CS 242 Final Examination Spring 2014

May 9, 11 AM to 12:50 PM

1. Write a propositional logic expression F involving variables p, q and r that is true only if at least two of the three variables are true.

2. Express each of these system specifications using predicates, quantifiers, and logical connectives. First create a set of relevant propositional or predicate symbols (such as: Let $P(x)$ be "disk x has more than 10 kilobytes of free space").

   (a) At least one person in the world with a blue shirt can swim if they are taught.

   (b) Each student that did not have a scholar ship or a rich acquaintance had to pay for college.

3. Identify the error or errors in this argument that incorrectly shows if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x (P(x) \wedge Q(x))$ is true.

   (a) $\exists x P(x) \wedge \exists x Q(x)$ Premise

   (b) $\exists x P(x)$ Simplification from (1)

   (c) $P(c)$ Existential instantiation from (2)

   (d) $\exists x Q(x)$ Simplification from (1)

   (e) $Q(c)$ Existential instantiation from (4)

   (f) $P(c) \vee Q(c)$ Conjunction from (3) and (5)

   (g) $\exists x (P(x) \vee Q(x))$ Existential generalization

4. Show $(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$.

5. Determine if the following functions are one to one or onto. Justify your answer.

   - The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$
   - The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$
   - The function $f : \mathbb{Z} \to \mathbb{N}$ defined by $f(x) = x^2$

6. Find the solution of the recurrence relation $a_n = 1.02 \cdot a_{n-1}$, $a_0 = 10000$, i.e., give a formula for a_n in terms of n. Can you think of an application in which this type of recurrence relations is used.

7. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 7 & 10 \\ 0 & 1 & 2 \end{bmatrix}$

   then calculate $A * (-2B)$. Can you add A and B? Can you add A^T and B?

8. Recall the greedy algorithm to convert a given amount using the standard coin denominations. Give a currency set that will not always convert change optimally. Then show an example amount that does not work optimally with this currency set.

9. How many ways are there to arrange the letters A, B, C, D and E such that A never comes immediately after E ?

10. What is the minimum number of students that must be in a class so at least four have last names that begin with the same letter?

11. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with five members if it must have 2 more women than men? You don't need to simplify your answer.

12. How many ways are there to choose 5 surf boards if there are 5 identical 6'3" short boards, 5 identical 6'8" short boards, 5 identical 9'3" long boards?

13. Use mathematical induction to show that $\frac{1}{1.3} + \frac{1}{3.5} + \ . \ . \ . + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ whenever n is a positive integer.

14. Define a rooted binary tree as a tree with a node designated as root and in which each non-leaf node has *exactly* two children. (a) Show that the number of nodes in a binary tree is always odd. (Use induction.) (b) Give an expression for the number of leaves in a binary tree with $n = 2k - 1$ nodes? Prove that your expression is correct by induction.

15. What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

16. Show using both ways (algebraic and combinatorial) if n and k are integers greater than 1, then: $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$.

17. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?

18. In bridge, the 52 cards of a standard deck are dealt to four players. Howmany differentways are there to deal bridge hands to four players?

19. Exercises 13 and 14, page 704, Exercise 34, page 705 of the text.

20. Exercises 1-5, 11, page 686 of the text.

21. Exercises 35-44, page 677 of the text.

22. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree. (Use pigeon-hole principle.)

23. Can a simple graph exist with 15 vertices each of degree five? Justify your answer.

24. Consider an undirected graph G whose vertices are labeled by a binary string of length 4 and in which there is an edge from vertex u to v if the label of u can be obtained from the label of v by switching a pair of adjacent bits. (For example, there will be an edge from the vertex labeled 010 to the vertex labeled 001.)

25. What is the number of vertices in G? What is the number of edges in G?

26. How many connected components does G have?

27. A chain letter starts when a person sends a letter to five others. Each person who receives the letter either sends it to five other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter, and how many do not send it out?