1. If we want to approximate
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$
by a partial sum, how many terms are needed to get an error below 10^{-6}?

   We know that the error with n terms is less than or equal to a_{n+1}, where $a_n = 1/n^3$. When $n = 100$, we get $a_n = 10^{-6}$; that means that if we took 99 terms, we'd get an error less than or equal to 10^{-6}. So, the "official" answer is that you need 99 terms.

   If you want to read the question with "below" meaning "strictly less than", then you would argue that you need 100 terms instead of 99.

   If that's a detail you're asking yourself on an exam, be sure to say something about that. If you're asking it in real life, you should consider the broader context of the problem.

2. Label each of the following as "absolutely convergent," "conditionally convergent but not absolutely convergent," or "divergent." (All sums are from $n = 1$ to ∞.)

   (a) $\sum(-1)^n$ - Diverges by divergence test.
   (b) $\sum(-1)^n/n$ - Conditionally convergent, not absolutely convergent.
   (c) $\sum(-1)^n/(n^2+n)$ - Absolutely converges, by comparison with $\sum 1/n^2$
   (d) $\sum(-1)^n/\ln(n)$ - Conditionally convergent, not absolutely convergent.
   (e) $\sum(-1)^n e^{-n}$ - Absolutely convergent (geometric series, with $r = 1/e$.)

3. Consider the series
$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$$
A classmate makes the argument that the series converges: "This is an alternating series, and the terms of that sequence converge to 0. Thus, by the alternating series test, the series converges."

   You write a program to find the first 50 terms of the series, and the series appears to converge. Yet, when you make this argument to your TA, they stop, think for a moment, and say the following:

“The series does converge. However...”

“your classmate hasn’t explained that $\sin(1/n)$ is monotonically decreasing. That’s not particularly obvious, even if it’s true. You can check this by taking a derivative, for example.”