INSTRUCTIONS

- You have 2 hours to complete the exam.
- The exam is open book and open notes. You may not use a computer, calculator, or anything responsive.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a brief explanation.
- Since some students will be taking the exam later, PLEASE DO NOT DISCUSS IT before discussion sections.

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For staff use only

<table>
<thead>
<tr>
<th>Q. 1</th>
<th>Q. 2</th>
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<tr>
<td>/12</td>
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1. (12 points) What would Python Do?

(a) (9 points) For each of the following expressions, write the value to which it evaluates. The first two rows have been provided as examples. If evaluation causes an error, write Error. If evaluation never completes, write Forever.

Assume that you have started Python 3 and executed the following statements:

```python
class Doge:
    much = "wow"
    def bark(self):
        return self.much

class Puppye(Doge):
    very = "meow"
    def __init__(self, friend):
        self.friend = friend
        self.much = [Doge.much, friend.much]
    
    def bark(self):
        return "puppye" + Doge.bark(self)[1]
```

```python
such = Doge()
such.much = "interesting"
fido = Puppye(such)
flora = Puppye(fido)
```

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluates to</th>
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<tr>
<td>abs(-3)</td>
<td>3</td>
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<tr>
<td>1/0</td>
<td>Error</td>
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<tr>
<td>Doge.much</td>
<td></td>
</tr>
<tr>
<td>Puppye.much</td>
<td></td>
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<tr>
<td>such.very</td>
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<td>such.bark()[0:2]</td>
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<tr>
<td>fido.bark()</td>
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<tr>
<td>fido.friend.much</td>
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<td>fido.friend.bark()</td>
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<td>flora.friend.bark()</td>
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<tr>
<td>flora.friend.friend.much</td>
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(b) (3 points) Consider the following Python function:

```python
def foo(a, b):
    return map(lambda x: a[x//2] if x % 2 == 0 else b[x//2], range(len(a+b)))
```

What is the value of `tuple(foo( (10, 5, 25), (8, 7, 11) ))`?

Ans: ________________________________
2. (15 points) Assorted Diagrams

(a) (8 points) Fill in the environment diagram that results from executing the following segment until the entire program is finished, an error occurs, or all frames are filled. You may not need to use all of the spaces or frames. A complete answer will:

- Add all missing names, labels, and parent annotations to all local frames.
- Add all missing values created during execution.
- Show the return value for each local frame resulting from a call that has completed when execution stops (leave other return values blank).

```python
a = 0
def april(fools):
    a = 2
def joke(a):
    nonlocal fools
    fools = lambda y: a + 4
    return a
if fools(777) == 1:
    joke(a + 9)
return fools(april)
april(lambda x: a + 1)
```

Global Frame

```
Global Frame

a

april

func april(fools)
```

Return Value

```
Return Value

Return Value

Return Value

Return Value
```

```
Return Value

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Return Value

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Return Value

Return Value
```
(b) (5 points) Draw the box-and-pointer diagram that results from executing the following code:

```python
q = [1, 2]
s = [1, 2, [3]]
t = [4, [s, 5], 6]
u = [t]
```

We've already drawn the diagram for `q` as an example.

(c) (2 points) Now assume the following line is executed after all of the statements above are executed.

```python
u.append(u)
```

Show the result on your diagram. Mark any added list cells with an asterisk (*) to show what you change.
3. (8 points) Data Abstraction
Albert loves to create salads, and decides to create functional abstractions representing vegetables and salads.

(a) (2 points) Unfortunately, Albert only had time to finish writing the constructors for his abstract data types. Help him out by writing in the corresponding selector functions.

```python
def make_vegetable(name, color):
    """Returns a representation of a vegetable with given NAME and COLOR, which must both be strings."""
    return [[name], [color]]

def get_name(vegetable):
    return ________________

def get_color(vegetable):
    return ________________

def make_salad(name, vegetables):
    """Return a salad having the given NAME, and constituent VEGETABLES (a list of vegetables)."""
    return [[name], vegetables]

def get_salad_name(salad):
    return ________________

def get_vegetables(salad):
    return ________________
```
Condensation of the methods from part (a) for reference:

```python
def make_vegetable(name, color): return [[name], [color]]
def get_name(vegetable): ...
def get_color(vegetable): ...
def make_salad(name, vegetables): return [[name], vegetables]
def get_salad_name(salad): ...
def get_vegetables(salad): ...
```

(b) **(5 points)** Steven decides to use Albert’s data abstractions to create a delicious lunch. However, he doesn’t feel confident that he understands how to use Albert’s abstract data types. Help him out by both:

- Crossing out any mistakes or data abstraction violations he has made.
- Writing in code that correctly conveys what he was trying to do, either on the line of code itself or below the line of code where the issue occurs.

```python
ingredients = [ make_vegetable(["lettuce"], ["green"]), ["carrot"], ["orange"]] 
first_vegetable_color = ingredients[0][1][0]
lunch = make_salad( "simple_salad" , ingredients + [make_vegetable("cucumber", "green")])
salad_name = lunch[0][0]
for xxxxx in lunch[1]: #Line 5. See part (c)
    vegetable_name = get_salad_name(xxxxx) # Line 6
    vegetable_color = xxxxx[1][0]
    print("From " + salad_name + ", I have a " + vegetable_color + " " + vegetable_name)
last_ingredient = get_vegetables(lunch)[len(get_vegetables(lunch)) - 1]
print( last_ingredient.name + " was a great finishing touch!")
```

(c) **(1 point)** What meaningful variable name could be used in place of `xxxxx` on lines 5 and 6?
4. (11 points) Programs

(a) (5 points) Given a nested list of rlists, return the sum of all integers weighted by their depth. In the rlist below, 1 and 2 are at depth 1, 3 is at depth 2, and 4 is at depth 3:

![Diagram of rlist]

Here is the relevant part of the rlist implementation from lecture:

```python
empty_rlist = ...
def rlist(first, rest = empty_rlist): ...
def first(r): ...
def rest(r): ...

Hint: Python's built-in method isinstance(avalue, aclass) will tell you if avalue belongs to aclass (or a subtype of aclass).
```

def nested_sum(rlst):
    '''Assuming RLST to be an rlist containing only rlist or integer values, the sum of all integers in RLST, weighted by their depth.
    >>> simple = rlist(1, rlist(4, rlist(6)))
    >>> nested_sum(simple) # 1*1 + 1*4 + 1*6
    11
    >>> harder = rlist(1, rlist(rlist(3, rlist(rlist(4))), rlist(2)))
    >>> nested_sum(harder) # 1*1 + 2*3 + 3*4 + 1*2
    21
    return sum_at_depth(rlst, 1)

def sum_at_depth(rlst, d):
    '''The sum of all integers in RLST, weighted by their depths, assuming that RLST itself is at depth D.''
    if
        return
    elif
        return
    else:
        # (Your answer need not use all lines.)
        return
(b) (6 points) In lecture, we defined a class `BinTree` for describing binary trees, supporting the following operations (`T` denotes a `BinTree` value):

**BinTree**(`LABEL, LEFT, RIGHT`) A binary tree whose label is `LABEL`, whose left child is `LEFT`, and whose right child is `RIGHT`.

**BinTree.empty_tree** The empty tree.

**T.is_empty** True iff `T` is the empty tree.

**T.label** The label of `T`.

**T.left** The left child of `T`.

**T.right** The right child of `T`.

Write a function `bound` that takes in a binary search tree containing real numbers, and two numbers `low` and `high`. It returns a new binary search tree that contains all and only the elements in the input tree that are between `low` and `high`, inclusive. So if `tree` is the tree on the left, `bound(tree, 4, 10)` will produce the tree on the right.

```python
def bound(tree, low, high):
    """A binary search tree containing exactly the elements in binary search tree TREE that are between LOW and HIGH, inclusive. Assume LOW<=HIGH."""

    if _________________________________:
        return _________________________________

    elif ________________________________:
        return ________________________________

    elif ________________________________:
        return ________________________________

    else:
        return BinTree(tree.label,
                        ________________________________)  
```
5. (1 point) **Sum of Human Knowledge**

It is a little-known fact that Santa Claus actually manages to avoid the bad arctic winter flying conditions by instead using a series of perfectly straight tunnels dug (presumably by magical means) directly through the earth from the north pole. One of them, in fact, emerges near Coffs Harbour in New South Wales at about latitude 30°S. On his bathroom scale at the north pole, Santa weighs in at 100kg (or, for those who have recently taken physics, 100kg-wt). What would the scale read if he used it while exactly halfway along this particular tunnel (±2%)?

6. (4 points) **The Big Theta**

Consider the following function (assume that parameter \( S \) is a list):

```python
def umatches(S):
    result = set()
    for item in S:
        if item in result:
            result.remove(item)
        else:
            result.add(item)
    return result
```

For (b)–(d) below, give the tightest bounds you can (i.e., preferably \( \Theta \)) for each):

(a) (1 point) Fill in the blank: The function `umatches` returns the set of all

(b) (1 point) Let’s assume that the operations of adding to, removing from, or checking containment in a set each take roughly constant time. Give an asymptotic bound on the worst-case time for `umatches` as a function of \( N = \text{len}(S) \).

(c) (1 point) Suppose that instead of having `result` be a set, we make it a list, (so that it is initialized to `[ ]` and we use `.append` to add an item). What now is the worst-case time bound? You can assume that `.append` is a constant-time operation, and `.remove` and the `in` operator require time that is \( \Theta(L) \) in the worst case, where \( L \) is the length of the list operated on. Since we never add an item that is already in the list, each value appears at most once, just as for a Python set.

(d) (1 point) Now suppose that we consider only cases where the number of different values in list \( S \) is at most 100, and we again use a list for `result`. What is the worst-case time now?