Teaching about Quantum Theory

By C. Gregory Hood

Near the end of the school year or the second semester of a standard physics course, we usually present our students with some rudiments of quantum theory. Invariably we include aspects of the wave-particle duality of radiation and matter. After months of logical and rather precise reasoning, we suddenly enter into discussions bordering on the mystical: of entities that sometimes act like waves and at other times like particles, and of entities whose character is determined by what kind of experiment we choose to make on them. The textbooks we use are not especially helpful in clarifying the topic and current reviews confound the matter even more.

The purpose of this article is to indicate another way of looking at “duality” (Part I) and to propose an alternative approach to the teaching of quantum theory at the introductory level (Part II).

Part I: Looking for Waves

The title of this section reminds me of the Greek cynic-philosopher Diogenes (412?–323 B.C.), who was said to have walked about during daylight with a lighted lamp looking for a man. I gather he did this as a way to parody Plato’s attempts to define “man.” Likewise, I now imagine myself walking about looking for “waves.” My test for a wave is a simple one: Can I find an example of wave motion that is not reducible to the collective effect of a large number of particles?

Clearly a wave motion along a taut string does not meet this test. From a microlevel viewpoint, the molecules of the string are bound together such that as one molecule (or group of molecules) is displaced, a restoring force acts to return the molecule (or group) to its former position and to displace nearby molecules. For a similar reason, sound waves in air do not provide evidence for “waves” either. The wavy motion associated with sound is due to the collective motion of air molecules. And the same must also hold for “waves” in water. The use of the familiar wave motion description is, from a microlevel viewpoint, an approximation of the actual discrete atomic and molecular interactions.

There is no denying the usefulness and accuracy of wave motion descriptions at the macrolevel, but this usefulness should not lead us to suppose that there exists a continuous entity moving through these media called a “wave.” The wave model describes a type of motion, not a thing and not a fundamental property of matter. Wave motion is no more characteristic of matter than is circular motion or simple harmonic motion or whatever named motion one chooses. The motion is determined by the underlying dynamics, not by the nature of entities moving. Sound is a wavy motion in air; there can be wavy motion along a taut string, but there are no waves. I find only particles.

The continuity we usually associate with everyday wavy motions such as these is analogous to the continuity we associate with normal matter at the macrolevel, and analogous to the continuous charge distributions and currents we assign to macrolevel electronics. All of these are approximations of the underlying particulate nature of matter.
“Matter and radiation are particulate. Period. The observed wavy motion of radiation and matter is an approximation—a macrolevel approximation of the random impacts of a large number of particles, which are governed by an underlying quantum dynamics.”

Of course, the classical test for “waves” in physics is the double-slit experiment with visible light. When light of one color is allowed to pass through the two-slit arrangement, a series of parallel, equally spaced colored bands interspersed with dark regions can be seen on a distant screen. As first demonstrated by Thomas Young around 1800, this was one of many “interference” phenomena that helped to discredit Newton’s particle view of light and establish Huygen’s wave theory of light in the nineteenth century. In fact, when we seek to convince our students of the “wave-like character” of light, we appeal to the two-slit interference pattern.

As is true of the approximate continuity of matter and the approximate continuity of electric charge, the assumed continuity of the interference pattern is only an approximation. Many textbooks contain one or more pictures of the building up of the apparently continuous interference pattern by the random impact of photons—light particles. Only in the limit in which the number of photons is very, very large does the pattern of random photon impacts and the continuous wave approximation begin to blend. In other words, if one is able to look closely enough, the two-slit interference pattern is the collective effect of a large number of particles (photons). I do not find “waves” here either.

It should be remarked, perhaps, that what I have said above applies equally well to Maxwell’s concept of electromagnetic waves. When a source emits electromagnetic radiation with spherical symmetry, for example, what does one detect when far from the source and if one looks closely? Particles! Photons! Only in the limit of large numbers of photons is the observed distribution approximated by the classical wave model. Maxwell’s model explains most macrolevel aspects of electromagnetic radiation exceedingly well; but its approximate character is revealed by its inability to account for the observed microlevel features of radiation.

Obviously a similar result follows for the assumed wave-like character of matter. Interference effects due to electrons, for example, are the result of the random impact of individual electrons. And only in the limit where the number of electrons is very large does the pattern become well approximated by the wave model. What is observed, when one looks closely, is a pattern due to the random impact of a large number of particles. There is no evidence of “waves” here. In effect, our normal world is so overpopulated with photons and electrons that our concept of radiation and matter becomes biased toward the wave model. If we lived in a world where we routinely received few photons, for example, and received them as individual impacts, the wave approximation would be less useful and less meaningful to us—and less persistent in our minds.

Richard Feynman summed it up better than I can:

“I want to emphasize that light comes in this form—particles. It is very important to know that light behaves like particles, especially for those of you who have gone to school, where you were probably told something about light behaving like waves. I’m telling you the way it does behave—like particles.”

Just as Diogenes could not find a living example of Plato’s “man,” I cannot find a wavy motion that is not due to the collective motion of a large number of particles. Thus, there may be wavy motion, but there are no waves. No continuous entities moving from place to place. Consequently, there can be no duality of matter and radiation. Matter and radiation are particulate. Period. The observed wavy motion of radiation and matter is an approximation—a macrolevel approximation of the random impacts of a large number of particles, which are governed by an underlying quantum dynamics.

**Part II: \( E = h\nu, \ p = h/\lambda, \) and All That**

The primary conceptual difficulty with the Young’s double-slit experiment is this: particles moving along definite trajectories cannot produce the large-photon-limit pattern we observe. If we accept that there are no waves, how are we to explain the approximately continuous pattern we do observe?

In his book *QED*, Feynman set about to show how the rules of quantum electrodynamics, when applied to photons, explained the familiar facts about radiation. The rules he uses are not in themselves explicable—they are part and parcel of quantum theory itself. Just as we cannot state why \( F = ma \), or why there is a force of gravity, or why electric charge is conserved, and so on, so must we accept quantum dynamics as our best explanation of microphenomena. Because of the symmetry between matter and radiation, similar quantum
rules apply to electrons and other particles of matter. However, we do not teach quantum electrodynamics in introductory physics, at least not at my college. So what should we teach, if not the usual wave-particle duality?

There are several important ingredients for a consistent, nonmystical presentation of quantum phenomena. The first is to present the quantum picture of matter and radiation before any discussions of optics or wave motion, including Maxwell’s electromagnetic waves. Imagine for a moment that you had learned quantum theory before these classical models. Clearly, the classical models would represent a large scale approximation to the quantum mechanical view. That is the picture we must paint for our students.

A second ingredient is to tell students (and demonstrate where possible) that matter and radiation are particulate in nature. The basic properties of the principal particles should then be described. It is important to include spin or intrinsic angular momentum as one of these properties.

The third step in my proposed presentation is to make it clear that all motions—whether circular or simple harmonic or wavy—are due to an underlying dynamical scheme. Students should be familiar with the dynamics underlying circular motion and simple harmonic motion at this point in a course. At the microlevel, the dynamical scheme is provided by quantum theory. At the introductory level, one is more or less forced to state some major results of quantum theory due to mathematical limitations. (In calculus-based courses, I see no reason why position and momentum operators could not be introduced and used for simple cases such as the particle-in-a-box.) Among these results I would cite (1) that bound particles can appear only in states in which variables such as energy, momentum, and angular momentum have discrete values (are quantized); (2) the hydrogen atom energy level scheme, using $E = -(13.6 \, \text{eV})/n^2$ or the original formula derived by Niels Bohr, should be discussed; however, I would shy away from any approximate semiclassical derivations, as often done with the Bohr model, so as to not be encumbered by pictures of classical orbits, etc.; and (3) I would discuss the emission and absorption of photons in atomic hydrogen.

In connection with item (3) above, I would stress that the classification of photons is based upon their energy—not the change from current practice here. Our eyes respond to photons in the two to three electron-volt range. X-ray photons have energies in thousands of electron volts, and radio photons have energies generally smaller than one millionth of an electron volt. (This is also a good place to discuss the potential biological damage of radiation.) In addition, it is important to stress that when an electron makes a transition from a higher energy state to a lower one, it not only loses energy but also angular momentum; this angular momentum appears as the spin or intrinsic angular momentum of the emitted photon. The spin can be either $(\pm h/2\pi)$ or $(-h/2\pi)$, corresponding to spin in the direction of motion or opposite to the direction of motion. (The connection between these spin states and the usual polarization of light might require too much time and abstract reasoning in most courses, so I will not consider that here.) Finally, photons also carry linear momentum and, according to the special theory of relativity, $p = E/c$, where $c$ is the speed of light.

Now is the point at which to introduce the double-slit experiment, both for photons and matter, like electrons. The point is to show that the impact of each particle on a detection screen is random; no existing theory can predict where an individual particle will land. The best that can be done is to predict the probability for the particle to land in some region on the screen. In quantum mechanics this probability is given by the square of a state function. Only in the limit when a very large number of particles pass through the double-slit arrangement is the pattern approximated by the techniques of wave motion. The wave-motion approach can then be discussed in the usual way, but students must firmly grasp that this is merely a macrolevel approximation and the wave-motion approach is incorrect in a number of respects. Nevertheless, the wave-motion approximation is a very useful way of treating optical phenomena at the normal level of experience where the number of photons is so large that the patterns derived can be treated as if they were continuous.

How are the energy and momentum of a photon, for example, related to the wave-motion approximation? This was discovered by Max Planck in 1900 and is given by $E = h\nu$, where $\nu$ is the wavelength appearing in the wave motion approximation. (I prefer to write the energy relation this way since wavelength has more concrete and visual meaning for students than does frequency.) Note carefully that the energy equation is a statement of association, not a relationship defining the nature of photons. Photons are particles; there are no “waves.” The equation is a way to relate the photon energy to the large-scale wave model approximation—no more and no less. Using $p = E/c$, it naturally follows that $p = h\nu$. For particles with mass, $p = E/c$ is not true, but $p = h\nu$ is valid.

And there you have it—quantum theory without wave-particle duality and without the mysticism that has usually surrounded our presentations.

References
1. For example, see the latest edition (4th) of Fundamentals of Physics by Halliday, Resnick, and Walker (Wiley, 1993), p. 1171 ff. I list this text, not because its presentation is that different from other standard books, but because it traditionally represents a model of what is thought to be an excellent introductory physics textbook. For a recent review of difficulties in understanding quantum theory, see J. Horgan, “Quantum Philosophy,” Sci. Am. July 1992, pp. 94–104.


3. For example, see Figure 44-18 on page 1173 of Fundamentals of Physics (Ref. 1) for a particularly fine example using electrons instead of visible light.

4. It is easy to forget, when one has just marveled at the economy of the wave model in explaining some optical phenomena, that the model is incorrect in several important respects: (1) there is no evidence of continuity except at the macrolevel; (2) particles of the same kind—whether photons or electrons—cannot annihilate one another as the notion of “interference” implies; (3)
photon energy is not related to the square of a wave amplitude; and (4) there is no physical meaning one can attach to wavelength or frequency for discrete objects; these must relate to the wavy motion approximation, not to the particles of matter or radiation. (Where is our adherence to the idea that no experiment can prove a theory correct, but one experiment can prove it wrong?)


6. I am appalled by some colleagues and some national curriculum writing groups who advocate not teaching angular momentum concepts. It is at the heart of all quantum phenomena, including the group behavior of photons and electrons. I trust this trend is only a temporary aberration.

7. Never, never refer to quantum theory or quantum mechanics as "wave mechanics," except for historical references!

8. Discussions of these matters, including the detection of photon spin, can be found in several books. Two are: Frauenfelder and Henley, *Sub-Atomic Physics* (Prentice-Hall, 1974), pp. 80–82; Richard Feynman, *Lectures on Physics* (Addison-Wesley, 1965), Vol. I, Chap. 33, and Vol. III, Chap. 11. Feynman gives an extensive discussion of the superposition of spin states as well. I might add that a class discussion of measuring photon angular momentum or electron spin would be a good way to introduce Planck's constant into quantum theory, especially when one does not pursue the operator approach.

9. Please avoid the phrase "wave" function.

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