Quiz 15

Remember to attempt the quiz in the normal testing environment. This quiz should take 10 minutes. Don't forget to show your work for yourself. Make sure you read the questions carefully.

Question 1

Determine if the following infinite series converges. If it converges, then find its sum.

$$\sum_{k=1}^{\infty} \frac{5+3^k}{4^k}$$

Solution 1

We should split the fraction and the sum apart, then use power series formulas

$$\sum_{k=1}^{\infty} \frac{5+3^k}{4^k}$$
$$= \sum_{k=1}^{\infty} \frac{5}{4^k} + \frac{3^k}{4^k}$$
$$= \sum_{k=1}^{\infty} \frac{5}{4^k} + \sum_{k=1}^{\infty} \frac{3^k}{4^k}$$
$$= 5\sum_{k=1}^{\infty} (\frac{1}{4})^k + \sum_{k=1}^{\infty} (\frac{3}{4})^k$$
$$= 5\frac{1/4}{1-1/4} + \frac{3/4}{1-3/4} = \frac{5/4}{3/4} + \frac{3/4}{1/4} = \frac{5}{3} + 3$$
$$= \mathbf{\frac{14}{3}}$$

Question 2

Determine whether each of the following are convergent or divergent (Hint: Use either the divergence test or the integral test for each)

(a)

$$\sum_{k=3}^{\infty} \frac{11k}{\ln(13k)}$$

Solution 2(a)

Use the divergence test. Since a polynomial like $11k$ grows much faster than a logarithm like $\ln(13k)$ (or by L'Hopital's Rule) we see

$$\lim_{k\longrightarrow\infty}(\frac{11k}{\ln(13k)}) = \lim_{k\longrightarrow\infty}(\frac{11}{1/k}) = \lim_{k\longrightarrow\infty} 11k = \infty$$

So the limit diverges, specifically it does not converge to zero. Thus by the divergence test, the sum ***diverges***. Recall that even if the limit were a finite number other than zero the conclusion would be the same.

(b)

$$\sum_{m=1}^{\infty} \frac{4m^3}{m^4+9}$$

Solution 2(b)

The divergence test is inconclusive here (limit is 0) so we will use the integral test.

$$\int_1^{\infty} \frac{4x^3}{x^4+9}dx$$

If we let $u = x^4 + 9$ and $du = 4x^3dx$ then we get the following (notice we change bounds)

$$\int_{10}^{\infty} \frac{1}{u}du$$

$$= \lim_{b\longrightarrow\infty} \int_{10}^{b} \frac{1}{u}du$$

$$= \lim_{b\longrightarrow\infty} \ln|u|\Big|_{10}^{b}$$

$$= \lim_{b\longrightarrow\infty} \ln|b| - \ln|10| = \infty$$

So the integral diverges. Thus by the integral test, the sum ***diverges***.

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$$

Solution 2(c)

The divergence test is inconclusive here (limit is 0) so we will use the integral test.

$$\int_2^\infty \frac{1}{x(\ln(x))^3} dx$$

If we let $u = ln(x)$ and $du = \frac{1}{x} dx$ then we get the following (notice we change bounds)

$$\int_{\ln(2)}^\infty \frac{1}{(u)^3} du$$

$$= \lim_{b \to \infty} \int_{\ln(2)}^b \frac{1}{u^3} du$$

$$= \lim_{b \to \infty} -\frac{1/2}{u^2}\Big|_{\ln(2)}^b$$

$$= \lim_{b \to \infty} -\frac{1}{2b^2} + \frac{1}{2(\ln(2))^2}$$

$$= 0 + \frac{1}{2(\ln(2))^2} = \frac{1}{2(\ln(2))^2}$$

So the integral converges to a finite number. Thus by the integral test, the series ***converges***.