QUIZ 23

1. Use series to evaluate

$$\lim_{x\to\infty} x^2(e^{-\frac{3}{x^2}} - 1) = ?$$

Sol) The Taylor series of e^x is given by

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Thus, by replacing x by $-\frac{3}{x^2}$,

$$e^{-\frac{3}{x^2}} = \sum_{n=0}^{\infty} \frac{1}{n!}(-\frac{3}{x^2})^n.$$

Thus,

$$x^2(e^{-\frac{3}{x^2}}) - 1 = x^2 \sum_{n=0}^{\infty} \frac{1}{n!}\frac{(-3)^n}{x^{2n}} - 1 = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{(-3)^n}{x^{2n}} x^2 - 1 = \sum_{n=1}^{\infty} \frac{1}{n!}\frac{(-3)^n}{x^{2(n-1)}}$$

By letting x go to ∞,

$$\lim_{x\to\infty} x^2(e^{-\frac{3}{x^2}} - 1) = \lim_{x\to\infty} \sum_{n=1}^{\infty} \frac{1}{n!}\frac{(-3)^n}{x^{2(n-1)}} = -3.$$

Last equality comes from the fact that every term following the second term goes to 0 as $x \to \infty$.

2. Convert the following equation to Cartesian coordinates. Describe the resulting curve.

$$r = \frac{9}{4\cos\theta + 3\sin\theta}$$

Sol) By multiplying $4\cos\theta + 3\sin\theta$,

$$(4r\cos\theta + 3r\sin\theta) = 9. \tag{0.1}$$

we know that in Cartesian coordinates,

$$x = r\cos\theta,\ y = r\sin\theta.$$

Hence, (0.1) is turned into

$$4x + 3y = 9.$$