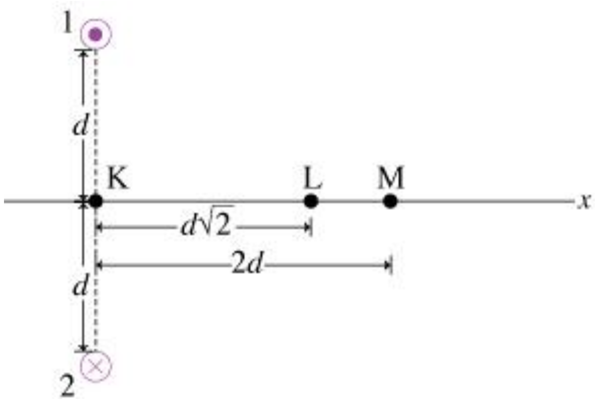
Below we provide solutions to some of MP questions which students seemed to have the most difficulty with. Note that we don't always work through the whole problem giving the final answer to each of the parts (you can find out what the answer was for each part within MP). Part of this is because many questions use randomized numbers. But mostly we want to focus on how you should be thinking about these problems.

2,6,7,8,10



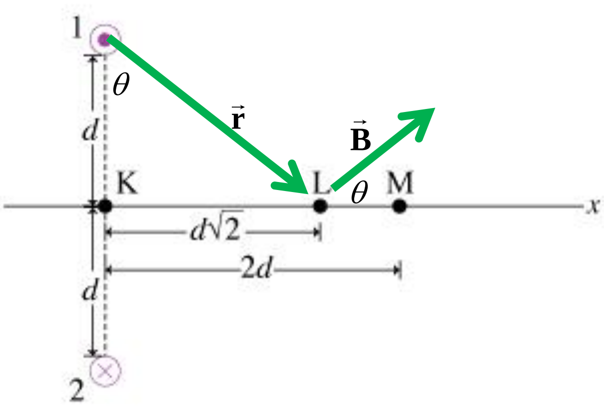
Q2 Magnetic Field from Two Wires

In this problem, you will be asked to calculate the magnetic field due to a set of two wires with antiparallel currents as shown in the diagram. Each of the wires carries a current of magnitude *I*. The current in wire 1 is directed out of the page and that in wire 2 is directed into the page. The distance between the wires is 2*d*. The x axis is perpendicular to the line connecting the wires and is equidistant from the wires.

At point K, directly between the wires, the fields from each are only along the x-axis. Using the right hand rule (RHR) wire 1 makes a field to the right and so does wire 2. So the total field is just the sum of those two, and since they are the same, twice the field of one:

$$\vec{\mathbf{B}}(\mathrm{K}) = 2 \cdot \frac{\mu_0 I}{2\pi d}\hat{\mathbf{i}} = \frac{\mu_0 I}{\pi d}\hat{\mathbf{i}}$$

Now when we go out to point L the cross product is no longer directly to the right. Instead it is at an angle. Let's look at the picture again:

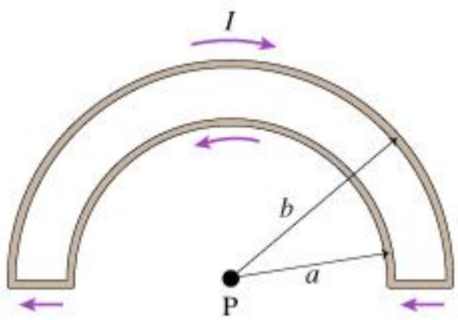


I've only drawn the B field from wire 1. The field from wire 2 is the mirror image of that across the x-axis. A little geometry + trigonometry helps you determine the angle:

$$B_{x,1} = B_1 \cos\theta = B_1 \frac{d}{r} = \frac{\mu_0 I}{2\pi r}\frac{d}{r} = \frac{\mu_0 I d}{2\pi\left(\sqrt{d^2 + \left(d\sqrt{2}\right)^2}\right)^2}$$

$$\vec{\mathbf{B}}(\mathrm{L}) = 2B_{x,1}\hat{\mathbf{i}} = 2\frac{\mu_0 I d}{2\pi 3d^2}\hat{\mathbf{i}} = \frac{\mu_0 I}{3\pi d}\hat{\mathbf{i}}$$

Again, the total field is just twice that from wire 1, but in this case only the x-component matters (the y-components cancel). A similar calculation gives us $\vec{\mathbf{B}}(M) = \mu_0 I / 5\pi d\,\hat{\mathbf{i}}$



Q6 Magnetic Field due to Semicircular Wires

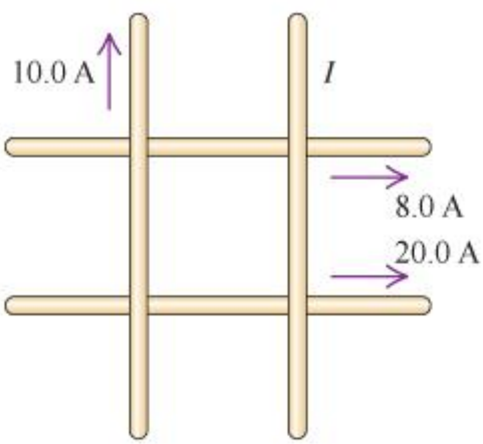
A loop of wire is in the shape of two concentric semicircles as shown. The inner circle has radius *a*; the outer circle has radius *b*. A current flows clockwise through the outer wire and counterclockwise through the inner wire. What is the magnitude,*B*, of the magnetic field at the center of the semicircles?

In class we calculated the magnetic field from a circle of wire to be $B = \frac{\mu_0 I}{2R}$

So we have two half circles. The outer one is making a B field INTO the page at point P and the inner one makes a field OUT of the page (use the RHR to see this). So they subtract. The two straight lengths of wire don't contribute to the field because they are running into/out of point P, meaning that I and r are parallel, so the cross-product in the Biot-Savart calculation goes to zero.

So we can just write down the answer (realizing we have two HALF circles):

$$\vec{\mathbf{B}} = \frac{1}{2}\left(\frac{\mu_0 I}{2a} - \frac{\mu_0 I}{2b}\right)\text{out} = \boxed{\frac{\mu_0 I}{4}\left(\frac{1}{a} - \frac{1}{b}\right)\text{out} = \vec{\mathbf{B}}}$$



Q7 Canceling a Magnetic Field

Four very long, current-carrying wires in the same plane intersect to form a square with side lengths of 39.0 cm $(\equiv s)$, as shown in the figure. Find the current that will make the magnetic field at the center of the square equal to zero.

These are all infinite wires the same distance from the point at which we want to cancel the field. Each either makes a field into or out of the page. We want the total in to equal the total out in order to cancel. So, using RHR, 10 A up makes a field IN, as does 8 A to the right. But 20 A to the right makes a field OUT (it is below the point while the 8A wire is above it, so the field directions are opposite). So we have 20 A making a field OUT, 18 A making a field IN, thus we need 2A more making a field in. That current would have to flow down the page. I = 2 A down!

Q8 Superconducting Dipole

A 98.0 A current circulates around a 1.80-mm-diameter superconducting ring. What is the ring's magnetic dipole moment and what is the on-axis magnetic field strength 5.90 cm from the ring?

The dipole moment is easily calculated for any flat circuit (in this case a ring) as:

$$\mu = I \cdot A = I\pi\left(\tfrac{d}{2}\right)^2 = (98.0\text{ A})\pi\left(\tfrac{1}{2}1.80\text{mm}\right)^2 = 2.49\times10^{-4}\,\text{A}\cdot\text{m}^2$$

Note that the unit is $\text{A}\cdot\text{m}^2$ not Am^2 which mastering physics would interpret as $(\text{Am})^2$ and have no idea how to deal with since Am is not a unit.

Now what is the field from a dipole on its axis? You can derive this specifically for a ring (the derivation is similar to what we did in class for the field at the center of a ring, but more difficult because you need to think about the components of the vectors). Using equation 32.9 we have: $\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{2\vec{\boldsymbol{\mu}}}{z^3}$

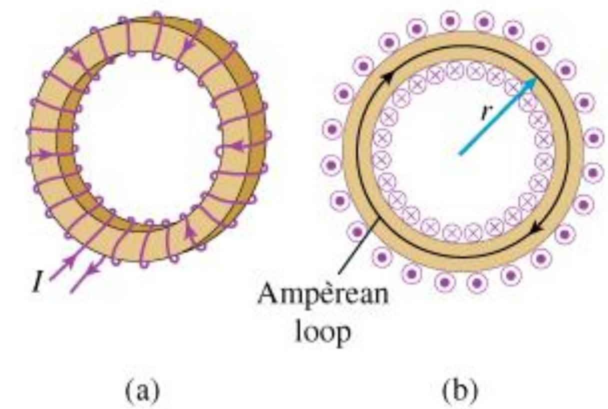
Note that it falls off like z^3, the same as electric fields from dipoles. Now that is for "point dipoles" (when you are a far distance away). The exact formula for a ring is a little up the page, and the z gets replaced with the distance from a point on the ring rather than from its center. Here we are pretty far away (about 60 times the radius) so those two should be about the same.

Using the numbers we were given we have:

$$B \approx \frac{\mu_0}{4\pi}\frac{2\mu}{z^3} = \left(10^{-7}\,\tfrac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{2\left(2.49\times10^{-4}\,\text{A}\cdot\text{m}^2\right)}{(5.90\text{ cm})^3} = \boxed{2.43\times10^{-7}\text{ T} = B}$$

Q10 Magnetic Field Inside a Toroid

Calculate the magnetic field inside a toroid.



To do this we use Ampere's law, taking advantage of the symmetry that the B field is constant along a loop inside the solenoid. Thus we have:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \cdot 2\pi r = \mu_0 I_{\text{enc}} = \mu_0 NI \Rightarrow \boxed{\vec{\mathbf{B}}_{\text{toroid}}(r) = \frac{\mu_0 NI}{2\pi r}\left(-\hat{\boldsymbol{\theta}}\right)}$$

where the negative unit vector there means clockwise (as pictured)

Now we usually pretend that we are in an ideal situation where the current only goes into and out of the page (in cross section) but in reality, of course, the current is going around the toroid (clockwise) and this effect current loop (radius R) is going to make a field into the page at the loop's center with the field strength of $\boxed{B_{\text{center}} = \frac{\mu_0 I}{2R}}$. Note that this is negligibly small compared to the desired field, as we don't get the times N multiplier and R > r so that also decreases the field strength. But the idealization that the field is zero outside the core of the toroid is just that – an idealization – and is *not* in practice true.