The University of Lahore
CS & IT Department

CS4349 Digital Logic Design
Mid Term Exam (Fall 2016)

Date: 19/11/2016

Time Allowed: 90 minutes Max. Marks: 30

Question No. 1: [1.5 + 1.5 = 3 Marks]

(a) Convert the following binary numbers to decimal: 1001101, 1010011.101 and 10101110.1001

Sol.:

$$(1001101)_2 = 2^6 + 2^3 + 2^2 + 2^0 = 77$$
$$(1010011.101)_2 = 2^6 + 2^4 + 2^1 + 2^0 + 2^{-1} + 2^{-3} = 83.625$$
$$(10101110.1001)_2 = 2^7 + 2^5 + 2^3 + 2^2 + 2^1 + 2^{-1} + 2^{-4} = 174.5625$$

(b) Determine the radix **r** of following number.

$(BEE)_r = (2699)_{10}$

Sol.:

$$(BEE)_r = (2699)_{10}$$
$$11 \times r^2 + 14 \times r^1 + 14 \times r^0 = 2699$$
$$11 \times r^2 + 14 \times r - 2685 = 0$$

By the quadratic equation: $r = 15$ or ≈ -16.27

ANSWER: $r = 15$

Question No. 2: Represent the decimal numbers 694 and 835 in BCD. And then show the steps necessary to form their sum. **[2 Marks]**

Sol.:

$(694)_{10} = (0110\ 1001\ 0100)_{BCD}$

$(835)_{10} = (1000\ 0011\ 0101)_{BCD}$

	1 ←		
	0110	1001	0100
	+1000	+0011	+0101
	1111	1100	1001
	+0110	+0110	+0000
0001	0101	1 0010	1001

Question No. 3: Show the configuration that represents the decimal number 255 in

(a) Binary
(b) BCD
(c) ASCII
(d) ASCII with odd parity **[1+1+1+1 = 4 Marks]**

Sol.:

a) $(11111111)_2$
b) $(0010\ 0101\ 0101)_{BCD}$
c) $011\ 0010 \quad 011\ 0101 \quad 011\ 0101_{ASCII}$
d) $0011\ 0010 \quad 1011\ 0101 \quad 1011\ 0101_{\text{ASCII with Odd Parity}}$

Question No. 4: Prove the identity by means of Boolean Algebraic Manipulation. **[2+2 = 4 Marks]**

(a) Y + X' Z + XY' = X + Y + Z

Sol.:

$$Y + \bar{X}Z + X\bar{Y} = X + Y + Z$$
$$= Y + X\bar{Y} + \bar{X}Z$$
$$= (Y + X)(Y + \bar{Y}) + \bar{X}Z$$
$$= Y + X + \bar{X}Z$$
$$= Y + (X + \bar{X})(X + Z)$$
$$= X + Y + Z$$

(b) X' Y' + Y' Z + XZ + XY + YZ' = X' Y' + XZ + YZ'

Sol.:

$$\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} \quad = \quad \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + \overline{Y}Z(X + \overline{X}) + XZ + XY + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + X\overline{Y}Z + \overline{X}\,\overline{Y}Z + XZ + XY + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y}(1 + Z) + X\overline{Y}Z + XZ + XY + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + XZ(1 + \overline{Y}) + XY + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + XZ + XY(Z + \overline{Z}) + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + XZ + XYZ + Y\overline{Z}(1 + X)$$

$$= \overline{X}\,\overline{Y} + XZ(1 + Y) + Y\overline{Z}$$

$$= \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$$

Question No. 5: Reduce the following Boolean expressions to the indicate number of literals: **[1+1+1+1 = 4 Marks]**

(a) X’ Y’ + XYZ + X’ Y to three literals.

Sol.:

$$\overline{X}\,\overline{Y} + XYZ + \overline{X}Y = \overline{X} + XYZ = (\overline{X} + XY)(\overline{X} + Z) = (\overline{X} + X)(\overline{X} + Y)(\overline{X} + Z)$$

$$= (\overline{X} + Y)(\overline{X} + Z) = \overline{X} + YZ$$

(b) X + Y (Z + (X + Z)’) to two literals.

Sol.:

$$X + Y(Z + \overline{X + Z}) = X + Y(Z + \overline{X}\,\overline{Z}) = X + Y(Z + \overline{X})(Z + \overline{Z}) = X + YZ + \overline{X}Y$$

$$= (X + \overline{X})(X + Y) + YZ = X + Y + YZ = X + Y$$

(c) W’ X (Z’ + Y’ Z) + X (W + W’ YZ) to one literal.

Sol.:

$$\overline{W}X(\overline{Z} + \overline{Y}Z) + X(W + \overline{W}YZ) = \overline{W}X\overline{Z} + \overline{W}X\overline{Y}Z + WX + \overline{W}XYZ$$

$$= \overline{W}X\overline{Z} + \overline{W}XZ + WX = \overline{W}X + WX = X$$

(d) (AB + A’ B’)(C’ D’ + CD) + (AC)’ to four literals.

Sol.:

$$(AB + \overline{A}\,\overline{B})(\overline{C}\,\overline{D} + CD) + \overline{AC} = AB\overline{C}\,\overline{D} + ABCD + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A} + \overline{C}$$

$$= ABCD + \overline{A} + \overline{C} = \overline{A} + \overline{C} + A(BCD) = \overline{A} + \overline{C} + C(BD) = \overline{A} + \overline{C} + BD$$

Question No. 6: For the Boolean functions E and F, as given in the following truth table: **[1+1+1+1+1 = 5 Marks]**

X	Y	Z	E	F
0	0	0	**0**	**1**
0	0	1	**1**	**0**
0	1	0	**1**	**1**
0	1	1	**0**	**0**
1	0	0	**1**	**1**
1	0	1	**0**	**0**
1	1	0	**1**	**0**
1	1	1	**0**	**1**

a) List the minterms and maxterms of each function.

Sol.:

a) $E = \Sigma m(1, 2, 4, 6) = \Pi M(0, 3, 5, 7)$, $F = \Sigma m(0, 2, 4, 7) = \Pi M(1, 3, 5, 6)$

b) List the minterms of E' and F'.

Sol.:

b) $\bar{E} = \Sigma m(0, 3, 5, 7)$, $\bar{F} = \Sigma m(1, 3, 5, 6)$

c) List the minterms of E + F and E.F

Sol.:

c) $E + F = \Sigma m(0, 1, 2, 4, 6, 7)$, $E \cdot F = \Sigma m(2, 4)$

d) Express E and F in sum of minterms algebraic form.

Sol.:

d) $E = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$, $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$

e) Simplify E and F to expressions with a minimum of literals.

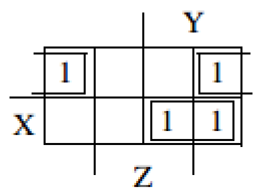
Sol.:

e) $E = \bar{Z}(X + Y) + \bar{X}\bar{Y}Z$, $F = \bar{Z}(\bar{X} + \bar{Y}) + XYZ$

Question No. 7: Optimize the following Boolean expressions using a map. **[2+2+2 = 6 Marks]**

(a) X’ Z’ + Y Z’ + X Y Z

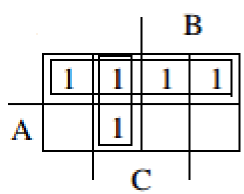
Sol.:



$\bar{X}\bar{Z} + XY$

(b) A’ B + B’ C + A’ B’ C’

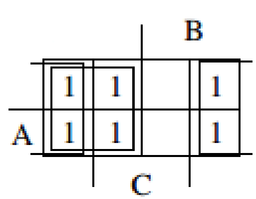
Sol.:



$\bar{A} + C\bar{B}$

(c) A’ B’ + A C’ + B’ C + A’ B C’

Sol.:

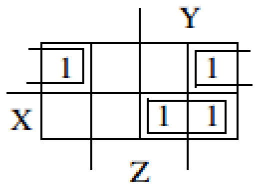


$\bar{B} + \bar{C}$

Question No. 8: Optimize the following Boolean function using three variable map.

F (X, Y, Z) = ∑(0, 2, 6, 7) **[2 Marks]**

Sol.:



$\bar{X}\bar{Z} + XY$