(p)$

9. Let $X \subset \mathbb{R}$ and $f : X \to X$ be a continuous function.

(a) Suppose $X = [0, 1]$. By using the Intermediate Value Theorem, show that there exists $x^* \in X$ such that $f(x^*) = x^*$.

(b) In each of the cases below, determine whether there exists $x^* \in X$ such that $f(x^*) = x^*$. Justify your claim by either providing a proof or a counter-example.

i. $X = (0, 1)$ and f is continuous.

ii. $X = [0, 1] \cup [2, 3]$ and f is continuous.

iii. $X = [0, 1]$ but f is not continuous.

(c) Let $f_i : [0, 1] \to [0, 1], i = 1, 2, \ldots, m$, be a collection of m continuous functions. Prove that there exists $x^* \in [0, 1]$ such that $\sum_{i=1}^{m} f_i(x^*) = mx^*$

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1. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with a uniform distribution on $(0, \theta]$ where $\theta > 0$.
   (a) Write down the joint probability density function of $X_1, \ldots X_n$
   (b) Suppose x_i is a realization of X_i, for each $i = 1, \ldots, n$ And suppose the value of θ is unknown. Find the value of θ that maximizes the joint p.d.f. in part (a) given that $x_1, \ldots, x_n$ have been observed. (This is called the maximum likelihood estimate of θ.)
   (c) Consider the function: $f(x, y) = x^2 + y^2 - 2x$
      i. Find the maximum value of f over the region $\{(x, y) \mid 2x^2 + 3y^2 - 2x \leq 100\}$
      ii. Find the minimum value of f over the region $\{(x, y) \mid 2x^2 + 3y^2 - 2x \geq 100\}$

2. A tournament consists of n players and all possible $C(n, 2) = \frac{n(n-1)}{2}$ pairwise matches between them. There are no ties in a match: in any match, one of the two players wins. The score of a player is the number of matches she wins out of all her $(n-1)$ matches in the tournament. Denote the score vector of the tournament as $s \equiv (s_1, \ldots, s_n)$ and assume without loss of generality $s_1 \geq s_2 \geq \ldots \geq s_n$
   (a) For any $2 \leq k \leq n$, show that $s_1 + \ldots + s_k \geq C(k, 2)$, where $C(k, 2) = \frac{k(k-1)}{2}$
   (b) Suppose $n > 3$ and players 1,2,3 win every match against players in $\{4, \ldots, n\}$. Find the value of $s_4 + \ldots + s_n$?
   (c) Suppose $s_n = s_0, s_{n-1} = s_0 + 1, s_{n-2} = s_0 + 2$ for some positive integer s_0 and $n \geq 3$. Show that
   $$s_0 \leq \frac{(n-2)(n-3)}{2n}$$
   (d) A tournament generates a score vector s such that
   $$s_j - s_{j+1} = 1 \text{ for all } j \in \{1, \ldots, n-1\}$$
   What is the score vector of this tournament? For every Player j, who does Player j beat in this tournament?
   (e) Suppose there are six players, i.e., $n = 6$. There is a tournament such that each player has a score of at least two and difference in scores of any two players is not more than one. What is the score vector of this tournament? Construct a tournament (describing who beats who) which generates this score vector.

3. Consider the following equation in x
   $$(x-1)(x-2)\cdots(x-n) = k$$
   where $n > 1$ is a positive integer and k is a real number. Argue whether the following statements are true or false by providing a proof or a counter example.
   (a) Suppose $n = 2$. There is a real solution to Equation (1) for every value of k

(b) Suppose $n = 3$. There is a real solution to Equation (1) for every value of k.

(c) For all $k \geq 0$ and for every positive integer $n > 1$, there is a real solution to Equation (1).

(d) For all $k < 0$ and for every odd positive integer $n > 1$, there is a real solution to Equation (1)

(e) For all $k < 0$, there is some even positive integer n such that a real solution to Equation (1) exists.

4. Consider an economy inhabited by identical agents of size 1. A representative agent's preference over consumption (c) and labour supply (l) is given by the utility function

$$u(c, l) = c^{\alpha}(24 - l)^{1-\alpha}, 0 < \alpha < 1$$

Production of the consumption good c is given by the production function $c = Al$, where $A > 0$ is the productivity of labour. Both the commodity market and labour market are perfectly competitive: the buyers and sellers take the price as given while taking demand and supply decisions. Let us denote the hourly wage rate by $w > 0$ and price of the consumption good by $p > 0$

(a) Competitive Equilibrium: A competitive equilibrium is given by the allocation of consumption and labour, $\left(c^{CE}, l^{CE}\right)$, and the relative price ratio, $\frac{w}{p}$, such that, given w and p, a representative agent decides her labour supply, l^S, and consumption demand, c^D, to maximize her utility; a firm decides its labour demand, l^D, and supply of consumption good, c^S, to maximize its profit; and, finally, both the commodity market and labour market clear, that is, $l^D = l^S$ and $c^D = c^S$

i. Set up the representative agent's utility maximization problem. Write down the first order conditions for this maximization problem and determine l^S and c^D as functions of w and p

ii. Set up a firm's profit maximization problem. Determine l^D and c^S as functions of w and p

iii. Determine the competitive equilibrium allocation, $\left(c^{CE}, l^{CE}\right)$ and the relative price ratio, $\frac{w}{p}$.

(b) Pareto efficient allocation: For this economy define the concept of a Pareto efficient allocation of consumption and labour. Find out a Pareto efficient allocation of consumption and labour in this economy. Provide a clear explanation.

5. (a) Ms. A's income consists of Rs. 1,00,000 per year from pension plus the earnings from whatever she sells of the 2.000 kilograms of rice she harvests annually from her farm. She spends this income on rice (x) and on all other expenses (y). All other expenses are measured in Rupees, so that the price of y is Rs. 1. Last year rice was sold for Rs. 20 per kilogram, and Ms. A's rice consumption was 2,000 kilograms, just the amount produced on her farm. This year the price of rice is Rs. 30 per kilogram. Ms. A has standard convex preferences over rice and all other expenses. Answer the following two questions without referring to any utility function or indifference curves.

i. What will happen to her rice consumption this year increase, decrease, or remain the same? Give a clear explanation for your answer.

ii. Will she be better or worse off this year compared to last year? Explain clearly.

(b) There are two goods a and y. Mr. B has standard convex preferences over the two goods. He has endowments of $e_x > 0$ units of good x and $e_y > 0$ units of good y. He does not have any other source of income. When the price of good y is Rs. 1 and the price of good x is Rs. p_x he decides neither to buy nor to sell good x.

i. Suppose that, for good x, the prices have become Rs. $p_L < p_x$ if an individual is a seller and Rs. $p_H > p_x$ if an individual is a buyer. The price of good y remains Rs. 1 no matter whether an individual buys or sells good y. Write down the equation of the new budget constraint and draw it labelling the important points clearly.

ii. Will Mr. B buy or sell good x ? By how much? Give a clear explanation for your answer without referring to any utility function or indifference curves.

6. Consider a moneylender who faces two types of potential borrowers: the safe type and the risky type. Each type of borrower needs a loan of the same size L to invest in some project. The borrower can repay only if the investment provides sufficient returns to cover the repayment. Suppose the safe type is always able to secure return R from the investment, where $R > L$. On the other hand, the risky type is an uncertain prospect, he can obtain a higher return R' (where $R' > R$) but only with probability p. With probability $1 - p$, the investment backfires and he gets a return of zero. The money lender has enough funds to lend to just one applicant and there are two of them, one risky and one safe. Each borrower knows his own type, but the moneylender does not know the borrower's type. He just knows that one is safe and the other is risky. Since the moneylender has enough funds to lend to just one applicant, when both the borrowers apply for the loan, he gives the loan randomly to one of them say by tossing a coin. Assume that the lender supplies the loan from his own resources and his opportunity cost is zero.

(a) What is the highest interest rate, call it i_s, for which the safe borrower wants the loan? What is the highest interest rate, i_r, for which the risky borrower wants the loan? Who is willing to pay a higher interest rate, the risky borrower or the safe borrower?

(b) The lender's objective is to maximize his expected profit. Argue clearly that the lender's effective choice is between two interest rates, i_s and i_r (That is, argue that the lender will not choose any interest rate strictiy lower than min $\{i_r, i_s\}$, any interest rate strictly higher than max $\{i_r, i_n\}$, or any interest rate strictly in between i_r and i_n.)

(c) Argue that when the lender charges i_r, his expected profit is given by $p(1 + i_r)L - L$. Derive, with a clear argument, the expression of lender's expected profit when he charges i_s

(d) An equilibrium with credit rationing occurs when, at the equilibrium interest rate, some borrowers who want to obtain loans are unable to do so; however, lenders also not raise the interest rate to eliminate the excess demand. Explain clearly that we have an equilibrium with credit rationing when

$$p < \frac{R}{2R' - R}$$

7. Consider an economy where identical agents (of mass 1) live for two periods: youth (period 1) and old age (period 2). The utility function of a representative agent born at time t is given by

$$u\left(c_{1,t}, c_{2,t+1}\right) = \log\left(c_{1,t}\right) + \beta \log\left(c_{2,t+1}\right)$$

where c_1 denotes consumption in youth, c_2 denotes consumption in old age, and $0 < \beta < 1$ is the discount factor reflecting her time preference. In her youth the representative agent supplies her endowment of 1 unit of labour inelastically and receives the market-determined wage rate w_1. So in her youth the agent faces the budget constraint $c_{1,t} + s_t = w_0$ where s_t denotes her savings. When old, she just consumes her savings from youth plus the interest earning on her savings, $s_t r_{t+1}$, where r_{t+1} is the market-determined interest rate in period $t + 1$. That is, when old, her budget constraint is $c_{2,t+1} = \left(1 + r_{t+1}\right) s_t$

   (a) Set up the agent's utility maximization problem by showing her choice variables clearly.

   (b) Write down the first order conditions for this maximization problem and derive the savings function. Explain how savings. s_t. if it does, depends on the interest rate r_{t+1}

   The production function of the economy is given by $Y_t = AK_t^\alpha L_t^{1-\alpha}$ $0 < \alpha < 1$, where K and L denote the amounts of capital and labour in the economy, respectively. Capital depreciates fully after use, that is. the rate of depreciation of capital is one. Factor markets being competitive, the equilibrium factor prices are given by their respective marginal products.

   (c) Derive the equilibrium wage rate (w_t) of the economy in terms of K_t.[Keep in mind that the mass of agents is 1 and each agent supplies her endowment of 1 unit of labour inelastically.]

   The role of the financial sector (banks, stock market, and so on) is to mobilize the savings of households to bring it for effective use by the production sector. But the financial sector does not work well and a fraction $0 < \theta < 1$ of aggregate savings gets lost (vanishes in thin air) in the process of intermediation.

   (d) Derive the law of motion of capital (that is. express capital in period $t + 1, K_{t+1}$, in terms of capital in period t, K_t)

   (e) Derive the steady state anount of capital of the economy.

   (f) How does the steady state amount of capital depend on the inefficiency of the financial sector θ?

8. Consider a Solow-Swan model with learning by doing. Assume that the production function is of the form

$$Y_t = K_t^\alpha \left(A_t L_t\right)^{1-\alpha}$$

where A is the level of technological progress and grows at the rate $g > 0, L$ is the population with grows at the rate $n > 0, K$ is the capital stock, Y is GDP, and $\alpha \in [0, 1]$. Assume that

$$\dot{K} = sY - \delta K$$

Define $Z = \frac{K}{AL}$ as the capital labor ratio in efficiency units. Let output per worker be given by $Q = AZ^{\alpha}$. The parameter $s \in [0, 1]$ denotes the savings rate. The parameter $\delta \in [0, 1]$ denotes the depreciation on capital.

(a) Derive an expression for $\frac{\dot{Z}}{2}$

(b) Instead of assuming that the rate of technological progress is constant (g), now assume that the instantaneous increase in A is proportional to output per worker, i.e., there is learning by doing

$$\dot{A} = \gamma Q$$

Show that the law of motion of capital is given by

$$\dot{Z} = (s - \gamma Z)Z^{\alpha} - (\delta + n)Z$$

(c) Draw a diagram describing the dynamics of growth in the model with learning by doing. Plot Z on the $x-$ axis, and the appropriate functions on the $y-$ axis

(d) In contrast to the model with no learning by doing, does an increase in the investment rate raise the balanced-growth rate? What does this tell you about the change in policy having level effects versus growth effects in the with learning by doing in contrast to the model when there is no learning by doing? Show your answer using the diagram in part (c).

9. Suppose households who live till T periods maximize $\sum_{n=1}^{T} \beta^{n-1} \ln(c_n)$ Where c_n represents their income in period $n = t, t+1, 1+2 \ldots T$ and β is a parameter with $0 < \beta < 1$. Suppose per period income and the saving of households are y_n and s_n respectively and the activity starts from the beginning of their life t. Further, the net interest rate on saving in between any two periods is exogenously fixed at r and so the gross rate of return is $1 + r$. Households have only two activities in every period - consuming and saving.

(a) Write down the sequence of budget constraints fone for each period) and the aggregate budget constraint derived from these periodic budget constraints where on the left hand side, consumption levels for all the periods appear, and on the right hand side, income in all periods appears.

(b) Under what condition between β and r, is the optimal solution for the above problem yield constant consumption, $\widetilde{C}$, in every period?

(c) Suppose the condition that you derive in (b) holds. Then answer the following questions in (c) and (d)

i. For a transitory change in income in period t only, calculate the change in the constant level of consumption, $\tilde{C}$.

ii. For a transitory change in income in period $t + k$ only; calculate the change in the constant level of consumption, $\widetilde{C}$.

(d) For a permanent change in income (assume the same amount of income change in all periods), calculate the change in the constant fevel of consumption, $\tilde{C}$. Compare this value derived with part c (i).