Quiz 22 Solution

1. First, calculate the derivative of several times: $(f(x) = x\cos x + 2\sin 2x)$

   $$f'(x) = \cos x - x\sin x + 2\cos 2x$$
   $$f''(x) = -2\sin x - x\cos x - 4\sin 2x$$
   $$f^{(3)}(x) = -3\cos x + x\sin x - 8\cos 2x$$
   $$f^{(4)}(x) = 4\sin x + \cos x + 16\sin 2x$$

   Then plug in $x = 0$, you can observe derivatives of even times vanish. Derivatives of odd times ($n = 2k+1$), well, there are two situations.

   When k is even, $k = 2p$ (eg: $n = 1$), $f^{(2k+1)}(0) = (2k+1) + 2^{2k+1}$

   When k is odd, $k = 2p+1$ (eg: $n = 3$), $f^{(2k+1)}(0) = -(2k+1) - 2^{2k+1}$

   Then we conclude $f^{(2k+1)}(0) = (-1)^k(2k+1+2^{2k+1})$, hence the Maclaurin series is $\sum_{k=0}^{\infty}(-1)^k \frac{2k+1+2^{2k+1}}{(2k+1)!}x^{2k+1}$

   (PS: Generally, it is possible to find out $f^{(n)}(x)$ without plugging in $x = 0$, but that costs time in an exam and induction is needed.)

2. First, calculate the derivative of several times: $(f(x) = \int \ln(1+2x)dx)$

   $$f'(x) = \ln(1+2x)$$
   $$f''(x) = 2(1+2x)^{-1}$$
   $$f^{(3)}(x) = -2^2(1+2x)^{-2}$$
   $$f^{(4)}(x) = 2\cdot 2^3(1+2x)^{-3}$$
   $$f^{(5)}(x) = -2\cdot 3\cdot 2^4(1+2x)^{-4}$$

   Then plug in $x = 0$. There are two things can be observed. First, derivatives don't vanish except $n = 1$. Second, derivatives of even times are positive and derivatives of odd times are negative.

   Then we conclude $f^{(n+2)}(0) = (-1)^n 2^{n+1} n!$ and the Maclaurin series is

   $$\sum_{n=0}^{\infty}(-1)^n \frac{2^{n+1}}{(n+1)(n+2)}x^{n+2}$$