Quiz 21 Solutions

April 17, 2020

1. Find the power series representation for $\frac{x}{x^2-1}$ centered at 0 and use this to determine the power series representation for $\ln(x^2-1)$.

Sketch of Solution: Observe that $\frac{1}{x^2-1} = \frac{1}{1-(2-x^2)}$ so, the power series representation of $\frac{1}{x^2-1} = \sum_{k=0}^{\infty} \frac{(x^2-1)^k}{k!}$. Now, notice that the derivative of $\ln(x^2-1)$ is $\frac{2x}{x^2-1}$

2. We know the power series representation for e^x centered at 0 is $\sum_{k=0}^{\infty} \frac{x^k}{k!}$. Using this, determine the power series representation for $2xe^x$ centered at 0 and determine the radius of convergence.

Sketch of Solution: Because $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, we know that $2xe^x = 2x \cdot \sum_{k=0}^{\infty} \frac{x^k}{k!} = 2 \cdot \sum_{k=0}^{\infty} \frac{x^{k+1}}{k!}$. Since $2xe^x$ exists for any value of x, we know that the interval of convergence for $2xe^x$ is $(-\infty, \infty)$. Therefore, the radius of convergence is ∞.

3. Determine the radius of convergence for the following power series representation: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

Sketch of Solution: Similar to 2