CS 61A- Problem Set 1

March 11, 2014

Instructions: This is meant to provide extra programming practice if you so desire. This is not required and not graded in any way (in particular, the cheating policies don't matter so feel free to share code for this between yourselves). In addition, these are rather difficult to solve, so you should collaborate and solve them.

Do not post anything about a problem if you haven't been thinking about it for at least 30 minutes. If you solve it in that time, then that problem was too easy and you shouldn't give it away. If you don't, then you should probably ask for hints and help.

Work in groups! These problems aren't easy. Get into study groups and work on them- you'll definitely learn a lot from each other.

Also, all of these have numerical answers, but you should use Python scripts to do most of the computation (these are *not* meant to be done by hand).

1. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of M or N below some number LIMIT.

   (Source: http://www.michellerubyhwang.com/cs61a/)

2. Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

   $$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...$$

   By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

   (Source: https://projecteuler.net/problem=2)

3. A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 9199.

   Find the largest palindrome made from the product of two 3-digit numbers.

   (Source: https://projecteuler.net/problem=4)

4. A number chain is created by continuously adding the square of the digits in a number to form a new number until it has been seen before.

   For example,

   $$44 \to 32 \to 13 \to 10 \to 1 \to 1$$
   $$85 \to 89 \to 145 \to 42 \to 20 \to 4 \to 16 \to 37 \to 58 \to 89$$

   Therefore any chain that arrives at 1 or 89 will become stuck in an endless loop. What is most amazing is that EVERY starting number will eventually arrive at 1 or 89.

   How many starting numbers below ten million will arrive at 89?

   (Source: https://projecteuler.net/problem=92)

5. The most naive way of computing n^{15} requires fourteen multiplications:

   $$n \times n \times ... \times n = n^{15}$$

   But using a "binary" method you can compute it in six multiplications:

   $$n \times n = n^2$$
   $$n^2 \times n^2 = n^4$$
   $$n^4 \times n^4 = n^8$$
   $$n^8 \times n^4 = n^{12}$$
   $$n^{12} \times n^2 = n^{14}$$
   $$n^{14} \times n = n^{15}$$

   However it is yet possible to compute it in only five multiplications:

   $$n \times n = n^2$$
   $$n^2 \times n = n^3$$
   $$n^3 \times n^3 = n^6$$
   $$n^6 \times n^6 = n^{12}$$
   $$n^{12} \times n^3 = n^{15}$$

   We shall define $m(k)$ to be the minimum number of multiplications to compute n^k; for example $m(15) = 5$.

   For $1 \leq k \leq 200$, find the sum of $m(k)$.

   (Source https://projecteuler.net/problem=122)